We calculate the Gaussian radius parameters of the pion-emitting source in high energy heavy ion collisions, assuming a first order phase transition between a thermalized Quark-Gluon-Plasma (QGP) and a hadron gas. Such a model leads to a very long-lived dissipative hadronic rescattering phase which dominates the properties of the two-pion correlation functions. The radii are found to depend only weakly on the thermalization time $\tau_t$, the critical temperature $T_c$ (and thus the latent heat), and the specific entropy of the QGP. The dissipative hadronic stage enforces large variations of the pion emission times around the mean value. Therefore, the model calculations suggest a rapid increase of $R_{\text{out}}/R_{\text{side}}$ as a function of $K_T$ if a thermalized QGP were formed.

Bose-Einstein correlations in multiparticle production processes [1] provide valuable information on the space-time dynamics of fundamental interactions [2–4]. In particular, lattice QCD calculations predict the occurrence of a phase transition at high temperature, and it is hoped that correlations of identical pions produced in high energy collisions of heavy ions lead to a better understanding of the properties of that phase transition (for a review on QGP signatures, see [5]). A first order phase transition leads to a prolonged hadronization time as compared to a cross-over or ideal hadron gas with no phase transition, and has been related to unusually large Hanbury-Brown–Twiss (HBT) radii [6–9]. This phenomenon should then depend on the hadronization (critical) temperature $T_c$ and the latent heat of the transition. For recent reviews on this topic we refer to [10,11].

Here, we investigate if and how HBT radii, characterizing the pion source in the final state when all strong interactions are frozen, depend on the properties of the QGP and the hadronization temperature. The QGP is modeled as an ideal fluid undergoing hydrodynamic expansion with a Bag model equation of state [12], eventually hadronizing via a first order phase transition [9,13]. For simplicity, cylindrically symmetric transverse expansion and longitudinally boost-invariant scaling flow are assumed [9,13,14]. This approximation should be reasonable for central collisions at high energy, and around midrapidity. The model reproduces the measured $p_T$-spectra and rapidity densities of a variety of hadrons at $\sqrt{s} = 17.4A$ GeV (CERN-SPS energy), when assuming the standard thermalization (proper) time $\tau_t = 1$ fm/c, and an entropy per net baryon ratio of $s/\rho_B = 45$ [14–16].

Due to the higher density at midrapidity, thermalization may be faster at BNL-RHIC energies – here we assume $\tau_t = 0.6$ fm/c and $s/\rho_B = 200$. The energy density and baryon density are initially distributed in the transverse plane according to a so-called “wounded nucleon” distribution with transverse radius $R_T = 6$ fm. For further details, we refer to refs. [14–16].

We shall first discuss the radii of the $\pi^{-}\pi^{-}$ correlation functions at hadronization. From the hydrodynamical solution in the forward light cone we determine the hadronization hypersurface $\sigma^\mu$, which is essentially a surface of constant temperature $T = T_c = 0$ since the net baryon density is small after the central region has passed through the mixed phase. On that surface, the two-particle correlation function is given by [8,9]

$$C_2(\mathbf{p}_1, \mathbf{p}_2) = 1 + N\int d\sigma \cdot Ke^{i\sigma \cdot f}(\mathbf{u} \cdot K/T)^2$$

(1)

This assumes a chaotic (incoherent) and large source. $f$ denotes a Bose distribution function. The normalization factor $N$ is given by the product of the invariant single-inclusive distribution of $\pi^{-}$ evaluated at momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, respectively. $u^\mu$ denotes the four-velocity of the fluid on the hadronization surface $\sigma^\mu$, and $K^\mu = (p_1^\mu + p_2^\mu)/2$, $q^\mu = p_1^\mu - p_2^\mu$ are the average four-momentum and the relative four-momentum of the pion pair, respectively. For midrapidity pions $K_{||} = 0$. Note that eq. (1) accounts for the direct pions only but not for decays of hadronic resonances (like $p^0 \to \pi^+\pi^-$ etc.), which are known to affect the correlation functions [8,10,11,17] and which will be included below.

