Hadrons in dense and hot matter: implications of chiral symmetry restoration

Guy Chanfray
Institut de Physique Nucléaire de Lyon, IN2P3-CNRS et Université Claude-Bernard Lyon I, 43 Bd du 11 Novembre 1918, 69622 Villeurbanne Cedex, France

Recent developments on medium modifications of hadron properties in dense and hot hadronic matter are discussed. I will focus in particular on the behavior of spectral functions associated to collective scalar-isoscalar modes, kaons and vector mesons from ordinary nuclear matter to highly excited matter produced in relativistic heavy-ion collisions from SIS to SPS energies. Various theoretical approaches are presented in connection with the interpretation of experimental data. Special emphasis will be put on the role of chiral dynamics and chiral symmetry restoration. I will discuss in particular to which extent the broadening of the rho meson peak signals the onset of chiral symmetry restoration.

1. Introduction

The problem of the in-medium modifications of hadron properties have motivated numerous works in the recent years both at the experimental and theoretical levels. One major goal of this rapidly developing field is the very fundamental problem of progressive chiral symmetry restoration with increasing temperature and/or baryonic density. In this talk I will try to draw some conclusions on what has been established recently starting from very much discussed examples (rho and sigma mesons, kaons). It will be also emphasized that relevant information can be obtained from variety of sources ranging from intermediate energy physics in the GeV range probing ordinary nuclear matter up to ultrarelativistic heavy-ion collisions probing hadronic matter under extreme conditions.

2. Chiral symmetry: breaking and restoration

Asymptotic freedom and color confinement are usually considered as the most prominent properties of our theory of strong interaction, Quantum Chromodynamics (QCD). However QCD also possesses an almost exact symmetry, the $SU(2)_L \otimes SU(2)_R$ chiral symmetry which is certainly the most important key for the understanding of many phenomena in low energy hadron physics. This symmetry originates from the fact that the QCD Lagrangian is almost invariant under the separate flavor $SU(2)$ transformations of right-handed $q_R = (u_R, d_R)$ and left-handed $q_L = (u_L, d_L)$ light quark fields $u$ and $d$:

$q_R \rightarrow e^{i\vec{\tau}.\vec{\sigma}_R/2} q_R$, \quad $q_L \rightarrow e^{i\vec{\tau}.\vec{\sigma}_L/2} q_L$.
The small explicit violation of chiral symmetry is given by the mass term of the QCD Lagrangian which is $\mathcal{L}_{\chi_{SB}} = -m_q(\bar{u}u + \bar{d}d)$, neglecting isospin violation. The averaged light quark mass $m_q = (m_u + m_d)/2 \leq 10$ MeV, the scale of explicit chiral symmetry breaking, has to be compared with typical hadron masses of order 1 GeV, indicating that the symmetry is excellent and in the exact chiral limit ($m_q = 0$) left-handed and right-handed quarks decouple. From the associated left-handed and right-handed conserved currents, one usually introduces two linear combinations, the vector and axial currents:

$$V^\mu_k = \bar{q} \gamma^\mu \frac{\tau_k}{2} q, \quad A^\mu_k = \bar{q} \gamma^\mu \gamma_5 \frac{\tau_k}{2} q$$

The corresponding charges $Q^{V}_k$ and $Q^{A}_k$ commute with the QCD Hamiltonian. However at variance with the vector charges (which actually coincide with the isospin operators) the axial charges of the QCD vacuum are not zero: $Q^{A}_k |0\rangle \neq 0$. Hence the QCD vacuum does not possess the symmetry of the vacuum i.e. chiral symmetry is spontaneously broken (SCSB). This key property of the QCD vacuum is evidenced by a set of remarkable properties listed below.

- The appearance of (nearly) massless goldstone bosons: the pions with extremely small mass compared to other hadrons.

- The building-up of a chiral quark condensate: $\langle \bar{q}q \rangle = \langle \bar{u}u + \bar{d}d \rangle / 2$ which explicitly mixes, in the broken vacuum, left-handed and right-handed quarks ($\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle / 2$).

Another order parameter at the hadronic scale is the pion decay constant $f_\pi = 94$ MeV which is related to the quark condensate by the Gell-Mann-Oakes-Renner relation: $-2m_q < \bar{q}q >_{\text{vac}} = m_\pi^2 f_\pi^2$ valid to leading order in the current quark mass. It leads to a large negative value $< \bar{q}q >_{\text{vac}} \approx -(240 \text{ MeV})^3$ indicating strong dynamical breaking of chiral symmetry.

- The absence of parity doublets. The normal Wigner realization of chiral symmetry would imply a doubling of the hadron spectrum. Each hadron would have a “chiral partner” with opposite parity and (nearly) the same mass. This is obviously not the case since the possible chiral partners (such as $\pi(140) - \sigma(400 - 1200)$, $\rho(770) - a_1(1260)$, $N(940) - N^*(1535)$) show a large mass splitting $\Delta M = 500$ MeV.

