ABSTRACT

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If a Kerr black hole is connected to a disk orbiting around it by a magnetic field, by the black hole and the disk, the Kerr black hole rotates faster than the disk, and angular momentum is extracted from the disk. The magnetic field transfers angular momentum from the disk to the Kerr black hole. The black hole accretes a portion of the disk in addition to disk accretion. The black hole accretes an effective source of the Kerr black hole accretion in a specific region which is close to the center of the disk. The magnetic field breaks most radiation coming from a region which is close to the center of the disk. The Kerr black hole is different from the normal black hole, the radiated radiation from the black hole with the efficiency of the disk, if the efficiency is no longer smaller than the efficiency of the disk, the radiated radiation is very different from the normal black hole. For a specific example, the black hole is connected to a disk orbiting around it by a magnetic field.
1. Introduction

In the standard theory of an accretion disk around a black hole it is assumed that there is no coupling between the disk and the central black hole (Pringle and Rees 1972; Shakura and Sunayev 1973; Novikov and Thorne 1973; Lynden-Bell and Pringle 1974). However, in the presence of a magnetic field, a magnetic coupling between the disk and the black hole could exist and play an important role in the balance and transportation of energy and angular momentum (Zel’dovich and Schwartzman, quoted in Thorne 1974; Thorne, Price, and Macdonald 1986; Blandford 1998, 1999; van Putten 1999; Gruzinov 1999; Li 2000c; Li and Paczyński 2000; Brown et al 2000). Though this issue was commented by Zel’dovich and Schwartzman more than twenty years ago and by many other people afterwards, a detailed and quantitative calculation has not appeared until Li (2000c). In the absence of the magnetic coupling, the energy source for the radiation of the disk is the gravitational energy of the disk (i.e., the gravitational binding energy between the disk and the black hole). But, if the magnetic coupling exists\(^1\) and the black hole is rotating, the rotational energy of the black hole provides an additional energy source for the radiation of the disk. With the magnetic coupling, the black hole exerts a torque on the disk, which transfers energy and angular momentum between the black hole and the disk. If the black hole rotates faster than the disk, energy and angular momentum are extracted from the black hole and transferred to the disk. The energy deposited into the disk is eventually radiated away by the disk, which will increase the efficiency of the disk and make the disk brighter than usual. If the black hole rotates slower than the disk, energy and angular momentum are transferred from the disk to the black hole, which will lower the efficiency of the disk and make the disk dimmer than usual. Therefore, the magnetic coupling between the black hole and the disk has important effects on the radiation properties of the disk. In this paper we investigate these effects in detail.

To be specific, we consider a model that a thin Keplerian disk rotates around a Kerr black hole in the equatorial plane, and a magnetic field connects the black hole to the disk (Li 2000c). This model is a variant of the standard Blandford-Znajek mechanism (Blandford and Znajek 1977; Macdonald and Thorne 1982; Phinney 1983a,b; Thorne, Price, and Macdonald 1986). In the standard Blandford-Znajek mechanism, the magnetic field threading the black hole is assumed to close on a load which could be very far from the black hole. Due to the large scale involved and the fact that the physics in the load region is ill understood, in some sense the Blandford-Znajek model is not well defined and thus the involved physics is

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\(^1\)In this paper we do not explore why a magnetic connection between a black hole and a disk can exist or if such a magnetic connection can exist. Instead, we assume a magnetic connection between a black hole and a disk exists, and explore the consequences of the magnetic connection.
extremely complicated. Since the load is far from the black hole, the magnetic field suffers from the screw instability (Li 2000b). Due to the lack of knowledge of the physics in the load region, we do not have a good model for the load. The way people usually adopt is to assume the load is a resistor whose resistance is roughly equal to the resistance of the black hole (Macdonald and Thorne 1982; Phinney 1983a,b; Thorne, Price, and Macdonald 1986). Since the load is so far from the black hole that the load cannot be casually connected to the black hole, it is hard to understand how the load can conspire with the black hole to have equal resistances and satisfy the impedance matching condition (Punsly and Coroniti 1990).

Compared with the standard Blandford-Znajek mechanism, the model we consider here is simpler and relatively well defined. In our model, the magnetic field lines threading the black hole close on the disk rather than a remote load. The disk is much better understood than the remote load though the magnetohydrodynamics (MHD) of the disk is still very complicated (Balbus and Hawley 1998; Miller and Stone 1999; Hawley and Krolik 2000). In most cases the disk can be treated as fully ionized and thus its resistance is negligible compared with the resistance of the black hole which is several hundred Ohms. In other words, it is a good approximation to assume that the disk is perfectly conducting and the magnetic field lines are frozen in the disk. Certainly MHD instabilities have important effects on the dynamics of the disk and the magnetic field – in fact people believe that the MHD instabilities play an important role in transporting angular momentum within the disk (Balbus and Hawley 1998), but here we choose to ignore this topic since the problem of MHD instabilities is so complicated that a detailed discussion will be beyond the scope of the paper.

We assume that the disk is thin and Keplerian, lies in the equatorial plane of the black hole with the inner boundary being at the marginally stable orbit (Lynden-Bell 1969; Bardeen 1970; Novikov and Thorne 1973). This requires that, the magnetic field is so weak that its influence on the dynamics of the particles in the disk is negligible. But, the role played a weak magnetic field in the balance and transportation of energy and angular momentum may be important. (In section 6 we will justify the approximation of a weak magnetic field in detail.) With this “weak magnetic field approximation”, the particles in the disk move around the central black hole on circular orbits with a superposition of a small radial inflow motion. If the inflow time-scale is much longer than the dynamical (rotational) time-scale of the disk, the disk and the magnetic field can be in a quasi-steady state even though the magnetic field slowly moves toward the central black hole along with the accretion. By “quasi-steady state” we mean that any macroscopic quantity at a given radius in the disk slowly changes with time: the integrated change within one rotational period of the disk is much smaller than the quantity itself. With the assumption that the disk is thin and Keplerian, and the magnetic field and the disk are in a quasi-steady state, we will solve
the conservation equations of energy and angular momentum, and calculate the radiation flux of the disk, the internal viscous torque of the disk, and compare the results with those predicted by the standard theory of an accretion disk.

Recently, several people have considered the magnetic coupling between the disk and the material in the transition region between the disk and the black hole horizon, and have argued that such a coupling produces a torque at the inner boundary of the disk (Krolik 1999; Gammie 1999; Agol and Krolik 2000; Hawley and Krolik 2000; Krolik 2001). We emphasize that our model is distinctly different from theirs: (1) In our model, the magnetic field connects the disk to the black hole, thus the torque on the disk is produced by the black hole; in their model, the magnetic field connects the disk to the material in the transition region, thus the torque on the disk is produced by the material in the transition region. (2) In our model, the magnetic field lines are assumed to connect the disk surfaces to the black hole horizon at intermediate latitudes; while in their model, the magnetic field is in the plane of the disk within the inner edge of the disk thus magnetic reconnection can easily take place (Blandford 2000). (3) In our model, the magnetic field can touch the disk over a range of radii; while in their model, the magnetic field is attached to the disk only at the inner boundary of the disk (Agol and Krolik 2000). (4) In our model, we will show that the torque produced by the black hole propagates outward only and thus the torque at the inner boundary of the disk is zero; in their model, the torque is non-zero at the inner boundary of the disk and extends into the transition region (Hawley and Krolik 2000; Krolik 2001). (5) In our model, energy and angular momentum can be extracted from the black hole even in the case with no accretion; in their model, in order to extract energy and angular momentum from the black hole accretion must exist so that negative energy and angular momentum are carried into the black hole horizon by the accretion material (Gammie 1999).

The paper is organized as follows: In section 2 we discuss the transfer of energy and angular momentum between the black hole and the disk arising from the magnetic coupling between them. In section 3 we solve the conservation equations of energy and angular momentum for a quasi-steady and thin Keplerian accretion disk torqued by a black hole and calculate the radiation flux, the internal viscous torque, and the total power of the disk. In section 4 we argue that, with magnetic coupling to a rapidly rotating black hole, a disk can radiate without accretion. Such a non-accretion disk is purely powered by the rotational energy of the black hole and has an infinite efficiency. In section 5 we study a specific example: the magnetic field lines touch the disk at a circle of a constant radius, and look for the consequences of the magnetic coupling. We show that, the radial profile of the radiation flux produced by the magnetic coupling is very different from that produced by accretion. In section 6, we justify the assumptions made in the paper, and discuss the limitations of our model which include the ignorance of the instabilities of the disk and the magnetic field
and the ignorance of the radiation captured by the black hole and the radiation returning to the disk. In section 7 we draw our conclusions.

Throughout the paper we use the geometric units $G = c = 1$ and the Boyer-Lindquist coordinates $(t, r, \theta, \phi)$ (Misner, Thorne, and Wheeler 1973; Wald 1984).

2. Transfer of Energy and Angular Momentum by the Magnetic Coupling

In the presence of a magnetic field a black hole behaves like a conductor with a surface resistivity $R_H = 4\pi \approx 377$ Ohms (Znajek 1978; Damour 1978; Carter 1979). So, when a black hole rotates in an external magnetic field, an electromotive force (EMF) is induced on its horizon (Macdonald and Thorne 1982; Thorne, Price, and Macdonald 1986). As Blandford and Znajek (1977) were first to note, the voltage drop along the magnetic field lines induced by the black hole rotation is huge enough to give rise a cascade production of electron-positron pairs, thereby producing a highly conducting electron-positron pair plasma (magnetosphere or corona) around the black hole and the disk. Assume the magnetic field lines threading the black hole go through the magnetosphere and close on the disk which rotates around the black hole. Since the plasma disk is a good conductor, the rotation of the disk induces an EMF on the disk (Macdonald and Thorne 1982; Li 2000a).