One usually employs a coordinate system in which the long axis ($z$) is chosen parallel to the beam axis, and where transverse space is spanned by the out ($x$) and side ($y$) axes. The out direction is defined to be parallel to the transverse momentum vector $\mathbf{K}_{\text{T}} = (\mathbf{p}_{1\text{T}}+\mathbf{p}_{2\text{T}})/2$ of the pair, and accordingly the side direction is perpendicular to the out direction.

From eq.(1), the inverse widths of the correlation function are obtained as $R_{\text{out}} = \sqrt{\ln 2}/q_{\text{out}}^*$ and $R_{\text{side}} = \sqrt{\ln 2}/q_{\text{side}}^*$ where $q_{\text{out}}^*$, $q_{\text{side}}^*$ are defined by $C_2(q_{\text{out}}^*, q_{\text{side}}^*) = C_2(q_{\text{side}}^*, q_{\text{out}}^*) = 1.5$. It has been suggested that the ratio $R_{\text{out}}/R_{\text{side}}$ should increase strongly once the initial entropy density $s_i$ becomes substantially larger than that of the hadronic gas at $T_c$ [9]. Indeed, Fig. 1 shows that $R_{\text{out}}/R_{\text{side}}$ is much larger if...
$T_c$ is low, such that entropy conservation dictates a long hadronization time. The closer $T_c$ is to the initial temperature $T_i (\approx 300 \text{ MeV}$ for the BNL-RHIC initial conditions), the faster $T_c$ is reached from above. For 1+1 dimensional isentropic scaling expansion the time to complete hadronization is given by $\tau_H/\tau_i = s_i/s_H(T_c)$, where $s_H(T_c)$ is the entropy density of the hadronic phase at $T_c$ (for simplicity of the argument, we disregard the finite net baryon density here). Of course, $s_H(T_c)$ increases with $T_c$ and so the hadronization time decreases. As $R_{\text{out}}$ is proportional to the duration of pion emission [6], it must therefore decrease as $T_c$ increases.

Also, the decrease of $R_{\text{out}}/R_{\text{side}}$ towards large transverse momentum $K_T$ is faster for low $T_c$, as transverse collective flow has more time to develop during the lifetime of the QGP.

We now proceed to include hadronic rescattering following hadronization. To describe the evolution of the hadrons towards freeze-out obviously requires to go beyond perfect-fluid dynamics. Here, we employ a semi-classical transport model that treats each particular hadronic reaction channel (formation and decay of hadronic resonance states and $2 \rightarrow n$ scattering) explicitly [18,19]. The transition at hadronization is performed by matching (on average over many events) the energy-momentum tensors and conserved currents of the hydrodynamic solution and of the microscopic transport model, respectively (for details, see [16]). The microscopic model propagates each individual hadron along a classical trajectory, and performs $2 \rightarrow n$ and $1 \rightarrow m$ processes stochastically. E.g., the total meson-meson cross section includes a 5 mb elastic contribution as well as resonance excitation, which dominates the cross section, with the total and partial $\sqrt{s}$-dependent decay widths $\Gamma_{\text{tot}}$ and $\Gamma_{R \rightarrow MM'}$. (Here, $\sqrt{s}$ refers to the cm-energy of the hadron-hadron scattering.) The full decay width $\Gamma_{\text{tot}}(m)$ of a resonance is defined as the sum of all partial decay widths and depends on the mass of the excited resonance. The sum extends over all resonance states which have a decay channel into $MM'$. The pole masses and partial decay widths are taken from the Review of Particle Properties [20]. The pion-pion cross section for example is dominated by the $\rho(770)$ resonance, with additional contributions from higher energy states such as $f_0(780)$. $\pi K$ scattering is either elastic or proceeds through formation of mainly a $K^*(892)$ resonance. (Meson-baryon elastic and resonant scattering is also taken into account, but proves to be less important in the present context.) In this way, a good description of elastic and total pion cross sections in vacuum is obtained [19]. Collective medium-induced effects on the pion scattering, see e.g. [21], are neglected at present because in a thermalized state at $T = T_c$ there is less than one $\pi^{-}$ per phase space cell $d^3x d^3p/(2\pi)^3$.

