Split Multiplets, Coupling Unification
and Extra Dimension

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Abstract

We study a gauge coupling unification scenario based on a non-supersymmetric 5-dimensional model. Through an orbifold compactification, we obtain the Standard Model with split multiplets on a 4-dimensional wall, which is compatible with a grand unification.
The Standard Model (SM) has been established as an effective theory below the weak scale. One of intriguing trials beyond the SM is to unify gauge interactions under a simple group such as $SU(5)$.[1] This scenario is very attractive,[2] but it suffers from several problems in the simplest version. First problem is that gauge coupling constants do not meet at a high-energy scale based on the desert hypothesis.[3] Second problem concerns that a dangerous proton decay is induced by an exchange of $X$ and $Y$ gauge bosons.[4] Last problem regards that the weak scale is not stabilized by quantum corrections (the gauge hierarchy problem).[5]

The introduction of supersymmetry (SUSY) solves the first [6] and the third problems.[7] A supersymmetric grand unified theory (SUSY GUT) is an attractive candidate of a high-energy theory,[8] but the proton stability is threatened due to a contribution from the dimension 5 operator in the minimal SUSY $SU(5)$ GUT.[9, 10] Recently stronger constraints have been obtained from the analysis including a Higgsino dressing diagram with right-handed matter fields.[11]

Recently a new possibility[12] has been proposed to solve the above problems. Starting from 5-dimensional (5D) SUSY $SU(5)$ model, we have obtained a low-energy theory with particles of the minimal supersymmetric standard model (MSSM) on a 4D wall through compactification upon $S^1/(Z_2 \times Z'_2)$. The proton stability is guaranteed due to a suppression factor in the coupling to the Kaluza-Klein modes if our 4D wall fluctuates flexibly.

In this letter, we propose another possibility to solve the first and second problems based on a 5D model without SUSY. The gauge coupling unification is realized by the introduction of extra multiplets which split after an orbifold compactification.† The splitting originates from a non-universal parity assignment on a compact space among components in each multiplet. The proton decay is suppressed enough with a suppression factor in the coupling to the Kaluza-Klein excitations if our 4D wall fluctuates pliantly. In the following, we will derive a model discussed in Ref.[15] as an example of a low-energy theory from 5D $SU(5)$ model through compactification upon $S^1/(Z_2 \times Z'_2)$.

*The gauge hierarchy problem is solved partially in a sense that the hierarchy is stabilized against radiative corrections perturbatively due to non-renormalization theorem, although the origin at the tree level is not understood.

†There are several works with respect to $SU(5)$ grand unification under an assumption that there are split multiplets.[13, 14, 15]
The space-time is assumed to be factorized into a product of 4D Minkowski space-time $M^4$ and the orbifold $S^1/(Z_2 \times Z_2')$ whose coordinates are denoted by $x^\mu$ ($\mu = 0, 1, 2, 3$) and $y(=x^5)$, respectively. The 5D notation $x^M$ $(M = 0, 1, 2, 3, 5)$ is also used. The orbifold is regarded as an interval with a distance of $\pi R/2$. There are two 4D walls placed at fixed points $y = -\pi R/2$ and $y = 0$ (or $y' = 0$ and $y' = \pi R/2$) on $S^1/(Z_2 \times Z_2')$ where $y' \equiv y + \pi R/2$.

We assume that the 5D gauge boson $A_M(x^\mu, y)$ and four kinds of scalar fields $\Phi_R(x^\mu, y)$ ($R = 5, 5', 10, \bar{10}$) exist in the bulk $M^4 \times S^1/(Z_2 \times Z_2')$. The fields $A_M$ and $\Phi_R$ form an adjoint representation $24$ and a representation $R$ of $SU(5)$, respectively. We assume that our visible world is one of 4D walls (We choose the wall fixed at $y = 0$ as the visible one and call it wall I) and that three families of quarks and leptons, $3\{\psi^5 + \psi_{10}\}$, are located on wall $I$. (Here and hereafter we suppress the family index.) The representations of $\psi^5$ and $\psi_{10}$ are $5$ and $10$ of $SU(5)$, respectively. Note that matter fields contain no excited states along the $S^1/(Z_2 \times Z_2')$ direction.