The black hole and the disk form a closed electric circuit, an electric current flows along the magnetic field lines in the magnetosphere\(^2\) and close itself in the disk and the black hole. Suppose the disk and the black hole rotates in the same direction, then the black hole’s EMF, $E_H$, and the disk’s EMF, $E_D$, have opposite signs. The direction of the electric current, and in turn the direction of the transfer of energy and angular momentum, is determined by the sign of $E_H + E_D$. If $E_H + E_D > 0$, the black hole’s EMF dominates the disk’s EMF, so the black hole “charges” the disk: energy and angular momentum are transferred from the black hole to the disk. If $E_H + E_D < 0$, the disk’s EMF dominates the black hole’s EMF, so the disk “charges” the black hole, energy and angular momentum are transferred from the disk to the black hole. If $E_H + E_D = 0$, the black hole’s EMF balances the disk’s EMF, then no energy and angular momentum are transferred between the black hole and the disk (Li 2000c).

It is straightforward to calculate the electromagnetic power and torque on the disk, using the standard electromagnetism. For the specific case that the magnetic field lines

\(^2\)It is assumed that in the magnetosphere the resistivity along the magnetic field lines is negligible, while the resistivity perpendicular to the magnetic field lines is large. Thus the electric current flows along the magnetic field lines in the magnetosphere without dissipation.
touch the disk at a single radius, the calculations are carried out by Li (2000c). In this case, the sign of $\mathcal{E}_H + \mathcal{E}_D$ (and thus the direction of the transfer of energy and angular momentum) is determined by the sign of $\Omega_H - \Omega_D$, where $\Omega_H$ is the angular velocity of the black hole which is constant over the black hole horizon, $\Omega_D$ is the angular velocity of the disk at the radius where the magnetic field touches the disk. If $\Omega_H > \Omega_D$, i.e. if the black hole rotates faster than the disk, energy and angular momentum are transferred from the black hole to the disk. If $\Omega_H < \Omega_D$, i.e. if the black hole rotates slower than the disk, energy and angular momentum are transferred from the disk to the black hole. If $\Omega_H = \Omega_D$, there is no transfer of energy and angular momentum between the black hole and the disk. For fixed values of the magnetic flux, the mass and the angular momentum of the black hole, and the resistance of the black hole, the power peaks at $\Omega_D = \Omega_H/2$ (Li 2000c).

If the magnetic field is distributed over a differentially rotating disk, the formulae given in Li (2000c) for the EMF of the disk, the power, and the torque on the disk should be replaced with integrations over the radius of the disk. Assume the magnetic field is stationary and axisymmetric, and touches the disk at radii ranging from $r_1$ to $r_2$, then, the total EMF induced on the disk is

$$\mathcal{E}_D = \frac{1}{2\pi} \int_{r_1}^{r_2} \Omega_D \frac{d\Psi_{HD}}{dr} dr,$$

where $\Psi_{HD} = \Psi_{HD}(r)$ is the magnetic flux through a surface whose boundary is a circle with a constant $r$ in the disk. In such a case, an infinite number of adjacent infinitesimal poloidal electric current loops flow between the black hole and the disk along the magnetic field lines connecting them. Each infinitesimal current loop produces an infinitesimal power and an infinitesimal torque on the disk, whose summation gives the total power and the total torque on the disk. Thus, assuming the disk is perfectly conducting, the total power produced by the black hole on the disk is

$$P_{HD} = \frac{1}{4\pi^2} \int_{r_1}^{r_2} \left( \frac{d\Psi_{HD}}{dr} \right)^2 \frac{\Omega_D \left( \Omega_H - \Omega_D \right)}{-dZ_H/dr} dr,$$

where we have treated the black hole’s resistance $Z_H$ as a function of the disk’s radius, which is defined by a map from the black hole horizon to the disk surface given by the magnetic field lines. Similarly, the total torque produced by the black hole on the disk is

$$T_{HD} = \frac{1}{4\pi^2} \int_{r_1}^{r_2} \left( \frac{d\Psi_{HD}}{dr} \right)^2 \frac{\Omega_H - \Omega_D}{-dZ_H/dr} dr.$$

From equation (2) and equation (3) we have $dP_{HD} = \Omega_D dT_{HD}$, or

$$P_{HD} = \int_{r_1}^{r_2} \Omega_D \frac{dT_{HD}}{dr} dr,$$
which follows from the assumption that the disk is perfectly conducting so the magnetic field is frozen in the disk.

For a Kerr black hole of mass $M_H$ and angular momentum $J_H = M_H a$, we have (Thorne, Price, and Macdonald 1986)

$$\frac{dZ_H}{d\theta} = \frac{R_H}{2\pi} \frac{r_H^2 + a^2 \cos^2 \theta}{(r_H^2 + a^2) \sin \theta},$$

where $r_H = M_H + \sqrt{M_H^2 - a^2}$ is the radius of the outer horizon of the black hole, $\theta$ is the polar angle coordinate on the horizon. Then, $dZ_H/dr$ can be calculated through

$$\frac{dZ_H}{dr} = \frac{dZ_H}{d\theta} \frac{d\theta}{dr},$$

where $\theta = \theta(r)$ is a map between the $\theta$ coordinate on the horizon and the $r$ coordinate on the disk, which is induced by the magnetic field lines connecting the disk to the horizon. Since $d\theta/dr < 0$ and $dZ_H/d\theta > 0$, we have $dZ_H/dr < 0$.

3. Quasi-Steady State for an Accretion Disk Torqued by a Black Hole

Of particular interest is the case of an accretion disk in a steady state, which is simple enough for solving the equations of energy conservation and angular momentum conservation and has been well studied when the magnetic coupling between the black hole and disk does not exist (Pringle and Rees 1972; Shakura and Sunyaev 1973; Novikov and Thorne 1973; Lynden-Bell and Pringle 1974; Pagel and Thorne 1974; Thorne 1974). In a steady state, the material of the disk advected with accretion toward smaller radii is balanced by the material advected from larger radii. Any macroscopic (statistically averaged) quantity of the disk remains (approximately) unchanged for a long period of time during accretion (say, over a time interval $> 100$ rotation periods of the disk). When there is a magnetic field connecting the black hole to the disk, the disk and the magnetic field cannot be in an exactly steady state unless there is no accretion at all. When there is accretion, the magnetic field frozen in the disk slowly moves toward the central black hole as accretion goes on, which raises the question how long a configuration of magnetic connection can be maintained. Due to the complex topologies of the magnetic field, the magnetic field advected with accretion toward smaller radii, which connects the black hole to the disk, does not have to be balanced by the magnetic field advected from larger radii which also connects the black hole to the disk. However, if the radial velocity of particles in the disk is much smaller than their rotational velocity, the inflow time-scale is much larger than the dynamical time-scale. Then, it is reasonable to assume that within one rotation period, the global configuration of the magnetic field is
almost unchanged, and the overall change in a macroscopic quantity at a given radius in the disk is much smaller than the quantity itself. When this condition is satisfied, we say that the disk and the magnetic field are in a quasi-steady state. In a quasi-steady state, a macroscopic quantity at a given radius may change significantly over a long period of time and the magnetic connection between the black hole and the disk may disappear at last, but they are approximately unchanged within one rotation period of the disk. In this section we solve the equations for energy conservation and angular momentum conservation for such a quasi-steady state disk magnetically coupled to a black hole.

For a steady, axisymmetric, and thin Keplerian disk around a Kerr black hole, the general relativistic equations of energy conservation and angular momentum conservation have been investigated in detail by Novikov and Thorne (1973); Page and Thorne (1974); and Thorne (1974). Assume the magnetic field is weak so that its influence on the dynamics of disk particles is negligible, then a thin Keplerian disk is a good approximation\(^3\). With the magnetic coupling between the black hole and the disk being taken into account, for a quasi-steady state the conservation of angular momentum is described by

\[
\frac{d}{dr} \left( \dot{M}_D L^+ - g \right) = 4\pi r \left( FL^+ - H \right),
\]

where \( \dot{M}_D \equiv dM_D/dt \) is the accretion rate of mass (measured by an observer at infinity; we use the convention \( \dot{M}_D > 0 \) for accretion), \( L^+ \) is the specific angular momentum of a particle in the disk, \( g \) is the internal viscous torque of the disk, \( F \) is the energy flux radiated away from the surface of the disk (measured by an observer co-rotating with the disk), and \( H \) is the flux of angular momentum transferred from the black hole to the disk by the magnetic field. The conservation of energy is described by

\[
\frac{d}{dr} \left( \dot{M}_D E^+ - g \Omega_D \right) = 4\pi r \left( FE^+ - H\Omega_D \right),
\]

where \( E^+ \) is the specific energy of a particle in the disk. When \( H = 0 \), i.e., there is no magnetic coupling, equation (7) and equation (8) return to the equations derived by Novikov and Thorne (1973), and Page and Thorne (1974). \( E^+ \) is related to \( L^+ \) by (Page and Thorne 1974)

\[
\frac{dE^+}{dr} = \Omega_D \frac{dL^+}{dr}.
\]

\(^3\)A magnetic field with negligible influence on the dynamics of the particles in the disk can play an important role in the balance and transportation of energy and angular momentum. In section 6 we will justify the weak magnetic field approximation and the importance of a weak magnetic field in the balance and transportation of energy and angular momentum in detail.
The flux of angular momentum transferred from the black hole to the disk, $H$, is defined by the torque $T_{HD}$ produced by the black hole on the disk

$$T_{HD} \equiv 2\pi \int_{r_1}^{r_2} H r dr.$$  \hfill (10)

Comparing equation (10) with equation (3), we have

$$H = \frac{1}{8\pi^3} \left( \frac{d \Psi_{HD}}{dr} \right)^2 \frac{\Omega_H - \Omega_D}{-r dZ_H/dr},$$  \hfill (11)

if the magnetic field is smoothly distributed on the disk.