When hadronic matter is heated and compressed, initially confined quarks and gluons start to percolate between the hadrons to be finally liberated. This picture is supported by lattice simulations showing that strongly interacting matter exhibits a sudden change in energy- and entropy-density (possibly constituting a true phase transition) within a narrow temperature window around $T_c = 170$ MeV. This transition is accompanied by a sharp decrease of the quark condensate indicating chiral symmetry restoration. However far before the critical region, partial restoration should follow through the simple presence of hadrons. Indeed, inside the hadrons the scalar density, originating either from the valence quarks or from the pion scalar density (the virtual pion cloud), is positive hence decreasing the quark condensate. Said differently, the presence of hadrons locally restores
chiral symmetry. This statement can be made quantitative since, to leading order in hadron densities, the condensate evolves according to [1,2]:

$$R = \frac{\langle \langle \bar{q}q \rangle \rangle_{\rho_h, T}}{\langle \langle \bar{q}q \rangle \rangle_{\text{vac}}} = 1 - \sum_h \frac{\rho_h \Sigma_h}{\frac{f_h^2}{2} m^2_\pi}.$$ (2)

Each hadron species present with scalar density $\rho_h$ contributes to the dropping of the condensate through a characteristic quantity $\Sigma_h$ directly related to the integrated quark scalar density inside the hadron $h$: $\Sigma_h/m = f_h \int h d r \langle h | \bar{u}u + \bar{d}d | h \rangle.$

In nuclear matter the relevant quantity is the nucleon sigma commutator $\Sigma_N \approx 45$ MeV. Putting the numbers together one finds a 30% restoration at normal nuclear matter density. The sigma commutator and the dropping of the chiral condensate can be estimated with effective theories in terms of the relevant degrees of freedom. It receives contribution from valence quarks, scalar field and virtual pion cloud. There is strong indication (from model calculations and analysis of photon data) showing that the major part of the nucleon sigma term comes from the pion cloud piece [3–5]: $\Sigma_N^{(\pi)} = \int_N d r \langle N | m^2_\pi \tilde{\Phi}^2_\pi/2 | N \rangle \approx 30$ MeV. The leading order formula (2), valid only for a non-interacting medium can be promoted to an exact one by replacing in nuclear matter the pion-nucleus sigma term by the full pion-nucleus sigma term per nucleon. Various higher order contributions have been examined. The role of short-range correlations has been found to be weak [6] but pion exchange contribution (i.e. the modification of the pion scalar-density $\langle \Phi^2_\pi \rangle$ itself related to the full longitudinal spin-isospin response and p-wave collective pionic modes) yields an increase of the pion-nucleon sigma term of about 5 MeV at normal density [7]. The acceleration of chiral symmetry restoration has not been found in the work of ref.[8] based on an effective chiral lagrangian. In the framework of relativistic theories short range repulsion (omega exchange) also yields a deceleration of symmetry restoration [9].

Despite the fact that the condensate is not an experimental observable, it is hardly conceivable that such a strong modification of the QCD vacuum should not have spectacular consequences on hadronic properties, namely on the hadronic spectral functions. A number of works have been devoted to the possible link between the evolution of the masses and the condensates using various models (sigma models, NJL model,...) most of the time at the mean field level. This activity has culminated with the universal scaling laws proposed by Brown and rho, where hadron masses drop together with quark and gluon condensates [10]. However, the link between the evolution of the masses and the condensates cannot be an absolute one. For instance the pionic piece of the quark condensate does not contribute to the evolution of the mass [11]. It manifests in a more subtle way through the mixing of the vector and axial-vector correlators. More generally the modification of hadronic spectral functions is certainly not restricted to the shift of centroids of mass distributions. In that respect we will discuss how chiral dynamics may generate a softening/sharpening or a broadening of hadronic spectral functions with some specific and highly debated examples (sigma, kaon, and rho mesons). The key question is the relationship between the observed reshaping and chiral symmetry restoration. One possible strategy to obtain this crucial connection is to make a simultaneous study of the spectral functions associated with chiral partners. A very important example is the rho meson and the axial-vector meson $a_1$ and we will see that there is a mixing of the associated current correlators through the presence of the pion scalar density as already
3. Scalar-isoscalar modes in nuclear matter

There are at least two excellent reasons to study the in-medium modifications of the pion-pion interaction in the scalar-isoscalar channel both being related to fundamental questions in present-day nuclear physics. The first one relies on the binding energy of nuclear matter since a modification of the correlated two-pion exchange may have some deep consequences on the saturation mechanism [12]. The second one is the direct connection with chiral symmetry restoration. Such a restoration implies that there must be a softening of a collective scalar-isoscalar mode, usually called the sigma meson, which becomes degenerate with its chiral partner \( i.e. \) the pion at full restoration density, even if this meson is not well identified in the vacuum. This also implies that at some density the sigma meson spectral function should exhibit a spectacular enhancement near the two-pion threshold. This effect can be seen as a precursor effect of chiral symmetry restoration associated with large fluctuations of the quark condensate near phase transition [13].