The distribution of freeze-out points of pions in the forward light-cone is rather broad in time [22,23]. Freeze-out occurs in a four-dimensional region within the forward light-cone rather than on a three-dimensional “hypersurface”. The single-particle distributions at hadronization are not altered very much during the dissipative hadronic phase because hard collisions are rare [15,16]. (In other words, the pressure $p$ is small and moreover $-pdV$ mechanical work is largely compensated by $+TdS$ entropy production.) There are, however, numerous soft collisions [22], characterizing the dissipative evolution that approaches freeze out. That means that the hadronic system, particularly when starting from a state of local equilibrium at hadronization, disintegrates rather slowly rather than emitting a “flash” of pions in an instantaneous decay. This is fundamentally different from the “explosive” hadron production from the decay of a classical background field at the confinement transition via parametric resonance [24].

The solution of the microscopic transport provides the classical phase space distribution of the hadrons at the points of their last (strong) interaction. Bose-Einstein correlations are introduced a posteriori by identifying the phase space distribution at freeze-out with the Wigner density of the source [11,17,25], $S(x, K)$. Corrections arise if the pions undergo a stage of “cascading” from the space-time point of their production to the point of their last interaction [26]. However, from the above mentioned model for pion rescattering we find that only $\sim 15 - 20\%$ of the pions in the final state freeze out after an elastic scattering. Rather, most pions emerge from the fragmentation of a hadronic resonance, or are emitted directly from the hadronization hypersurface. Thus the relative phases of the pions at freeze-out can be considered to a good approximation to be random. A more detailed discussion of the relative contributions will be
given elsewhere [27].

We focus here on the so-called Gaussian radius parameters, which are obtained from a saddle-point integration over $S(x,K)$ [11,28]. We have also computed the complete two-pion correlation functions without relying on the approximate saddle-point integration, but have found this simplified treatment sufficient to illustrate the essence of our findings. The correlation functions themselves, the value of the intercept, and the effects of introducing finite momentum resolutions (as in the experimental analysis) will be discussed in [27].

The HBT-radius parameters characterizing the Gaussian ansatz

$$R^2_{\text{side}}(K_T) = \langle \hat{y}^2 \rangle(K_T),$$

$$R^2_{\text{out}}(K_T) = \langle (\hat{x} - \beta \hat{t})^2 \rangle(K_T),$$

$$R^2_{\text{long}}(K_T) = \langle (\hat{z} - \beta \hat{t})^2 \rangle(K_T),$$

with the space-time coordinates $\hat{y} = x \cdot \Gamma(K_T)$ relative to the momentum dependent effective source centers $\Gamma(K_T) = (x^\mu(K_T))$. The average in (3)-(5) is taken over the emission function, i.e. $\langle f(K) \rangle = \int d^4x f(x)S(x,K)/\int d^4x S(x,K)$. Due to the given azimuthal symmetry of the source the emission function is symmetric under a sign flip in the side direction, hence $y = 0$. Since we investigate a symmetric rapidity interval, $y = 0$ as well. In the osl system $\beta = (\beta_x,0,\beta_z)$, where $\beta = K/E_K$ and $E_K = \sqrt{m^2 + K^2}$.