In a quasi-steady state, $\dot{M}_D$ is constant throughout the disk (the conservation of mass). Then, equation (7) and equation (8) can be solved for $F$ and $g$ by using equation (9). The solutions are

$$F = \frac{\dot{M}_D}{4\pi r} f + \frac{1}{4\pi r} \left( \frac{-d\Omega_D}{dr} \right) (E^+ - \Omega_D L^+)^{-2} \times$$

$$\left[ 4\pi \int_{r_{ms}}^{r} (E^+ - \Omega_D L^+) H r dr + g_{ms} (E_{ms}^+ - \Omega_{ms} L_{ms}^+) \right],$$  \hfill (12)

and

$$g = \frac{E^+ - \Omega_D L^+}{-d\Omega_D/dr} \frac{4\pi r F}{4\pi r F},$$  \hfill (13)

where the subscript “$ms$” denotes the marginally stable orbit which is assumed to be the inner boundary of a thin Keplerian disk, $g_{ms}$ is an integration constant, $f$ is defined by

$$f \equiv -\frac{d\Omega_D}{dr} (E^+ - \Omega_D L^+)^{-2} \int_{r_{ms}}^{r} (E^+ - \Omega_D L^+) \frac{dL^+}{dr} dr.$$  \hfill (14)

The integration of $f$ has been worked out by Page and Thorne (1974) and is given by their equation (15n), we do not display it here since it is lengthy. It is easy to check that $f(r = r_{ms}) = 0$ and $f(r \gg r_{ms}) \approx \frac{3M_H}{r^4}.$

Suppose the magnetic field is distributed on the disk from $r = r_1$ to $r = r_2$, where $r_2 > r_1 \geq r_{ms}$, then

$$\int_{r_{ms}}^{r} (E^+ - \Omega_D L^+) H r dr = \begin{cases} 0 & \text{if } r \leq r_1 \\ \int_{r_1}^{r_2} (E^+ - \Omega_D L^+) H r dr & \text{if } r_1 < r < r_2 \\ \int_{r_1}^{r_2} (E^+ - \Omega_D L^+) H r dr & \text{if } r \geq r_2 \end{cases}.$$  \hfill (15)

At the inner boundary of the disk, where $r = r_{ms} \leq r_1$, we have $g = g_{ms}$. For a thin Keplerian disk, the “no-torque inner boundary condition” is an excellent approximation (Novikov and
Thorne 1973; Muchotrzeb and Paczyński 1982; Abramowicz and Kato 1989; Paczyński 2000; Armitage, Reynold, and Chiang 2000). Though recently this boundary condition has been challenged when there is a magnetic connection between the disk and the material in the transition region (Krolik 1999; Hawley and Krolik 2000), in this paper we do not consider such a magnetic connection thus we adopt the “no-torque inner boundary condition”. The appearance of a magnetic coupling between the black hole and the disk does not introduce a torque at the inner boundary. This is because of that the integration in equation (15) always vanishes at \( r = r_m \) for any \( H \). In other words, the torque produced by the magnetic coupling propagates outward only in the disk. Therefore, in the following discussion we take \( g_m = 0 \). Then we have

\[
F = \frac{1}{4 \pi r} \left[ \dot{M}_D f + 4 \pi \left( - \frac{d \Omega_D}{dr} \right) (E^+ - \Omega_D L^+)^{-1} \int_{r_m}^{r} (E^+ - \Omega_D L^+) H_r dr \right], \tag{16}
\]

and

\[
g = \frac{E^+ - \Omega_D L^+}{-d \Omega_D/dr} \dot{M}_D f + 4 \pi (E^+ - \Omega_D L^+)^{-1} \int_{r_m}^{r} (E^+ - \Omega_D L^+) H_r dr. \tag{17}
\]

Equation (16) gives the radiation flux of the disk, equation (17) gives the internal viscous torque of the disk. The first terms on the right hand sides of equation (16) and equation (17) are the familiar results for a standard accretion disk (Page and Thorne 1974).

The integration of equation (8) gives the power (i.e., the luminosity) of the disk, which is the energy radiated by the disk per unit time as measured by an observer at infinity (Thorne 1974)

\[
\mathcal{L} \equiv 2 \int_{r_m}^{r} E^+ F^2 \pi rdr = \int_{r_m}^{r} \left[ \frac{d}{dr} \left( \dot{M}_D E^+ - g \Omega_D \right) + 4 \pi r \Omega_D \right] dr
\]

\[
= \dot{M}_D \left( 1 - E^+_m \right) + 4 \pi \int_{r_m}^{r} \Omega_D rdr, \tag{18}
\]

where in the first line on the right hand side the factor 2 accounts for the fact that a disk has two surfaces, in the third line we have used the boundary conditions \( E^+(r \to \infty) = 1 \), \( g \Omega_D(r \to \infty) = 0 \), and \( g \Omega_D(r = r_m) = 0 \). The power of the black hole, which is the energy transferred from the black hole to the disk per unit time as measured by an observer at infinity, is

\[
\mathcal{L}_{HD} \equiv 2 \mathcal{P}_{HD} = 4 \pi \int_{r_m}^{r} \Omega_D rdr = 4 \pi \int_{r_m}^{r} \Omega_D rdr, \tag{19}
\]
so equation (18) can be written as

\[ \mathcal{L} = \dot{M}_D \epsilon_0 + \mathcal{L}_{HD}, \]  

(20)

where

\[ \epsilon_0 = 1 - E_{m,s}^+, \]  

(21)

is the efficiency of accretion, i.e., the efficiency of the disk when the magnetic coupling between the black hole and the disk does not exist. For a Schwarzschild black hole, \( \epsilon_0 \approx 0.06 \); for a Kerr black hole of \( s \equiv a/M_H = 0.998 \), \( \epsilon_0 \approx 0.32 \); for an extreme Kerr black hole of \( s = 1 \), \( \epsilon_0 \approx 0.42 \) (Thorne 1974). For any thin Keplerian disk around a Kerr black hole, \( \epsilon_0 < 0.42 \) always.

Equation (20) describes the global balance of energy for a quasi-steady accretion disk magnetically coupled to a Kerr black hole. \( \mathcal{L} \) is the total power of the disk. \( \dot{M}_D \epsilon_0 \) represents the rate of change in the gravitational energy of the disk, as in the standard theory of an accretion disk. \( \mathcal{L}_{HD} \) represents the rate of energy transferred from the black hole to the disk, which is absent in the standard theory of an accretion disk.

From equation (20), the total efficiency of the disk is

\[ \epsilon \equiv \frac{\mathcal{L}}{\dot{M}_D} = \epsilon_0 + \frac{\mathcal{L}_{HD}}{\dot{M}_D}. \]  

(22)

If \( \mathcal{L}_{HD} = 0 \), i.e. there is no energy transfer between the black hole and the disk by the magnetic coupling — as the case in the standard theory of an accretion disk, \( \epsilon = \epsilon_0 \) and the power of the disk purely comes from the gravitational energy of the disk. If \( \mathcal{L}_{HD} > 0 \), which is the case when the black hole rotates faster than the disk, energy is transferred from the black hole to the disk. Then there are two sources for the power of the disk: one is the gravitational energy of the disk (represented by \( \dot{M}_D \epsilon_0 \)), the other is the rotational energy of the black hole (represented by \( \mathcal{L}_{HD} \)). The efficiency of the disk is increased by the magnetic coupling: \( \epsilon > \epsilon_0 \). For very small \( \dot{M}_D \), an efficiency \( \epsilon \gg 1 \) can be achieved. If there is no accretion at all, the total efficiency of the disk is infinite. If \( \mathcal{L}_{HD} < 0 \), which is the case when the black hole rotates slower than the disk, energy is transferred from the disk to the black hole. Then, the efficiency of the disk is decreased by the magnetic coupling: \( \epsilon < \epsilon_0 \). Since \( \mathcal{L} \) cannot be negative, we must have

\[ \dot{M}_D \epsilon_0 \geq -\mathcal{L}_{HD}. \]  

(23)
4. Radiation without Accretion: A Disk Powered by a Black Hole

For a disk magnetically coupled to a rapidly rotating black hole, an extremely interesting feature is that the disk can radiate without accretion. This can be seen directly from equation (16) and equation (18): when \( \dot{M}_D = 0 \), \( F \) is non-negative over the disk and \( \mathcal{L} \) is positive if

\[
\int_{r_{ms}}^r \left( E^+ - \Omega_D L^+ \right) H r dr \geq 0
\]

for any \( r > r_{ms} \), and

\[
\int_{r_{ms}}^\infty \left( E^+ - \Omega_D L^+ \right) H r dr > 0.
\]

From equation (11), the sign of \( H \) is determined by the sign of \( \Omega_H - \Omega_D \) since \( dZ_H/dr < 0 \). Since \( \Omega_H \) is constant over the horizon of the black hole and \( d\Omega_D/dr < 0 \) for \( r > r_{ms} \) over the disk, conditions (24) and (25) are equivalent to the requirement that \( H \geq 0 \) over the disk and \( H > 0 \) at least over an interval of \( r > r_{ms} \).