The medium effect which has been first proposed is a consequence of the modification of the two-pion propagator and unitarized \( \pi\pi \) interaction from the softening of the pion dispersion relation by p-wave coupling to \( p-h \) and \( \Delta-h \) states [14]. The existence of the collective pionic modes produces a strong accumulation of strength near the two-pion threshold in the scalar-isoscalar channel around \( \rho_0 \). On the experimental side the CHAOS collaboration has measured at TRIUMF the invariant mass distribution of the produced pion pair in \( A(\pi^+,\pi^+\pi^-) \) and \( A(\pi^+,\pi^+\pi^+) \) for incoming pions of energy 283 MeV (fig.1) [15]. In the \( \pi^+\pi^- \) channel a strongly A-dependent accumulation of strength is growing up from hydrogen to lead. This effect is not present in the \( \pi^+\pi^+ \) channel which is purely isospin \( I = 2 \) contrary to the \( \pi^+\pi^- \) channel predominantly isoscalar. Since the angular distribution shows that the \( \pi^+\pi^- \) pair is almost in a pure s-wave state, this process probes scalar-isoscalar modes inside the nucleus. According to the first realistic calculations [16], this reshaping of the strength coming from the p-wave pionic collective modes may provide a partial explanation of the CHAOS data. These last results have been questioned in another recent paper where it is found that pion absorption forces the reaction to occur at very peripheral density [17]. Hence the effect of chiral symmetry restoration has to be included on top of p-wave pionic effects to reach a better description of the data. This has been achieved in recent works using the linear sigma model implemented by adding a one-parameter form factor \( v(k) \) to fit the phase shifts in vacuum once the scattering amplitude is unitarized [18]. The (in-medium) unitarized scalar-isoscalar \( \pi\pi \) T matrix (in the CM frame and at total energy \( E \) of the pion pair) has been taken as [16,18] :

\[
\langle k, -k|T(E)|k', -k'\rangle = v(k)v(k') \frac{6\lambda(E^2 - m^2_\pi)}{1 - 3\lambda\Sigma(E)} \left( E^2 - m^2_\pi - \frac{6\lambda^2f^2_\pi\Sigma(E)}{1 - 3\lambda\Sigma(E)} \right)^{-1}
\]

(3)

where \( \lambda \) is the \( \sigma\pi\pi \) coupling and the last factor in (3) is nothing but the unitarized sigma meson propagator \( D_\sigma(E) \) \( i.e \) with two-pion loop). The p-wave collective effects are embedded in the two-pion loop :

\[
\Sigma(E) = \int \frac{d\mathbf{q}}{(2\pi)^3} v(q) \int \frac{id\mathbf{q}_0}{2\pi} D_\pi(\mathbf{q}, q_0) D_\pi(-\mathbf{q}, E - q_0).
\]

(4)
Figure 1. Invariant mass spectrum for the $\pi^+ \rightarrow \pi^+\pi^-$ and $\pi^+ \rightarrow \pi^+\pi^+\pi^-$ mass distributions on various nuclei measured by the CHAOS collaboration [15]. Diagrams (dots) are the results of phase-space simulations for the pion production in $\pi A \rightarrow \pi\pi[A-1]$ reactions.

The pion propagator $D_\pi(q, q_0)$ is calculated in a standard nuclear matter approach and incorporates the p-wave coupling of the pion to delta-hole states with short-range screening described by the usual $g'_{\Delta\Delta} = 0.5$ parameter. Chiral symmetry restoration can be accounted for by dropping the sigma mass according to: $m_\sigma(\rho) = m_\sigma(1 - \alpha \rho/\rho_0)$. Such a density dependence very naturally arises in this model from the tadpole graph where the sigma meson directly couples to the nuclear density. In [13] a value of $\alpha$ in the range of 0.2 to 0.3 was found. The result for the invariant-mass distribution $Im D_\pi(E_{\pi\pi})$ is shown in fig.2 at saturation density. One observes a dramatic downward shift of the mass distribution as compared to the vacuum. The low-energy enhancement, already present without sigma-mass modification ($\alpha = 0$) and induced by the density dependence of the pion loop, is strongly reinforced as the in-medium $\sigma$-meson mass is included. For $\alpha = 0.2$ and $\alpha = 0.3$ the peak height is increased by a factor 2 and 4 respectively. Similarly for the T-matrix, a sizable effect can be noticed in its imaginary part which might be sufficient to explain the findings of the CHAOS collaboration. To facilitate the comparison with theoretical calculations the experimental group has presented his data in the form of a
composite ratio [15]:

\[ C_{A\pi\pi}^{A} = (M_{A\pi\pi}^{A}/\sigma_{T}^{A}) / (M_{A\pi\pi}^{N}/\sigma_{T}^{N}) \]  \hspace{1cm} (5)

