Constraints on the tachyon condensate from anomalous symmetries

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Abstract

Using anomalous symmetries of the cubic string field theory vertex we derive set of relations between the coefficients of the tachyon condensate. They are in agreement with the results obtained from level truncation approximation.

1 Introduction

Since the original formulation of the Sen’s conjecture [1] there has been significant progress in understanding the nonperturbative aspects of the string field theory. Initially the existence of translationally invariant vacuum with conjectured energy density was established numerically [2, 3] to a rather high accuracy by the level expansion method [4] in the Witten’s cubic string field theory [5, 6, 7]. More recently the Sen’s conjecture has been proved rigorously in the framework of background independent string field theory [8, 9, 10]. Nevertheless it seems worth continuing to look for the exact tachyon condensate in the original cubic string field theory since it can teach us many things [15].

Various insights into the nature of the tachyon condensate has already been obtained in [11, 12, 13, 14, 15]. One particular suggestion for the exact form of the condensate

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based on the noncommutative geometric nature of the string field action was made in [16, 17]. Independently of this recent suggestion we will show in this letter, that we can obtain some new exact information about the tachyon condensate. We point out that some anomalous symmetries [6, 19, 12] of the string field theory vertex can be used to derive an infinite set of identities for any string field which solves the equations of motion. We will see that at level \( n \) we get \( n \) additional constraints on the coefficients of the string field which are in reasonable agreement with the explicit results from level truncation scheme [2, 3]. It would be very interesting if one could find even further symmetries which would then fix all the coefficients completely.

## 2 Anomalous symmetries

The string field theory action as given by [5, 6, 7] takes the form of noncommutative Chern-Simons action

\[
S[\Psi] = -\frac{1}{\alpha' g_0^2} \left( \frac{1}{2} \langle \Psi, Q \Psi \rangle + \frac{1}{3} \langle \Psi, \Psi \ast \Psi \rangle \right)
\]

with the noncommutative multiplication defined by

\[
\Psi_1 \ast \Psi_2 = \text{bpz} (\langle V | \Psi_1 \otimes \Psi_2 \rangle),
\]

where \( \text{bpz} \) denotes the \( \text{bpz} \) conjugation in conformal field theory and the vertex \( \langle V \rangle \) was reviewed in the oscillator formulation [18, 16] and studied in a background independent manner in [12].

From [6, 19, 12] we know that the vertex \( \langle V \rangle \) satisfies certain identities. For us will be important in particular the following ones for \( n \) even

\[
\langle V | \sum_{i=1}^{3} (L^{(i)}_{-n} - L^{(i)}_{n}) = 3k^x_n \langle V |,
\]

\[
\langle V | \sum_{i=1}^{3} (J^{(i)}_{-n} + J^{(i)}_{n}) = 3(h^g_{n} + 3\delta_{n,0}) \langle V |,
\]

where \( L_n \) and \( J_n \) denote matter Virasoro and ghost current generators respectively. The constants \( k^x_n \) and \( h^g_{n} \) take for \( n \) even the following values

\[
k^x_n = \frac{13 \cdot 5}{2^7} \cdot \frac{n}{2^2} (-1)^{\frac{n}{2}},
\]

\[
h^g_{n} = (-1)^{\frac{n}{2}}.
\]

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For $n$ odd there would be extra signs between the generators in (3) and the right hand side would vanish. We are not interested in this case since it will not lead to any information about the tachyon condensate. Note that the additional term on the right hand side of the second equation in (3) accounts for the nontensor character of the ghost number current.

Let us study now the variation of the action (1) under the infinitesimal variations of the string field

$$
\delta \Psi = (L_{-n} - L_n - k^x_n) \Psi,
\delta \Psi = (J_{-n} + J_n - h^{gh}_m - 3\delta_{n,0}) \Psi
$$

respectively. Under these variations the cubic term in the action is obviously invariant due to the invariance of the vertex (3). On the other hand we know that the total action should also be invariant as long as $\Psi$ satisfies equations of motion. Combining these two facts we get from the kinetic term

$$
\langle \Psi | [Q, L_n] | \Psi \rangle = -k^x_n \langle \Psi | Q | \Psi \rangle,
\langle \Psi | [Q, J_n] | \Psi \rangle = h^{gh}_n \langle \Psi | Q | \Psi \rangle.
$$

Let us note that both commutators on the left hand side are modes of conformal primary fields, the latter being minus the BRST current $J^B$.

3 Explicit checks

To compare the above formulas with the results obtained in level expansion scheme in [2, 3] one should first of all impose the Siegel gauge condition $b_0 | \Psi \rangle = 0$ on the string field and simplify the commutators. For the first equation of (6) one has simply

$$
[Q, L_n] = -nc_0 L_n + \cdots
$$

where the dots stand for terms which do not contribute. For the second equation one can use a little trick. Write the left hand side as

$$
\langle \Psi | [Q, J_n] | \Psi \rangle = -\langle \Psi | \{J^B_n, b_0 \} c_0 | \Psi \rangle
$$
where we used the facts that \([Q, J_n] = -J_n^B\) and \(b_0|\Psi\rangle = 0\). The anticommutator can be easily evaluated using the operator product expansion (see e.g. [20]). Both formulas (6) thus simplify in the Siegel gauge to

\[
\langle \Psi | c_0 L_n | \Psi \rangle = \frac{1}{n} k_n x_n \langle \Psi | c_0 L_0^{tot} | \Psi \rangle,
\]
\[
\langle \Psi | c_0 (nJ_n + L_n^{tot}) | \Psi \rangle = -h_n^{gh} \langle \Psi | c_0 L_0^{tot} | \Psi \rangle,
\]
(9)

where \(L_n^{tot}\) denotes the total Virasoro generator. These identities can be easily checked for the numerical values obtained in [2, 3]. Let us define \(r_n^{L,J}\) to be the ratio of the left and right hand sides of the first or second equation of (9) respectively. Then inserting for simplicity the values for the string field coefficients from [2] obtained at the level (4,8) we get the following results

\[
r_2^L = 1.069, \quad r_4^L = 1.044, \\
r_2^J = 1.004, \quad r_4^J = 0.939.
\]

We see that the above identities are preserved within 7\%. This can be compared with the value of the potential which is for the same values about 1.4\% away from the expected value. This discrepancy in the errors by a factor of five does not necessarily mean that there are mistakes neither in the derivation nor in the numerical evaluation. In fact we know that the convergence properties of the level truncation approximation depends rather strongly on what kind of calculation we are doing. In an unpublished work we have studied the properties of the string field algebra unity \(|I\rangle\) in the level truncation using the universal recursive methods of [12]. Keeping only terms up to level 8 in the unity \(|I\rangle\) and during the whole calculation we got for example

\[
L_{-2} |0\rangle \ast |I\rangle = 0.990 L_{-2} |0\rangle + 0.108 L_{-2}^{tot} |0\rangle - 0.196 L_{-2}^{tot} L_{-2}^{tot} |0\rangle + \cdots, \\
L_{-2}^{tot} |0\rangle \ast |I\rangle = 0.990 L_{-2}^{tot} |0\rangle + 0.009 L_{-2}^{tot} L_{-2}^{tot} |0\rangle + \cdots,
\]
(10)

where the dots stand for terms which are relatively smaller or of higher levels where one can understand bigger errors. Looking at these values one might wonder whether after all the string algebra unity is unity also for the state \(L_{-2} |0\rangle\). The experience from calculations at lower levels where the errors are much bigger suggests that it really converges, hopefully to the correct state. The fact that calculations involving matter Virasoro generators converge much more slowly can be easily traced back to the presence of the Virasoro anomaly.
References