For such a non-accretion disk, the radiation flux and the internal viscous torque are respectively

\[
F = \frac{1}{r} \left( -\frac{d\Omega_D}{dr} \right) (E^+ - \Omega_D L^+)^{-2} \int_{r_{ms}}^r \left( E^+ - \Omega_D L^+ \right) H r dr,
\]

and

\[
g = 4\pi (E^+ - \Omega_D L^+)^{-1} \int_{r_{ms}}^r \left( E^+ - \Omega_D L^+ \right) H r dr,
\]

where \( H \) is given by equation (11). The power of the disk comes purely from the rotational energy of the black hole

\[
\mathcal{L} = \mathcal{L}_{HD} = 4\pi \int_{r_{ms}}^\infty H \Omega_D r dr.
\]

Such a disk is powered by a black hole and has an infinite efficiency.

We emphasize that “radiation without accretion” is a specific feature of our model: a disk coupled to a black hole with a magnetic field. A standard accretion disk radiates only if the accretion rate is non-zero, and the efficiency of a standard accretion disk is always smaller than 0.42. (The maximum efficiency 0.42 can be reached only for a disk around an extreme Kerr black hole with \( a = M_H \).) For the model of a disk magnetically coupled to the material in the transition region, though it has been demonstrated that the efficiency of the disk is unbounded from above if the black hole rotates faster than the disk (Agol and Krolik 2000), a state with a zero accretion rate and a finite power can never be realized since in that model in order to extract energy from the black hole material with negative energy must fall into the black hole thus accretion must exist (Gammie 1999).
5. An Example: the Magnetic Field Touches the Disk at a Circle

Suppose the magnetic field touches the disk at a circle with a radius \( r = r_0 > r_{ms} \), i.e.

\[
H = A_0 \delta (r - r_0),
\]

where \( A_0 \) is a constant and \( \delta (x) \) is a Dirac \( \delta \)-function which satisfies

\[
\delta (x) = 0,
\]

for any \( x \neq 0 \), and

\[
\int_{-\infty}^{\infty} y(x) \delta (x) \, dx = y(0),
\]

for any smooth function \( y(x) \). Though this is a simple and highly ideal case, it is fundamental in understanding the effect of the magnetic coupling on the energetic process of the disk. For any given distribution of a magnetic field on the disk, the corresponding \( H(r) \) can always be written as

\[
H(r) = \int_{r_{ms}}^{\infty} H(r') \delta (r' - r) \, dr'.
\]

Since the energy flux and the internal torque given by equation (16) and equation (17) are linear functionals of \( H(r) \), the results for any given distribution of a magnetic field can be obtained from the results for the simple in equation (29) by linear superpositions. Furthermore, for a realistic case of a smooth distribution of a magnetic field with \( r_b - r_a \ll r_a \), where \( r_a \) and \( r_b \) are the radii of the boundaries of an annular region in the disk within which most of the magnetic field lines are concentrated, equation (29) is a good approximation for \( H \) if we take \( r_0 \approx (r_a + r_b)/2 \) and

\[
A_0 \approx \frac{2}{r_a + r_b} \int_{r_a}^{r_b} H r \, dr.
\]

Thus, the simple case given by equation (29) is meaningful not only in mathematics but also in practice.

Inserting equation (29) into equation (10) and comparing the result with equation (3) of Li (2000c), we obtain

\[
A_0 = \frac{1}{2\pi r_0} \left( \frac{\Delta \Psi_{HD}}{2\pi} \right)^2 \frac{\Omega_H - \Omega_0}{Z_H},
\]
where $\Omega_0 \equiv \Omega_D(r_0)$, $\Delta \Psi_{HD}$ is the magnetic flux connecting the disk to the black hole. The sign of $A_0$ is determined by the sign of $\Omega_H - \Omega_0$: $A_0 > 0$ if $\Omega_H > \Omega_0$; $A_0 < 0$ if $\Omega_H < \Omega_0$.

For a thin Keplerian disk around a Kerr black hole, The ratio of $\Omega_0/\Omega_H$ is

$$\frac{\Omega_0}{\Omega_H} = \frac{2\omega}{s} \left[1 + (1 - s^2)^{1/2}\right],$$  \hspace{1cm} (35)

where $s \equiv a/M_H$, $\omega \equiv M_H\Omega_0$ is a function of $s$ and $r_0/M_H$

$$\omega = \frac{1}{s + \left(\frac{r_0}{M_H}\right)^{3/2}}.$$  \hspace{1cm} (36)

The critical case of $\Omega_H = \Omega_0$ (i.e. $A_0 = 0$) is determined by

$$\omega\left(s, \frac{r_0}{M_H}\right) = \frac{s}{2\left[1 + (1 - s^2)^{1/2}\right]}.$$  \hspace{1cm} (37)

For any given value of $r_0/M_H$, or equivalently, for any given value of $r_0/r_{ms}$, equation (37) can be solved for $s$ to obtain the critical spin $s_c$. $\Omega_H > \Omega_0$ (thus $A_0 > 0$) for $s > s_c$, $\Omega_H < \Omega_0$ (thus $A_0 < 0$) for $s < s_c$. It is easy to show that $s_c$ is a monotonically decreasing function of $r_0/r_{ms}$. For $r_0/r_{ms} = 1$, $s_c \approx 0.3594$ \footnote{Another solution to equation (37) for $r_0/r_{ms} = 1$ is $s = 1$, but here we do not consider the case of an extreme Kerr black hole. In other words, we restrict ourselves to the cases with $0 \leq s < 1.$}; as $r_0/r_{ms}$ increases, $s_c$ decreases quickly. As $r_0/r_{ms} \to \infty$, $s_c \to 0$.

When $0.3594 < s < 1$, the black hole rotates faster than a particle in the disk at any radius ($\geq r_{ms}$). When $0 < s < 0.3594$, there exists a co-rotation radius in the disk defined by equation (37), beyond which the black hole rotates faster than the disk, within which the black hole rotates slower than the disk. When the magnetic field lines touch the disk beyond the co-rotation radius, energy and angular momentum are transferred from the black hole to the disk. When the magnetic field lines touch the disk inside the co-rotation radius, energy and angular momentum are transferred from the disk to the black hole. The co-rotation radius, which we denote by $r_c$, can be solved from equation (37)

$$r_c = M_H \left\{\frac{2}{s} \left[1 + (1 - s^2)^{1/2}\right] - s\right\}^{2/3}, \hspace{1cm} 0 < s < 0.3594.$$  \hspace{1cm} (38)

The ratio $r_c/r_{ms} = (r_c/M_H)(r_{ms}/M_H)^{-1}$ as a function of $s$ is plotted in Fig 1, from which we see that $r_c/r_{ms}$ is a monotonically decreasing function of $s$: $r_c \to r_{ms}$ as $s \to 0.3594$, $r_c \to \infty$ as $s \to 0$. The co-rotation radius does not exist when $s > 0.3594$ for which the black hole
always rotates faster than the disk. [Fig. 1 also shows \( s_c = s_c(r_0) \) if we replace the label of the horizontal axis with \( \log s_c \) and replace the label of the vertical axis with \( \log (r_0/r_{ms}) \).]

Inserting equation (29) into equation (16) and equation (17), we obtain

\[
F = \begin{cases} 
\frac{1}{4\pi r} \dot{M}_D f, \\
\frac{1}{4\pi r} \left[ \dot{M}_D f + 4\pi r_0 A_0 \left( \frac{E_0^+ - \Omega_0 L_0^+}{-\Omega_0^2} \right) \left( \frac{E^+ - \Omega_D L^+}{-\Omega_D^2} \right)^{-2} \right], 
\end{cases}
\]

\( r_{ms} \leq r < r_0 \), \hspace{1cm} \text{(39)}

and

\[
g = \begin{cases} 
E^+ - \Omega_D L^+, \\
E^+ - \Omega_D L^+ \frac{\dot{M}_D}{\Omega_D^2}, \\
\dot{M}_D f + 4\pi r_0 A_0 \left( \frac{E_0^+ - \Omega_0 L_0^+}{-\Omega_0^2} \right) \left( \frac{E^+ - \Omega_D L^+}{-\Omega_D^2} \right)^{-1}, 
\end{cases} \hspace{1cm} r_{ms} \leq r < r_0 \hspace{1cm} \text{(40)}
\]

The expressions for \( E^+ - \Omega_D L^+ \), \( d\Omega_D/dr \), and \( f \) can be found in Page and Thorne (1974). Equation (39) and equation (40) clearly show that the torque produced by the magnetic coupling at \( r = r_0 \) propagates outward only, thus the magnetic coupling between the black hole and disk has effects only in the region beyond the circle \( r = r_0 \) in the disk.