where \( \sigma_{T}^{A} (\sigma_{T}^{N}) \) is the measured total cross section of the \((\pi, 2\pi)\) process in nuclei (nucleon). As is apparent on fig.3, while a conventional calculation is able to reproduce the \(\pi^{+}\pi^{+}\) case, chiral symmetry restoration on top of p-wave effects is needed to reach a better agreement with data in the \(\pi^{+}\pi^{-}\) case. It is therefore very tempting to conclude that a precursor effect of chiral symmetry restoration has been seen in this experiment. However one has to keep in mind that this comparison with theoretical predictions is at best semi-quantitative and a full calculation for the absolute yields incorporating all the complicated reaction dynamics together with the relevant medium effects remains to be done before firm conclusions can be drawn. Nevertheless experimental efforts in that direction has certainly to be encouraged since the still poorly known in-medium scalar field is of utmost importance in present-day nuclear physics.

4. Kaons in dense matter

The strong interest for studying kaon production in relativistic heavy-ion collisions originates from two complementary reasons. The first one is the sensitivity of kaon prop-
Figure 3. The composite ratio $C^{A}_{\pi\pi}$ for $^{12}$C, $^{40}$Ca and $^{208}$Pb compared with theoretical calculations including p-wave pionic effects only (full curve), chiral symmetry restoration only (dash-dotted) and both effects (dashed). Taken from ref.[15].

Properties and propagation to the state of the produced matter and the second one is the fact that the basic kaon-nucleon interaction is governed to a large extent by the $SU(3)$ extension of chiral symmetry. Hence kaon production provides a unique opportunity to study chiral dynamics in a dense and hot medium. Recent data taken in particular at SIS by the KaoS and FOPI collaborations have demonstrated sizable medium effects; the details and experimental data can be found in the contribution of P.Senger in this present volume [19]. The absence of in-plan flow and the quite strong azimuthal emission of kaons ($K^+$) together with the unexpected high sub-threshold $K^-/K^+$ ratio favorize a strong in-medium attraction for the anti-kaons ($K^-$) and a more moderate but significant repulsion for the kaon ($K^+$). This effect has been predicted by many theoretical calculations based on chiral symmetric frameworks. From a chiral symmetry lagrangian taken at the mean-field level, one can easily establish the leading order terms modifying kaon and anti-kaon masses at finite proton and neutron densities:

\[
\Delta m_{K^+} = \frac{1}{2} \frac{1}{4 f^2_\pi} \left( 3 (\rho_p + \rho_n) + (\rho_p - \rho_n) \right) - \frac{\Sigma_{KN}}{2 m_K f^2_\pi} \rho \tag{6}
\]

\[
\Delta m_{K^-} = -\frac{1}{2} \frac{1}{4 f^2_\pi} \left( 3 (\rho_p + \rho_n) + (\rho_p - \rho_n) \right) - \frac{\Sigma_{KN}}{2 m_K f^2_\pi} \rho \tag{7}
\]

The first contribution arises from vector current interaction between the pseudo-scalar mesons and the nucleons and is determined by chiral symmetry alone (Weinberg-Tomozawa theorem). It gives the bulk of repulsion for the kaon and attraction for the anti-kaons. The second term of scalar nature is proportional to the not very well-known kaon-nucleon sigma term $\Sigma_{KN}$ and yields attraction in both cases; it is actually to next order in the chiral perturbation expansion and could be balanced by other repulsive terms at this order.
but without altering too much the strong mass splitting of \( S = 1 \) and \( S = -1 \) modes in matter.

For the specific case of \( K^+ \), there is a consensus between the various transport code approaches in favor of repulsion to account for the flow variables data \([20–22]\). For what concerns the \( K^+ \) spectra and excitation functions in the sub-threshold or near-threshold region there is also a consensus for the dominant role played by secondary processes \([22–24]\) such as \( \pi N \rightarrow YK \) or \( N\Delta \rightarrow NYK \) \((Y = \Lambda, \Sigma)\) which are very sensitive to modifications of in-medium masses. Although, there are still uncertainties for the input cross-sections (this is especially important for the \( N\Delta \) channel), transport code calculations also favor in-medium repulsion for the \( K^+ \) \([25,24]\). The sensitivity to the EOS has also been investigated. It turns out that when in-medium kaon mass is incorporated the sensitivity to the EOS decreases. In the case of soft EOS, the system reaches a higher density yielding a higher \( K^+ \) production; this effect can be counterbalanced by the larger increase of the \( K^+ \) mass increasing the threshold production. Nevertheless, according to \([26]\) detailed study of Au-Au versus C-C excitation functions with in-medium repulsion favors a soft EOS with compressibility of the of the order of 200 MeV.