![FIG. 2. HBT-radii $R_{\text{out}}$, $R_{\text{side}}$, $R_{\text{long}}$, and emission duration $\Delta \tau$ at freeze-out as a function of $K_T$, for central collisions at RHIC (above) and SPS (below) and $T_c \simeq 160$ MeV (left) and $T_c \simeq 200$ MeV (right).](image)

Fig. 2 shows the HBT-radius parameters and the duration of emission, $\Delta \tau = \sqrt{\langle t^2 \rangle}$ as obtained from eqs. (3)-(5) for the two different sets of initial conditions, corresponding to CERN-SPS and BNL-RHIC. In addition, we have varied the bag parameter $B$ from 380 MeV/fm$^3$ to 720 MeV/fm$^3$, corresponding to critical temperatures of $T_c \simeq 160$ MeV and $T_c \simeq 200$ MeV (at vanishing net baryon density), respectively. Note that within the Bag model this automatically corresponds to a large variation of the latent heat $4B$ as well. While a latent heat of about 3 GeV/fm$^3$ is much larger than that expected for QCD, it only serves the purpose of proving the (in)sensitivity of the HBT radii to variations of the phase transition parameters in this model. We also note that changing $T_c$ implies variation of the longitudinal and transverse flow profile on the hadronization hypersurface (which is the initial condition for the subsequent hadronic rescattering stage) over a broad range.

It is obvious from Fig. 2 that the HBT-radii are very similar for all cases considered. They depend only weakly on the specific entropy, on the critical temperature and latent heat for the transition, on the thermalization time $\tau_l$, or on the initial condition for the hadronic rescattering stage. Thus, the properties of the QGP-phase are not directly reflected in the HBT-radii, which are essentially determined by the large space-time volume of the hadronic rescattering stage. In the presence of a viscous hadronic stage, with soft hadronic scattering occurring over a long time-span after hadronization [22], the pion mean free path increases gradually towards freeze-out [29] and pions are emitted over a broad time interval from the entire volume. Hence, the RMS deviation of the emission time is large, see Fig. 2.

![FIG. 3. $R_{\text{out}}/R_{\text{side}}$ for RHIC initial conditions, as a function of $K_T$ at freeze-out (symbols) and at hadronization (lines).](image)

In Fig. 3 we compare $R_{\text{out}}/R_{\text{side}}$ at hadronization to that at freeze out. Clearly, up to $K_T \sim 200$ MeV/c $R_{\text{out}}/R_{\text{side}}$ is independent of $T_c$, or $s_H(T_c)$, if hadronic rescatterings are taken into account. Moreover, at higher $K_T$ the dependence on $T_c$ is even reversed: for high $T_c$ the $R_{\text{out}}/R_{\text{side}}$ ratio even exceeds that for low $T_c$. Higher $T_c$ speeds up hadronization but on the other hand prolongs the dissipative hadronic phase that dominates the HBT radii.

Experimental data [30] for central Pb+Pb collisions at $\sqrt{s} = 17.4$A GeV indicate smaller HBT radii than seen in Fig. 2, and $R_{\text{out}}/R_{\text{side}} \simeq 1.2$ up to $K_T \simeq 400$ MeV/c. The experimental centrality trigger can only roughly be
attributed to an impact parameter ($b < 3$ fm). The small but finite impact parameter leads to a smaller reaction volume, thus decreasing the radii if compared to ideal central collisions ($b = 0$ fm) as assumed in the present calculation. Moreover, experimental momentum resolutions or fit procedures could in effect reduce the extracted radii – these rather technical issues will be investigated in a detailed forthcoming study [27]. Finally, higher energies than at CERN-SPS may actually be required to produce a QGP state. For central collisions of Au nuclei at $\sqrt{s} = 130$A GeV, preliminary data yield $R_{out}/R_{side} \simeq 1.1$ at small $K_T$ [31]. Results from RHIC at higher $K_T$ will soon test whether a long-lived hadronic soft-re-scattering stage, associated with the formation and hadronization of an equilibrated QGP state is indeed formed in heavy ion collisions at the highest presently attainable energies.

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