5.1. \( \dot{M}_D = 0 \)

Substituting \( \dot{M}_D = 0 \) into equation (39) and equation (40) and using the expressions for \( E^+ - \Omega_D L^+ \) and \( d\Omega_D/dr \) given by Page and Thorne (1974), we obtain

\[
F = \frac{3T_0}{8\pi} \left( \frac{M_H}{r^7} \right)^{1/2} \frac{1}{1 - 3M/r + 2a \left( M/r^2 \right)^{1/2}} \vartheta (r - r_0), \hspace{1cm} \text{(41)}
\]

and

\[
g = \frac{T_0 \left[ 1 + a \left( M/r^2 \right)^{1/2} \right]}{\left[ 1 - 3M/r + 2a \left( M/r^2 \right)^{1/2} \right]^{1/2}} \vartheta (r - r_0), \hspace{1cm} \text{(42)}
\]

\(^5\text{A similar formula for } F \text{ was obtained by Agol and Krolik (2000), who treated a non-accretion disk magnetically connected to a black hole as the “infinite efficiency limit” of an accretion disk magnetically connected to the material in the transition region. However, we emphasize that in our model of a disk magnetically connected to a black hole the torque and the radiation flux of the disk are always zero at the inner boundary of the disk. This fact is manifested by the step functions in eq. (41) and eq. (42), which are absent in Agol and Krolik’s formula. Thus, as we have pointed out in the Introduction, the two models are distinctly different from each other.}\)
where $\vartheta$ is the step function

$$
\vartheta(r - r_0) = \begin{cases} 
1, & \text{if } r > r_0 \\
0, & \text{if } r < r_0 
\end{cases},
$$

and

$$
T_0 \equiv 4\pi r_0 A_0 \left( E_0^+ - \Omega_0 J_0^+ \right) \\
= 4\pi r_0 A_0 \left[ 1 + a \left( \frac{M}{r_0^3} \right)^{1/2} \right]^{-1} \left[ 1 - \frac{3M}{r_0^3} + 2a \left( \frac{M}{r_0^3} \right)^{1/2} \right]^{1/2} .
$$

Since $F$ and $g$ cannot be negative, such non-accretion solutions can exist only if $A_0 > 0$, which requires $\Omega_H > \Omega_0$, i.e. the black hole rotates faster than the disk at $r = r_0$. Examples of $F$ and $g$ are plotted in Fig. 2 and Fig. 3. These figures clearly show that the magnetic coupling takes effect only beyond the radius $r = r_0$, i.e. the torque produced by the magnetic coupling propagates only outward from $r = r_0$. Both $F$ and $g$ have sharp peaks at $r = r_0$, are zero for $r < r_0$. Since $r_{ms} < r_0$, the torque and the radiation flux are always zero at the inner boundary of the disk. At $r = r_0$, the radiation flux rises suddenly from zero to a sharp peak, then decreases quickly for $r > r_0$. At $r \gg r_0$, the radiation flux decreases with radius as $F \propto r^{-3.5}$. At $r = r_0$, the internal viscous torque rises suddenly from zero to a finite value, then decreases slowly for $r > r_0$ and approaches a constant at $r \gg r_0$.

The radial profile of the radiation flux produced by the magnetic coupling is very different from that produced by accretion. For a standard accretion disk, the radiation flux is zero at $r = r_{ms}$ (the inner boundary of the disk), gradually rises to a maximum at a radius beyond $r_{ms}$, then decreases slowly, and approaches $F \propto r^{-3}$ at large radii (Novikov and Thorne 1973; Page and Thorne 1974; Thorne 1974). While for a non-accretion disk magnetically coupled to a Kerr black hole, assume the magnetic field touches the disk at the inner boundary, then at $r = r_{ms}$ the radiation flux suddenly rises from zero to a sharp peak, then decreases quickly and approaches $F \propto r^{-3.5}$ at large radii. To compare the radiation profile of the magnetic coupling with the radiation profile of accretion, in Fig. 4 we plot both the radiation flux of a non-accretion disk magnetically coupled to a rapidly rotating black hole and the radiation flux of a standard accretion disk rotating around the same black hole. For the non-accretion disk, the magnetic field is assumed to touch the disk at the inner boundary (the marginally stable orbit). Obviously, the radiation flux of the non-accretion disk has a much steeper radial profile and a sharp peak closer to the center of the disk, compared to the radiation flux of the standard accretion disk. For the same models, in Fig. 5 we show

\footnote{At large radii the internal torque approaches $g \propto r^{1/2}$ for a standard thin accretion disk.}
the emissivity index defined by $\alpha \equiv -d \ln F/d \ln r$, which measures the slope of the radial emissivity profile in the disk. We see that, throughout the disk the emissivity index for the non-accretion disk with magnetic coupling is significantly bigger than the emissivity index for the standard accretion disk. At large radii, $\alpha$ approaches 3.5 for the non-accretion disk, 3 for the standard accretion disk.

Inserting equation (29) into equation (10) and equation (19), we get

$$T_{HD} = 2\pi A_0 r_0, \quad P_{HD} = 2\pi A_0 \Omega_0 r_0, \quad (45)$$

where $A_0$ is given by equation (34). As expected, $P_{HD} = T_{HD} \Omega_0$. Since a disk has two surfaces, the total power of the disk is $L_{HD} = 2P_{HD} = 4\pi A_0 \Omega_0 r_0$. The energy radiated per unit time from the region inside a circle of radius $r > r_0$ in the disk is

$$L_{HD}(< r) = 2 \int_{r_{ms}}^r E^+ F^2 \pi r^2 dr$$

$$= 4\pi \int_{r_{ms}}^r H \Omega_D r^2 dr - g\Omega_D$$

$$= 4\pi A_0 \Omega_0 r_0 \left( 1 - \frac{\Omega_D}{\Omega_0} \frac{E_0^+ - \Omega_0 L_0^+}{E^+ - \Omega_D L^+} \right), \quad (46)$$

where on the right hand side in the second line we have used equation (8) (taking $\dot{M}_D = 0$) and the boundary condition $g\Omega(r = r_{ms}) = 0$, in the third line we have used equation (29) and equation (40) (taking $\dot{M}_D = 0$). We can define a half-light radius $r_{1/2}$, within which the energy radiated per unit time by the disk is one half of the total power of the disk, i.e. $L_{HD}(< r_{1/2}) = \frac{1}{2} L_{HD}$. From equation (46), for a non-accretion disk magnetically coupled to a black hole, $r_{1/2}$ can be solved from

$$\left. \frac{E^+ - \Omega_D L^+}{\Omega_D} \right|_{r = r_{1/2}} = \frac{2}{\Omega_0} \frac{(E_0^+ - \Omega_0 L_0^+)}{\Omega_0} . \quad (47)$$

Similarly, for a standard accretion disk, the energy radiated per unit time from the region inside a circle of radius $r > r_{ms}$ in the disk is

$$L_{acc}(< r) = \dot{M}_D \left( E^+ - E_{ms}^+ - \frac{E^+ - \Omega_D L^+}{-d\Omega_D/dr} \Omega_D f \right). \quad (48)$$

The total power of an accretion disk is $L_{acc} = \dot{M}_D (1 - E_{ms}^+)$. Thus, the half-light radius of a standard accretion disk, which is defined by $L_{acc}(< r_{1/2}) = \frac{1}{2} L_{acc}$, can be solved from

$$\left[ E^+ - \frac{E^+ - \Omega_D L^+}{-d\Omega_D/dr} \Omega_D f \right]_{r = r_{1/2}} = \frac{1}{2} (1 + E_{ms}^+) , \quad (49)$$
where \( f \) is given by equation (15n) of Page and Thorne (1974).

We have calculated the half-light radius of a disk magnetically coupled to a Kerr black hole, assuming the disk has no accretion and the magnetic field touches the disk at the inner boundary. The results are shown in Fig. 6. For comparison, we have also calculated the half-light radius of a standard accretion disk around a Kerr black hole, the results are also shown in Fig. 6 with the dashed curve. From these results we see that, for a non-accretion disk magnetically coupled to a Kerr black hole with the magnetic field touching the disk at the inner boundary, most energy radiated by the disk comes from a region closer to the center of the disk, compared to the case of a standard accretion disk. A similar figure is shown by Agol and Krolik (2000, Fig. 1), and very similar results are obtained by them for a disk magnetically coupled to the material in the transition region. But we emphasize that in their model a state with a zero accretion rate and a finite power can never be realized, since in their model in order to extract energy from a black hole material with negative energy must fall into the black hole. And, in our figure the curve is broken at \( a/M_H = 0.3594 \) for the non-accretion disk, while in Agol and Krolik’s figure the curve is drawn without broken.

Suppose a Kerr black hole loses its energy and angular momentum through the magnetic coupling to a thin Keplerian disk with no accretion, with the magnetic field lines touching the disk at a circle of radius \( r = r_0 \). Then, the evolution of the black hole spin \( s = a/M_H = J_H/M_H \) is given by

\[
\frac{ds}{d \ln M_H} = \frac{1}{\omega} - 2s, \tag{50}
\]

where \( \omega \) is defined by equation (36), which is a function of \( s \) and \( r_0/r_{ms} \) (see the discussions below eq. [37]). If we know how \( r_0/r_{ms} \) evolves with \( s \), equation (50) can be integrated to obtain \( M_H = M_H(s) \). As an example, let us consider a Kerr black hole of initial mass \( M_{H0} \) and initial spin \( s = 0.998 \) – the spin of a canonical black hole (Thorne 1974). As the black hole spins down to \( s = s_c \) – the value when \( \Omega_H = \Omega_0 \) and thus the transfer of energy and angular momentum between the black hole and the disk stops, which is defined by equation (37), the total amount of energy extracted from the black hole by the disk, \( \Delta M_H = M_H(s = 0.998) - M_H(s = s_c) \), can be calculated by integrating equation (50). Then we can calculate the fraction of energy that can be extracted from the black hole

\[
\eta \equiv \frac{\Delta M_H}{M_{H0}} = 1 - \exp \int_{s_0}^{s_c} \frac{ds}{\omega - 1 - 2s}, \tag{51}
\]

where \( s_0 = 0.998 \). For simplicity, we assume \( r_0/r_{ms} \) keeps constant as \( s \) decreases. The corresponding \( \eta \) is plotted in Fig. 7 as a function of \( r_0/r_{ms} \). The figure shows that \( \eta \) decreases quickly with increasing \( r_0/r_{ms} \). The maximum fraction is reached when \( r_0 = r_{ms} \):
\( \eta_{\text{max}} = 0.152 \). Thus, the magnetic field lines that touch the inner edge of the disk are most efficient in extracting energy from the black hole—in the sense that the largest amount of energy can be extracted from the black hole\(^7\).