Transport calculations \([27,28,22]\) also show that sub-threshold \( K^- \) spectra are better reproduced if a dropping kaon mass of the type given by the mean field approach \((6,7)\) is incorporated. It has been realized that the situation is not so simple because the anti-kaon interaction in the medium is governed by a very rich and rather complex chiral dynamics as we will discuss below. To begin with, we have to understand a rather paradoxal situation. On one hand, although the basic \( K^-N \) Weinberg-Tomozawa is attractive, the scattering lengths imply a repulsive interaction in vacuum. On the other hand, kaonic atom data are compatible with a strongly attractive anti-kaon optical potential as large as 200 MeV once extrapolated at normal nuclear matter density. The solution of this paradox is the existence of a very peculiar object, the \( \Lambda(1405) \) resonance. Following ref. \([29–32]\) one can start with a coupled channel equation for the vacuum \( \bar{K}N \) scattering matrix, schematically written as:

\[
\langle \bar{K}N|T|\bar{K}N \rangle = \langle \bar{K}N|V_{WT}|\bar{K}N \rangle + \langle \bar{K}N|V_{WT}|MB \rangle G_{MB} \langle MB|T|\bar{K}N \rangle
\]  

(8)

where \( G_{MB} \), with \( M(\bar{K}, \pi) \) and \( B(N, \Lambda, \Sigma) \), is the meson-baryon propagator in the intermediate state. It turns out that the attractive Weinberg-Tomozawa interaction \( (V_{WT}) \) in the isospin \( I = 0 \) channel is sufficiently strong that it generates a pole at 27 MeV below the \( K^-p \) threshold. This pole correspond to a quasi-bound state, the \( \Lambda(1405) \) which decays into \( \pi\Sigma \) with a width of about 50 MeV. This is precisely the existence of this quasi-bound state what makes the \( K^-p \) interaction repulsive at threshold, while the Weinberg-Tomozawa amplitude is attractive. Going at finite density Pauli blocking starts to work in the intermediate \( \bar{K}N \) state and the quasi-bound state moves up above threshold. Hence the \( \Lambda(1405) \) dissolves already at very small density \([29]\) making the in-medium \( K^-N \) interaction attractive as seen in kaonic atom data. However, as pointed out by M. Lutz \([30]\), a self-consistent incorporation of the in-medium dressed \( K^- \) propagator in the intermediate state modifies the picture. The position of the resonance does not really move up but it considerably broadens. The in-medium \( K^- \) spectral function exhibits a two-level structure, the lower mode corresponding to the \( K^- \) pole branch and the upper to the \( \Lambda(1405) \)-hole branch with strength decreasing very fast with increasing density \((\text{fig.4})\).
Figure 4. $K^-$ spectral function as a function of energy for several densities. Left panel: influence of the $K^-$ momentum in the self-consistent calculation of ref. [30]. Right panel: zero momentum calculation of ref.[32] with the influence of in-medium dressing of the pion (c) on top of Pauli blocking (a) and in-medium dressed kaons (b).
Self-consistency makes the two-peak mode barely visible and if the modification of the pion dispersion relation is also included the $K^-$ completely melts with the $\Lambda(1405) - h$ branch even losing its status of quasi-particle [32]. Finally it has been emphasized that the observed enhancement of $K^-$ production has probably little relation with the dropping of the anti-kaon mass or in other words to its optical potential at zero momentum [31]. Indeed, in the conditions prevailing in heavy-ion collisions, the anti-kaons have a typical momentum of 300 MeV with respect to the matter rest frame. In that regime a self-consistent calculation shows little attraction if not repulsion and the medium effect might originate from the enhancement of cross-section of important secondary processes such as $\pi \Sigma \rightarrow K^- p$ [31]. Although, there is not yet a quantitative understanding of the $K^-$ production, the physics of the $K^-$ is a beautiful example of the very rich in-medium chiral dynamics. It is also an example of the strong reshaping and broadening of an hadronic spectral function which is also present but for different reasons in the case of the rho meson.

5. Dilepton production and the rho meson

Low mass dilepton production has been reported as being among the evidences for the formation of a new phase of matter in relativistic heavy-ion collisions at CERN/SPS [33]. In particular the CERES collaboration [34] has observed an important radiation in the invariant mass region 300 – 700 MeV/c beyond what is expected from the conventional sources able to explain the proton-nucleus data (fig.5). Since these conventional sources (the so-called hadronic cocktail) correspond to final state Dalitz decays ($\eta, \eta' \rightarrow \gamma e^+ e^-$, $\omega \rightarrow \pi^0 e^+ e^-$) and direct vector meson decays ($\rho, \omega, \Phi \rightarrow e^+ e^-$), one can conclude that this excess of radiation originates from the interacting fireball before freeze-out. Due to the very large number of produced pions the first candidate is the $\pi^+ \pi^- \rightarrow l \bar{l}$ annihilation process which is dynamically enhanced by the rho meson. Using vacuum meson properties many theoretical groups have included this process within (very) different models for the space-time evolution of $A - A$ reactions. Their results are in reasonable agreement with each other, but in disagreement with the data: the experimental spectra in the mass region 300 – 600 MeV/c are significantly underestimated and the rho peak itself has the tendency to be overestimated as seen from fig.6. Thus one came to the conclusion that strong medium effects yielding a flattening of the spectra are needed. This has motivated a considerable theoretical activity that I will now briefly describe. Dilepton production from hot and dense matter. The dilepton production rate (DPR) per unit 4-volume from a hot ($T = 1/\beta$) and dense medium is given by:

$$\frac{dN_{l\bar{l}}}{d^4xd^4q} = -\frac{\alpha^2}{6\pi^3M^2} \frac{1}{e^{\beta q_0} - 1} g_{\mu\nu} \left( -\frac{1}{\pi} Im\Pi_{V}^{\mu\nu} \right)$$