5.2. \( \dot{M}_D \neq 0 \)

In our model there are two torques acting on the disk: one is the external torque produced by the magnetic coupling to the black hole, the other is the internal torque which conveys angular momentum within the disk and dissipates energy. We do not discuss the origin of the internal torque, we only assume the internal torque exists and the disk automatically adjusts its internal torque so that quasi-steady state solutions exist. If the internal torque balances the external torque exactly—as assumed in section 5.1, which can be true only if the black hole rotates faster than the disk, then a steady state with no accretion is built. In such a case, the power of the disk comes from the rotational energy of the black hole, the disk has an infinite efficiency. If the disk has so much internal torque that cannot be balanced by the external torque, the excess internal torque will produce accretion\(^8\). In such a case, the power of the disk comes from both the rotational energy of the black hole and the gravitational energy of the disk.

In the case that there is accretion and the magnetic field touches the disk at a circle of radius \( r_0 \), the quasi-steady solutions are given by equation (39) and equation (40). The radiation flux is plotted in Fig. 8 and Fig. 9, respectively for the case that the black hole rotates faster than the disk (\( \Omega_H > \Omega_0 \)) and for the case that the black hole rotates slower than the disk (\( \Omega_H < \Omega_0 \)). In both cases, for \( r < r_0 \) the solutions are the same as those predicted by the standard theory of a thin Keplerian disk (Novikov and Thorne 1973; Page and Thorne 1974; Thorne 1974), in particular \( F \) and \( g \) are zero at \( r = r_{\text{in}} \). The extensions of the standard solutions to \( r > r_0 \) are shown with dashed curves, the positions of \( r_0 \) are shown with vertical dotted lines. Due to the magnetic coupling to the black hole, \( F \) and \( g \) are modified for \( r > r_0 \) by superposing the contribution of the magnetic coupling to the

---

\(^7\)We should note that there are two different quantities describing the energetic process for a black hole: the fraction of energy extraction, which describes how much energy can be extracted from a black hole in total; the power, which describes how much energy can be extracted from a black hole per unit time. A higher fraction does not imply a higher power, and vice versa. In fact, since \( P_{BD} \propto \Omega_D (\Omega_H - \Omega_D) \), \( P_{BD} \) peaks when the magnetic field lines touch the disk at the place where \( \Omega_D = \Omega_H/2 \) (Li 2000c).

\(^8\)Certainly there is yet another possibility: the disk does not have enough internal torque to remove the angular momentum deposited into the disk by the black hole, then the accretion flow may be reversed and thus steady state solutions do not exist (Blandford 1999).
standard solutions. For $r \gg r_0$, the radiation flux given by the standard theory decreases as $r^{-3}$, while the radiation flux contributed by the magnetic coupling decreases as $r^{-3.5}$. Thus, at large radii the radiation flux $F$ is dominated by the contribution of accretion. This is also true for the viscous torque in the disk, i.e., at large radii the torque in the disk is dominated by the contribution of accretion. When the black hole rotates faster than the disk – as the case shown in Fig. 8, the black hole pumps energy and angular momentum into the disk, the energy is locally dissipated in the disk and eventually radiated away, a bright annular “bump” is produced at $r = r_0$. When the black hole rotates slower than the disk – as the case shown in Fig. 9, the black hole extracts energy and angular momentum from the disk, and a dark annular “valley” is produced at $r = r_0$.

When the black hole rotates slower than the disk (i.e., $\Omega_H < \Omega_0$), $A_0$ is negative thus energy and angular momentum are transferred from the disk to the black hole. From equation (39), the minimum radiation flux at $r = r_0$ is

$$F_{\text{min}} = \frac{1}{4\pi r_0} \left[ \dot{M}_D f(r_0) + 4\pi r_0 A_0 \left( -\frac{d\Omega_0}{dr_0} \right) \left( E_0^+ - \Omega_0 L_0^+ \right)^{-1} \right],$$

(52)

which (and the minimum $g$ at $r = r_0$) is non-negative if and only if

$$4\pi r_0 |A_0| \leq \left| \frac{d\Omega_0}{dr_0} \right| \left( E_0^+ - \Omega_0 L_0^+ \right) \dot{M}_D f(r_0),$$

(53)

where we have used the fact that $d\Omega_D/dr < 0$ in the disk. Therefore, when the black hole rotates slower than the disk, quasi-steady solutions exist only if the condition in equation (53) is satisfied. In particular, since $f(r_{\text{mz}}) = 0$, no quasi-steady solutions exist if the magnetic field touches the disk at the inner boundary and the black hole rotates slower than the disk (i.e. $a/M_H < 0.3594$).

6. Weak Magnetic Field Assumption, Instabilities, and Photon Capture

For a non-accretion disk magnetically coupled to a black hole, a steady state can be established if the magnetic field is frozen in the disk and the angular momentum transferred from the black hole to the disk is steadily conveyed outward by an internal viscous torque. When there is accretion, an exactly steady state cannot exist since the magnetic field frozen in the disk slowly moves toward the central black hole with the accretion flow. However, if the inflow velocity of the disk is much smaller than the rotational velocity, we can expect the disk and the magnetic field to be in a quasi-steady state, which means that any macroscopic quantity at a given radius in the disk slowly changes with time: the overall change within one rotation period is negligible compared with the quantity itself. For example, the magnetic
flux within a given radius is $\Psi \sim Br^2$, the change of the magnetic flux within one rotation period is $\Delta \Psi \sim Brv_T T \sim Br^2(v_r/v_\phi)$, where $T \approx 2\pi r/v_\phi$ is the rotation period, $v_\phi$ is the rotational velocity of the disk, $v_r$ is the radial velocity of the particles in the disk (we adopt the convention that $v_r > 0$ for accretion). Obviously, we have $\Delta \Psi \ll \Psi$ for a disk with $v_r \ll v_\phi$. The assumption of a quasi-steady state is more or less similar to the assumption of “adiabatic invariance” usually discussed in galactic dynamics and quantum mechanics where it is assumed that a potential changes very slowly with time so that within one rotation/oscillation period the potential can be treated as unchanged. For a quasi-steady state, the magnetic connection between the black hole and the disk may evolve with time from the view of a long time interval, but within one rotation period the magnetic connection is approximately unchanged.

In searching for quasi-steady state solutions we have assumed that the magnetic field is weak so that its influence on the dynamics of particles in the disk is negligible, and thus a thin Keplerian disk is a good approximation. This “weak magnetic field assumption” requires that

$$\left| \nabla \left( B^2/8\pi \right) \right| \ll \rho |\vec{g}|$$

(54)

in the disk, where $\rho$ is the mass density of the disk, $\vec{g}$ is the gravitational acceleration produced by the black hole. Since $\left| \nabla \left( B^2/8\pi \right) \right| \sim B^2/r$, $|\vec{g}| \sim GM_H/r^2 \sim r_H/r^2$, equation (54) is equivalent to

$$B^2 \ll \rho c^2 \left( \frac{r_H}{r} \right),$$

(55)

where we have restored the speed of light in the equation. Equation (55) is the condition for the weak magnetic field assumption.

A weak magnetic field may play an important role in the balance and transportation of energy and angular momentum. From equation (2) of Li (2000c) and $\Psi_{HD} \sim Br^2$, we have

$$\mathcal{L}_{HD} = 2P_{HD} \sim B^2 r^2 \left( \frac{r}{r_H} \right)^{1/2}$$

(56)

for the case of $a/M_H \approx 1$, where we have used $\Delta Z_H \sim 1$. The accretion power of the disk is

$$\mathcal{L}_{acc} = \dot{M}_{D} \epsilon_0 \sim \rho r h v_r,$$

(57)

where $\epsilon_0$ is the efficiency of accretion ($\approx 0.32$ for $a/M_H = 0.998$, see eq. [21]), $h$ is the thickness of the disk. From equation (56) and equation (57), $\mathcal{L}_{HD} \gtrsim \mathcal{L}_{acc}$ if and only if

$$B^2 \gtrsim \rho c^2 \left( \frac{h}{r} \right) \left( \frac{r_H}{r} \right) \left( \frac{v_r}{v_\phi} \right),$$

(58)
where we have used $v_r \sim r^{-1/2}$. Equation (58) is the condition for the magnetic field to be important in the balance and transportation of energy and angular momentum.

For a thin and quasi-steady accretion disk, we have $h/r \ll 1$ and $v_r/v_\phi \ll 1$. In the disk we always have $r_H/r \lesssim 1$. Therefore, equation (58) is not a too stringent restriction on the values of $B$. To see this, we can compare equation (58) with equation (55). There is a large room for $B^2$ to satisfy both equation (55) and equation (58) if

$$\frac{v_r}{v_\phi} \ll \frac{r}{h}. \quad (59)$$

Equation (59) is always satisfied by a thin and quasi-steady disk.