(9)

where $M (M^2 = q_0^2 - q^2)$ is the invariant mass of the produced pair. Once the overall thermal factor has been extracted, the DPR is directly proportional to the imaginary part of the current-current correlation function:

$$\Pi_{V}^{\mu\nu}(q) = -i \int d^4x e^{-iqx} \langle \langle J^{\mu}(x), J^{\nu}(0) \rangle \rangle (T, \rho_B).$$

(10)
Figure 5. CERES/Na45 data on dilepton production data in central 158 AGeV Pb+Au collisions compared to a hadronic cocktail inferred from a thermal model.

For simplicity, we will concentrate on the (prevailing) isospin $I = 1$ (isovector) projection of the electromagnetic current:

$$
\mathcal{V}_\rho^\mu = \frac{1}{2} \left( \bar{u} \gamma^\mu u - \bar{d} \gamma^\mu d \right) \label{eq:11}
$$

which just coincides with the third component of the conserved vector current of chiral symmetry (see eq.1). We know from the well established Vector Dominance phenomenology (VDM) that the corresponding correlator is accurately saturated by the rho meson. This property is formally incorporated through the famous field-current identity $\mathcal{V}_\rho^\mu = (m_\rho^2/g_\rho) \rho^\mu$. Hence dilepton production allows to reach the imaginary part of the rho meson propagator, namely the in-medium rho meson spectral function. To have some insight about manifestation of chiral symmetry restoration this vector correlator should be studied simultaneously with the axial-vector correlator in which the properties of the chiral partner of the rho meson, namely the $a_1$ meson, are encoded. In the vacuum the SCSB manifests itself in the marked difference between the $\rho$ and the $a_1$ and the transition from the hadronic to the partonic regime (“duality threshold”) is characterized by the onset of perturbative QCD around $M_{dual} \simeq 1.5$ GeV. In the medium, full chiral symmetry
restoration requires the degeneracy of vector and axial correlators over the entire mass range.

Density expansion. Several approaches have been put forward to determine the spectral properties of vector mesons in the medium. One method, very usual in nuclear physics, is the low density expansion:

$$\Pi^{\mu \nu}(q, T, \mu) = \Pi^{\mu \nu}_{\text{vac}}(q) + \sum_h \rho_h \Pi^{\mu \nu}_h(q).$$

Taking the imaginary part, one obtains the contribution of hadron species $h$ present with density $\rho_h$ to the spectral function. The vacuum piece is extremely well known from $e^+e^-$ annihilation. The hadronic part is expected to be dominated by the lightest meson ($\pi$) and baryon ($N$). In the chiral reduction formalism [35], the hadronic matrix elements ($\Pi^{\mu \nu}_h$) can be inferred from a combination of empirical information ($\pi N$, $\rho N$ or $\gamma N$ data...) and chiral Ward identities. Although model independent in spirit, this framework does not allow to perform systematic resummations. Indeed it has the tendency to overestimate the rho meson peak itself because these higher order many-body effects are
absent. Nevertheless this approach explicitly contains the already mentioned axial-vector mixing that we will now discuss.

**Axial-Vector mixing.** In the medium the emission and the absorption of thermal (finite temperature) or virtual (finite density) pions is able to transform a vector current into an axial current. In other words, the response of the system to a vectorial probe contains an axial contamination mediated by the pions, the pure vector piece being quenched by the emission and absorption at the same point. Hence increasing temperature or density (i.e. increasing the pion scalar density) makes the axial-vector mixing more and more important until full restoration where axial and vector correlators become identical. This mixing has been formally proven at finite temperature in the chiral limit, using only chiral symmetry. The finite temperature correlators are described to order $T^2$ by the following mixing of zero-temperature correlators [36]:

\[
\Pi_{V}(q; T) = (1 - \epsilon) \Pi_{V}(q; T = 0) + \epsilon \Pi_{A}(q; T = 0)
\]

\[
\Pi_{A}(q; T) = (1 - \epsilon) \Pi_{A}(q; T = 0) + \epsilon \Pi_{V}(q; T = 0)
\]

where $\epsilon = T^2/6f_{\pi}^2$ is directly proportional to the scalar density of the thermal pions. This implies that, to this order, the masses of the $\rho$ and $a_1$ meson do not change although the order parameters (quark condensate and pion decay constant) are modified in contradiction with the BR scaling law. It is amusing to note that full mixing $\epsilon = 1/2$ corresponding to full symmetry restoration is realized at $T \simeq 160$ MeV very close to the lattice critical temperature. The above result has been extended beyond chiral limit in the chiral reduction formalism [35] in which the DPR writes :

\[
\frac{dR}{d^4xd^4q} = -\frac{\alpha^2}{\pi^3q^2} e^{i\phi} + 1 \left[ Im \Pi(q^2) - \frac{2}{f_{\pi}^2} \int \frac{dk}{(2\pi)^3} \frac{n(\omega_k)}{\omega_k} Im \Pi_V(q^2) \right] \\
+ \frac{1}{f_{\pi}^2} \int \frac{dk}{(2\pi)^3} \frac{n(\omega_k)}{\omega_k} \left( Im \Pi_A((q + k)^2) + Im \Pi_A((q - k)^2) \right) + ....
\]

(15)

The first term corresponds to the full electromagnetic correlator (with $\rho, \omega$ and $\phi$ pieces) and the second term exhibits the quenching of the (isovector-)vector correlator. The last term represents the axial-vector mixing beyond the soft pion limit ($k \to 0$). The integration over all the pion momenta yields a broadening of the (rho meson) spectrum which has to be understood as an unavoidable consequence of partial chiral symmetry restoration.

**QCD sum rule.** The QCD sum rule approach aims at an understanding of physical current-current correlation functions in terms of QCD by relating the observed (or calculated) hadron spectrum to fundamental condensates $C_n$ (quarks, gluons) i.e. to the non-perturbative QCD vacuum structure. For large space-like momenta ($Q^2 = -q^2$), OPE techniques lead to :

\[
\frac{\Pi(q^2 = -Q^2)}{Q^2} = \int_0^\infty ds \left( \frac{-1}{s} \right) \frac{Im \Pi(s)}{s + Q^2}
\]

\[
= \frac{d\nu}{12 \pi^2} \left[ -C_0 ln \left( \frac{Q^2}{\mu^2} \right) + \frac{C_1}{Q^2} + \frac{C_2}{Q^4} + \frac{C_3}{Q^6} + .... \right]
\]

(16)
I will not discuss here the technical difficulties and limitations of the method which can be applied both in the vacuum and in the medium. I will only emphasize that it provides a crucial test of consistency between hadronic correlators and fundamental properties such as chiral symmetry and its restoration. Although such a test should be systematically done, the approach is only of little predictive power. It has been found [37,38] that the generic decrease of the quark and gluon condensates on the right-hand-side is compatible with the phenomenologically left-hand-side if either (a) the vector meson masses decrease (together with small resonance widths as in the $\omega$ meson case) or (b) both width and mass increase (as found in most phenomenological models for the $\rho$ meson).

Dropping mass scenario. Early QCD sum rule analysis based on a sharp ansatz for the vector mesons [39] gave a decrease of the vector meson (rho and omega) of about 20% at normal nuclear matter density. At this time this result has been understood as being in favor of the scaling law proposed by Brown and Rho [10] on the basis of broken scale invariance in QCD:

$$\frac{f_\pi}{m_\pi} = \frac{m_\sigma}{m_\omega} = \frac{m_\rho}{m_\omega} = \left(\frac{<\bar{q}q>^*}{<\bar{q}q>}\right)^{1/3}. \tag{17}$$

The rho mass itself plays the role of an order parameter. Although these scaling laws contradict some low density results, chiral symmetry does not forbid the vanishing of the mass at full restoration. But, as already discussed, such a dropping mass scenario is certainly not a necessary consequence of chiral symmetry restoration. Although this scenario yields a reasonable agreement with data [40,41], other hadronic many-body approaches containing other aspects of chiral symmetry restoration without dropping mass are also able to reproduce the data.

Many-body approaches. More conservative approaches reside on standard many-body techniques to calculate the self-energy and consequently the rho meson spectral function. They are based on effective hadronic Lagrangians possessing chiral symmetry and incorporating vector dominance. The various parameters (coupling constants and form factor cutoffs) are constrained as much as possible by other data and phenomenology (decay rates, photoabsorption, $\rho N$ scattering [42],...). One is thus able to evaluate the in-medium modification of the rho meson from its coupling to the various many-body excitations of the dense and hot matter from which one gets the DPR at a given temperature and baryonic chemical potential. As we will see below, once various final state hadronic decays are incorporated on top of the interacting fireball contribution, such an approach is able to reproduce the enhancement of the DPR below the $\rho/\omega$ peak and the apparent depletion of the peak itself. The latter depletion has a pure many-body origin: the propagator formalism generates resummations to all orders which are totally absent in any kind of low-density expansion and/or in incoherent summations of various processes. The total thermal yield in heavy-ion reactions is obtained by a space-time integration over the density-temperature profile for a given collision system modeled within transport or hydrodynamics simulations. Another very successfull attempt is provided by a simple expanding thermal fireball [43] allowing to incorporate in a rather simple way hadronic many-body effects which are needed to obtain a consistent description of the data. In the most recent calculation [44] the trajectory starts at $(T,\rho_B)_{in}=(190 \text{ MeV}, 2.55\rho_0)$, goes
Figure 7. Dilepton invariant mass spectrum measured in central 158 AGeV Pb+Au collisions compared to theoretical calculations [44]. The dashed-dotted lines correspond to the hadronic cocktail (without $\rho$ decays), the dashed lines to the hadronic cocktail plus free $\pi\pi$ annihilation and the solid lines incorporate medium effects in the rho meson propagator (many-body approach). In the upper panel the long-dashed line is obtained within the dropping mass scenario and in the lower panel the long-dashed line is an exploratory calculation using lowest order $q\bar{q} \rightarrow ee$ annihilation rates only within the same expanding thermal fireball.
through the experimentally deduced point in hadro-chemical analysis [45] up to thermal freeze out \((T, \rho_B)_{fo}=(115 \text{ MeV}, 0.33 \rho_0)\). Transport calculations, where no assumption is made about the degree of thermalization, also get better agreement with data once in-medium spectral function is incorporated [46]. It is convenient at least qualitatively to separate temperature and baryonic density effects.