In all our analyses the instabilities of the magnetic field and the disk are ignored, but in practice they may be important. Because of the outwardly decreasing differential rotation, a plasma disk threaded by a weak magnetic field is subject to the Balbus-Hawley instability (Balbus and Hawley 1991, 1998). Within a few rotational periods the magnetic field lines in the disk become chaotic and tangled, the disk becomes turbulent. This magneto-rotational instability is assumed to play an important role in transporting angular momentum within the disk, which is important for producing accretion (Balbus and Hawley 1998; Hawley 2000; Hawley and Krolik 2000; Menou 2000). For a thin disk, the effect of magnetic reconnection becomes important if the poloidal magnetic field lines threading the disk are twisted too much by the rotation of the disk and the central object. This may put a restriction on the amplitude of the toroidal magnetic field near the plane of the disk, which in turn restricts the power of the energy and the angular momentum transferred between the central object and the disk (Ghosh and Lamb 1978, 1979a,b; Livio and Pringle 1992; Wang 1995). But due to the fact that a black hole has a large resistance, the situation here may not be so serious as in the case when the central object is a star. The screw instability of the magnetic field plays a similar role in limiting the amplitude of the toroidal magnetic field and in turn the power and torque of the central black hole (Gruzinov 1999; Li 2000b). Thus, considering the effect of magnetic reconnection or the screw instability, the actual power and the torque of the black hole may be somewhat smaller than those given by equation (2) and equation (3). If the instabilities are strong enough, the situation may change so dramatically that quasi-steady state solutions do not exist. All these effects of instabilities will be addressed in detail in future.

Not all photons emitted by the disk escape to infinity. Some return to the disk, some are captured by the black hole, due to the gravity of the black hole and the shape of the disk (Thorne 1974; Cunningham 1976; Agol and Krolik 2000). These effects are especially important in the inner disk region when $a/M$ is close to 1. The radiation returning to the disk is dissipated and re-radiated, and eventually reaches infinity or falls into the black hole.
The radiation flux and the overall power of the disk are affected by the photons captured by the black hole and the photons returning to the disk. The black hole's capture cross-section is greater for photons of negative angular momentum (angular momentum opposite to the spin of the black hole) than for photons of positive angular momentum (Godfrey 1970; Bardeen 1973). Thus, photon capture tends to spin down the black hole. Considering the effect of photon capture, Thorne (1974) has shown that a black hole will be prevented from spinning up beyond a limiting state with $a/M \approx 0.998$ through accretion from a thin Keplerian disk\(^9\). For a standard accretion disk without a magnetic coupling, depending on the spin of the black hole, among the energy radiated by the disk up to 6% is captured by the black hole, up to 28% returns to the disk. For a disk with a magnetic coupling, more energy is captured by the black hole or returns to the disk. In an extreme case when the magnetic field lines touch the disk at the inner boundary and there is no accretion, depending on the spin of the black hole, among the energy radiated by the disk up to 15% is captured by the black hole, up to 58% returns to the disk, as in the "infinite efficiency limit" case of a disk magnetically coupled to the material in the transition region (Agol and Krolik 2000). Considering the effect of radiation capture, the radiation flux at the inner part of the disk is moderately modified. However, in the case of a non-accretion disk magnetically coupled to a Kerr black hole, the returning radiation dominates at large radii since the radiation due to the magnetic coupling scales as $r^{-3.5}$ while the returning radiation scales as $r^{-3}$ (Agol and Krolik 2000). In our calculations the effects of photon capture (either captured by the black hole or returning to the disk) are ignored, we hope to consider them in future.

7. Conclusions

For an accretion disk magnetically coupled to a Kerr black hole, quasi-steady state solutions are obtained by assuming the inflow time-scale of particles in the disk is much longer than the rotational time-scale – as adopted in the standard theory of an accretion disk. Though the magnetic field frozen in the disk slowly moves toward the central black hole as accretion goes on, the inflow velocity of the magnetic field and the particles is much smaller than the rotation velocity of the disk. Thus, within one rotation period the magnetic field configuration can approximately be regarded as unchanged, and the assumption of a quasi-steady state can be applied. For a general distribution of the magnetic field connecting the disk to the black hole, the solutions for the radiation flux and the internal viscous torque

\(^9\)However, the limit $s = 0.998$ may be exceeded if a black hole is spun-up by an external torque, such as in black hole mergers (Agol and Krolik 2000), or through accretion from a thick disk (Abramowicz and Lasota 1980).
are given by equation (16) and equation (17), which are superpositions of the the contribution from accretion and the contribution from magnetic coupling. From the view of a long period of time (e.g. with many rotation periods), the radiation flux and the internal torque of the disk varies with time (the magnetic connection may even disappear finally), but equation (16) and equation (17) give the instant values of the radiation flux and the internal torque of the disk. These general solutions clearly show that for any distribution of a magnetic field on the disk, the torque produced by the magnetic coupling propagates outward only, thus the internal torque and the radiation flux are always zero at the inner boundary of the disk. Even for the extreme case that the magnetic field touches the disk at the inner boundary, the internal torque $g$ and the radiation flux $F$ of the disk are also zero at the inner boundary: beyond the marginally stable orbit, as $r$ decreases, $g$ and $F$ increase but suddenly drops to zero at $r = r_{ms}$. We emphasize that this feature differs from the case of a disk magnetically connected to the material in the transition region, where the magnetic stress is demonstrated to be non-zero at the inner boundary and extends into the transition region (Krolik 1999; Agol and Krolik 2000; Hawley and Krolik 2000; Krolik 2001).

If the black hole rotates faster than the disk, the black hole pumps energy and angular momentum into the disk through magnetic coupling, which increases the efficiency of the disk. Most interestingly, with the existence of the magnetic coupling, a disk can radiate without accretion: all the power of the the disk comes from the rotational energy of the black hole, such a non-accretion disk has an infinite efficiency. We have discussed in detail a simple but important case: the magnetic field touches the non-accretion disk at a single radius $r_0$ — or equivalently, the magnetic field is distributed on the disk within a narrow region around $r_0$. For this simple case, the radiation flux has a sharp peak at $r_0$: it is zero for $r < r_0$ and decreases quickly for $r > r_0$. At large radii, the radiation flux approaches $F \propto r^{-3.5}$. This behavior of the radiation flux is very different from that of a standard accretion disk, where the radiation flux spreads widely over radii and approaches $F \propto r^{-3}$ at large radii. We have compared the radiation flux of a non-accretion disk with the magnetic field touching the disk at the inner boundary (i.e. the marginally stable orbit), with the radiation flux of a standard accretion disk in detail. The results are summarized in Fig. 4 — Fig. 6. Clearly, the radiation profile of the non-accretion disk is very different from that of the standard accretion disk: the non-accretion disk magnetically coupled to a rapidly rotating black hole has a bigger emissivity index throughout the disk and a radiation region closer to the center of the disk. We have also shown that the magnetic field lines touching the disk at the inner boundary is most efficient in extracting energy from the black hole, though the power of the black hole peaks at $\Omega_D = \Omega_H/2$.

A non-accretion state can exist only if the internal torque of the disk exactly balances the external torque on the disk produced by the magnetic coupling. If the disk has so much
internal torque that cannot be balanced by the external torque, the excess internal torque will produce accretion. For an accretion disk with the magnetic field touching the disk at a single radius, a specific feature is that the radiation flux of the disk may have two maxima (Fig. 8 and Fig. 9). When the black hole rotates faster than the disk, the black hole pumps energy to the disk and the second maximum in the radiation flux is produced by adding a bright “bump” to the standard radiation flux (Fig. 8). When the black hole rotates slower than the disk, the black hole extracts energy from the disk – in such a case the efficiency of the disk is decreased by the magnetic coupling – and the second maximum in the radiation flux is produced by digging a dark “valley” in the standard radiation flux (Fig. 9). The position of the second maximum is determined by the position where the magnetic field touches the disk. If the magnetic field is smoothly distributed over the disk instead of touching the disk at a single radius or distributed on the disk within a narrow interval of radii, one of the two maxima in the radiation flux may be smeared out.

For a black hole with \( a/M_H \approx 1 \), the power of the black hole is approximately

\[
P_{HD} \approx B^2 r_H^2 c,
\]

if the magnetic field lines touch the disk close to the inner boundary \( (r_{in} = r_{ms} \approx r_H) \). If the energy deposited into the disk by the black hole is radiated away locally – either thermally or non-thermally, we can define an effective radiation temperature through

\[
T_{eff} \equiv \left( \frac{P_{HD}}{\sigma r_H^4} \right)^{1/4} \approx \left( \frac{B^2 c}{\sigma} \right)^{1/4} \approx 5 \times 10^5 \text{K} \left( \frac{B}{10^4 \text{Gauss}} \right)^{1/2},
\]

where \( \sigma \) is the Stephan-Boltzmann constant. Interestingly, this temperature depends only on the strength of the magnetic field on the disk. If the energy is radiated thermally, the radiation is in the UV to soft X-ray domain for \( B \approx 10^4 \) Gauss.