In a hot meson gas the first medium effect is the Bose enhancement of the \(\pi\pi\) annihilation or, in terms of the rho meson propagator, the temperature effect affecting the two-pion loop contribution to the rho self-energy. In addition \(\rho\)-meson scattering, in first rank \(\rho\pi \rightarrow \omega, a_1\), also significantly contributes to the rho self-energy. It is important to notice that the \(a_1\) piece is of axial-vector mixing nature and the corresponding contribution to the DPR can be seen as a consequence of partial chiral symmetry restoration.

Historically the first advocated baryonic density effect was the medium modification of the two-pion loop through p-wave coupling of the pion to \(\Delta\)-h states. This effect, usually referred as the pion cloud contribution, gives a significant enhancement of the DPR below the rho peak [47] and is mainly related to the coupling of the rho to dressed pions and \(\Delta\)-h states (the so-called pisobars). Again, the very important point is that it contains an explicit mixing between the vector and the axial baryonic current, as recently demonstrated [5]. Finally, the direct coupling of the rho to baryonic resonances having sizable coupling to the rho has been incorporated. In a many-body language the rho couples to \(N^* - h\) excitations building up the so-called rhosobars [48]. Among them the \(N^*(1520)\) plays a prominent role (since the coupling is of s-wave nature) and gives a very important contribution to the low mass enhancement [49]. Contrary to the case of the \(a_1\) and pion cloud contributions, the connection with chiral symmetry restoration of the \(N^*(1520)\) is not transparent. The resulting full spectrum which incorporates all the above effects within the expanding fireball model nicely accounts for the data (fig.7). A very important observation is that this spectrum is very flat and very close to a pure perturbative quark-gluon spectrum (see lower panel of fig.7). One possible conclusion is that chiral symmetry restoration manifests itself as a lowering of the quark hadron duality threshold from its free space value of 1.5 GeV down to 0.5 GeV near the phase boundary [44].

6. Conclusion

We have seen the prominent role of in-medium chiral dynamics to generate strong reshaping of hadronic spectral functions. Chiral symmetry restoration itself yields a softening and a sharpening of the scalar-isoscalar modes and the structure seen in \((\pi, \pi\pi)\) has been tentatively attributed to precursor effects of the restoration associated to strong fluctuations of the chiral condensate. However, most of the time one observes a broadening and a flattening of hadronic spectral functions. This is in particular true in the rho meson channel and convincing arguments based on detailed calculations yield to the conclusion that dilepton spectra in relativistic heavy-ion collisions probing the phase transition region constitute a possible signature of this restoration associated to a lowering of the quark-hadron duality threshold. Nevertheless it is clear that a strong effort has to be pursued to improve theoretical analysis in connection with present and forthcoming experimental data. For instance, it is not clear that the \((\pi^-, \pi^0\pi^0)\) data [50] can be described within the
theoretical framework of section 3. For what concerns dilepton production the connection of the resonance contribution (especially the $N^*(1520)$) with chiral symmetry restoration and axial-vector mixing remains to be fully elucidated although some suggestions have been proposed [51]. In addition, on a more practical side, the precise contribution of the omega meson should be separated to isolate the interesting medium effects relative to the rho meson channel. In that respect forthcoming high resolution measurements in a more baryon-dominated regime will certainly bring crucial information. In particular dilepton data obtained with the HADES detector at GSI will be of utmost importance for studying in the most favorable regime the baryonic medium effects which already seem to play a dominant role in the CERN/SPS regime.

Acknowledgments. I am especially grateful to D. Davesne and M. Ericson for critical comments on the manuscript. I have also benefited from productive conversations with P. Schuck, J. Wambach and J. Delorme.

REFERENCES
19. P. Senger, Contribution to these proceedings and references therein.
33. U. Heinz and M. Jacob, nucl-th/0002042.