The recent XMM-Newton observation of soft X-ray emission lines from two Narrow Line Seyfert 1 galaxies MCG –6-30-15 and Mrk 766 shows an extreme big emissivity index \( (\sim 4) \), which has been suggested to indicate that most of the line emission originates from the inner part of a relativistic accretion disk (Branduardi-Raymont et al 2001). From the calculations in section 5.1, we have seen that for a non-accretion disk magnetically coupled to a rapidly rotating black hole with the magnetic field touching the disk at the inner boundary, most radiation comes from a region more concentrated toward the center of the disk and a big emissivity index can be easily realized at any radius, compared to the case for a standard accretion disk. Thus, probably the observational results of XMM-Newton can be more easily explained with the model presented in this paper. Another observation which may be relevant to our model is the kilohertz quasi-periodic oscillations (kHz QPOs) in X-ray binaries, which
has been suggested to originate from the inner edge of a relativistic accretion disk (van der Klis 2000 and references therein).

The magnetic field connecting the black hole to the disk does not have to be axisymmetric. If a bunch of magnetic field lines connect the black hole to the disk and the feet of the magnetic field lines are concentrated in a small region in the disk, a hot spot will be produced on the disk surface if the black hole rotates faster than the disk. If $a/M_H \approx 1$, the temperature of the hot spot is approximately given by equation (61).

In our analyses the magnetic field has been assumed to be weak and the effect of instabilities of the disk plasma and the magnetic field has been ignored. The situation with a strong magnetic field may be significantly different from that has been discussed in this paper, at least the inner part of the disk will not be Keplerian any more and quasi-steady state solutions may not exist: accretion may be stopped and a state similar to the “propeller” phase for pulsars (Schwartzman 1971; Illarionov and Sunyaev 1975; Lipunov 1992) may be produced, or even the accretion flow is reversed (Blandford 1999). The instabilities of the disk plasma and the magnetic field may seriously affect the dynamics and energetics of the disk, which will make the situation much more complicated than that considered in this paper. The effect of photon capture caused by the black hole and the disk has also been ignored, which is expected to modify both the radiation flux and the total power of the disk. All these issues will be addressed in future.

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Fig. 1.— The ratio of the co-rotation radius to the marginally stable radius of a thin Keplerian disk, $r_c/r_{ms}$, as a function of the black hole spin, $s = a/M_H$. When $0 < s < 0.3594$, the co-rotation radius is given by equation (38), which is shown with the solid curve. Beyond the co-rotation radius the black hole rotates faster than the disk, within the co-rotation radius the black hole rotates slower than the disk. When $s > 0.3594$, the co-rotation radius does not exist: the black hole always rotates faster than the disk. [If the label of the horizontal axis is replaced with $\log s_c$ where $s_c$ is the critical spin of the black hole defined by equation (37), the label of the vertical axis is replaced with $\log (r_0/r_{ms})$ where $r_0$ is the radius where the magnetic field touches the disk, then the same solid curve gives the relation $s_c = s_c(r_0)$.]
Fig. 2.— The radiation flux of a thin Keplerian non-accretion disk coupled to a Kerr black hole with a magnetic field: the magnetic field touches the disk at a circle of radius $r = r_0 > r_{\text{ms}}$, the black hole rotates faster than the disk (i.e. $\Omega_H > \Omega_0$). The radius is in unit of $r_{\text{ms}}$—the radius of the marginally stable orbit, the radiation flux is in unit of $F_0 \equiv A_0/r_{\text{ms}}^2$, where $A_0$ is given by equation (34). The inner boundary of the disk is at $r = r_{\text{ms}}$, as indicated by the vertical dashed line. The radiation flux is shown with the solid curve. For $r < r_0$ (i.e. the left side of the vertical solid line), the radiation flux is zero. The radiation flux has a sharp peak at $r = r_0$, and decreases quickly for $r > r_0$. The radiation flux approaches $F \propto r^{-3.5}$ at $r \gg r_0$. 

\[ a = 0.95 \, M_\odot \]
\[ r_0 = 2 \, r_{\text{ms}} \]
Fig. 3.— The internal viscous torque of a thin Keplerian non-accretion disk coupled to a Kerr black hole with a magnetic field. The model is the same as that in Fig. 2: there is no accretion, the magnetic field touches the disk at a circle of radius $r = r_0 > r_{ms}$, and the black hole rotates faster than the disk. The radius is in unit of $r_{ms}$, the torque is in unit of $g_0 \equiv A_0 r_{ms}$. The inner boundary of the disk is at $r = r_{ms}$, as indicated with the vertical dashed line. The internal viscous torque is shown with the solid curve. For $r < r_0$ (i.e. the left side of the vertical solid line), the torque in the disk is zero. (Especially, the torque is zero at $r = r_{ms}$, which is true even for the case of $r_0$ approaching $r_{ms}$.) The torque rises suddenly at $r = r_0$, then decreases and approaches a constant at $r \gg r_0$. 

\[ a = 0.95 \, M_{\text{BH}} \]
\[ r_0 = 2 \, r_{ms} \]
Fig. 4.— Comparison of a non-accretion disk magnetically coupled to a rapidly rotating black hole with a standard accretion disk. The horizontal axis is the logarithm of the radius in the disk, which is in unit of $r_{ms}$ — the radius of the marginally stable orbit, which is assumed to be the inner edge of the disk. The vertical axis is the radiation flux of the disk, which is in unit of $F_0 \equiv A_0 / r_{ms}^2$. The solid curve is the radiation flux of a non-accretion disk magnetically coupled to a Kerr black hole of $a/M_H = 0.99$, where the magnetic field is assumed to touch the disk at the inner boundary. The dashed curve is the radiation flux of a standard accretion disk around a Kerr black hole of $a/M_H = 0.99$, where the accretion rate is assumed to be $\dot{M}_D = 200A_0$. 

\( a = 0.99 \, M_H \)

— Magnetic Coupling

— Accretion
Fig. 5.— The emissivity index: $\alpha \equiv -d \ln F/d \ln r$. The horizontal axis is the logarithm of the radius of the disk, which is in unit of $r_{ms}$. The solid curve is the emissivity index of a non-accretion disk magnetically coupled to a Kerr black hole of $a/M_H = 0.99$, where the magnetic field is assumed to touch the disk at the inner boundary. The dashed curve is the emissivity index of a standard accretion disk around a Kerr black hole of $a/M_H = 0.99$. Throughout the disk the emissivity index for magnetic coupling is significantly bigger than the emissivity index for accretion. At large radii, the emissivity index for magnetic coupling approaches 3.5, the emissivity index for accretion approaches 3.
Fig. 6.— The half-light radius of a disk, $r_{1/2}$, as a function of the black hole spin, $s = a/M_H$. The solid curve is the half-light radius of a non-accretion disk magnetically coupled to a Kerr black hole, where the magnetic field is assumed to touch the disk at the inner boundary (the marginally stable orbit). The solid curve starts at $s = 0.3594$ (marked by a thick dot), since energy is transferred from the black hole to the disk only if $s > 0.3594$. The dashed curve is the half-light radius of a standard accretion disk around a Kerr black hole, which exceeds the half-light radius of a non-accretion disk with magnetic coupling by approximately a half order of magnitude.
Fig. 7.— The fraction of energy that can be extracted from a Kerr black hole as a function of the radius where the magnetic field lines touch the disk. The disk’s inner boundary is at \( r = r_{m, s} \) – the marginally stable orbit as indicated by the vertical dashed line. There is no accretion, the magnetic field touches the disk at a circle with a radius \( r_0 > r_{m, s} \). The black hole is spun down by the magnetic coupling from \( s = 0.998 \), the spin of a canonical black hole, to \( s = s_c \), the spin at which extraction of energy and angular momentum from the black hole stops. During the evolution of the black hole spin the ratio \( r_0/r_{m, s} \) is assumed to keep unchanged.
Fig. 8.— The radiation flux of a thin Keplerian accretion disk coupled to a Kerr black hole with a magnetic field: the black hole rotates faster than the disk. The radius is in unit of $r_{m, s}$ – the radius of the marginally stable orbit. The radiation flux is in unit of $F_0 = A_0/r_{m, s}^2$. The magnetic field touches the disk at a circle of radius $r = r_0$ which is indicated with the vertical solid/dotted line. The solid curve shows the total radiation flux. For $r < r_0$ (i.e. the left side of the vertical solid/dashed line), the radiation flux is the same as that predicted by the standard theory of an accretion disk, whose extension beyond $r = r_0$ is shown with the dashed curve. The magnetic coupling produces a bright “bump” at $r = r_0$, as indicated by the sharp peak in the solid curve. [Parameters for the model: $a/M_H = 0.95$, $r_0/r_{m, s} = 2$, $\dot{M}_D = 80 A_0$.]
Fig. 9.— The radiation flux of a thin Keplerian accretion disk coupled to a Kerr black hole with a magnetic field: the black hole rotates slower than the disk. The radius is in unit of $r_{m,s}$ — the radius of the marginally stable orbit. The radiation flux is in unit of $F_0 \equiv A_0/r_{m,s}^2$. The magnetic field touches the disk at a circle with a radius $r = r_0$ which is indicated with the vertical solid/dotted line. The solid curve shows the total radiation flux. For $r < r_0$ (i.e. the left side of the vertical solid/dashed line), the radiation flux is the same as that predicted by the standard theory of an accretion disk, whose extension beyond $r = r_0$ is shown with the dashed curve. The magnetic coupling produces a dark “valley” at $r = r_0$, as indicated by the deep valley in the solid curve. [Parameters for the model: $a/M_H = 0$, $r_0/r_{m,s} = 2$, $\dot{M}_D = 320A_0$.]