The gauge invariant action is given by

$$
S = \int d^5x \left( -\frac{1}{2} tr F_{MN}^2 + \sum_R |D_M \Phi_R|^2 - V(\Phi_R) \right) 
+ \int d^4x \sum_{3\text{families}} (i \bar{\psi}_{10} \gamma^\mu D_\mu \psi_{10} + i \bar{\psi}_{\bar{5}} \gamma^\mu D_\mu \psi_{\bar{5}} 
+ f_{U(5)} \Phi_5 \bar{\psi}_{10} \psi_{10} + f_{D(5)} \Phi_{\bar{5}} \bar{\psi}_{\bar{5}} \psi_{\bar{5}} + f_{Q(5)} \Phi_{10} \bar{\psi}_{\bar{5}} \psi_{\bar{5}} + \text{h.c.}) 
$$

where $D_M \equiv \partial_M - i g(5) A_M(x^\mu, y)$, $g(5)$ is a 5D gauge coupling constant, and $f_{U(5)}, f_{D(5)}$ and $f_{Q(5)}$ are 5D Yukawa coupling matrices. In 4D action, the bulk fields $A_M$ and $\Phi_R$ are replaced by fields including the Nambu-Goldstone boson $\phi(x^\mu)$ at wall I such that $A_M(x^\mu, \phi(x^\mu))$ and $\Phi_R(x^\mu, \phi(x^\mu))$, respectively. The Lagrangian is invariant under the $Z_2 \times Z_2'$ transformation

$$
A_\mu(x^\mu, y) \rightarrow A_\mu(x^\mu, -y) = PA_\mu(x^\mu, y)P^{-1},
A_5(x^\mu, y) \rightarrow A_5(x^\mu, -y) = -PA_5(x^\mu, y)P^{-1},
$$

\[\text{Recently, Barbieri, Hall and Nomura have constructed a constrained standard model upon a compactification of 5D SUSY model on the orbifold } S^1/(Z_2 \times Z_2'). \text{[16]} \text{ They have used a } Z_2 \times Z_2' \text{ parity to reduce SUSY. There had been several works on the model building through a reduction of SUSY [17, 18, 19, 20] or a gauge symmetry [21] by the use of a } Z_2 \text{ parity. Attempts to construct GUT have been made through the dimensional reduction over coset space.}[22]\]
\[ \Phi_R(x^\mu, y) \rightarrow \Phi_R(x^\mu, -y) = P\Phi_R(x^\mu, y), \quad (R = 5, S) \]
\[ \Phi_R(x^\mu, y) \rightarrow \Phi_R(x^\mu, -y) = P\Phi_R(x^\mu, y)P^{-1}, \quad (R = 10, \overline{10}) \]  
(2)

where \( P \) and \( P' \) are 5 \times 5 matrices which satisfy \( P^2 = P'^2 = I \) (\( I \) is the unit matrix). Here \( A_M \) and \( \Phi_{10, \overline{10}} \) are expressed by 5 \times 5 symmetric and anti-symmetric matrices, respectively. The intrinsic \( Z_2 \times Z'_2 \) parity of each component is given by an eigenvalue of \( P \) and \( P' \).

When we take \( P = \text{diag}(1, 1, 1, 1, 1) \) and \( P' = \text{diag}(-1, -1, -1, 1, 1) \), the \( SU(5) \) gauge symmetry is reduced to that of the SM, \( G_{\text{SM}} \equiv SU(3) \times SU(2) \times U(1) \), in 4D theory. This is because the boundary conditions on \( S^1/(Z_2 \times Z'_2) \) given in (3) do not respect \( SU(5) \) symmetry, as we see from the relations for the gauge generators \( T^A \ (A = 1, 2, \ldots, 24) \),
\[ P'T^a P'^{-1} = T^a , \quad P'T^\bar{a} P'^{-1} = -T^{\bar{a}}. \]  
(4)

The \( T^a \)'s are gauge generators of \( G_{\text{SM}} \) and the \( T^{\bar{a}} \)'s are other gauge generators. The parity assignment and mass spectrum after compactification are given in Table 1. The scalar fields \( \Phi_R \) are broken up into several pieces such that
\[ \Phi_5 = \phi_C + \phi_W, \quad \Phi_S = \overline{\phi_C} + \overline{\phi_W}, \]
\[ \Phi_{10} = Q + \overline{U} + E, \quad \Phi_{\overline{10}} = \overline{Q} + U + E. \]  
(5)

In the second column, we give \( SU(3) \times SU(2) \) quantum numbers of 4D fields. In the third column, \((\pm, \pm)\) and \((\pm, \mp)\) denote eigenvalues \((\pm 1, \pm 1)\) and \((\pm 1, \mp 1)\) of \( Z_2 \times Z'_2 \) parity, respectively. The fields \( \phi_{\pm\pm}(x^\mu, y) \) and \( \phi_{\pm\mp}(x^\mu, y) \), whose values of intrinsic parity are \((\pm 1, \pm 1)\) and \((\pm 1, \mp 1)\), are Fourier expanded as
\[ \phi_{\pm\pm}(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{++}^{(2n)}(x^\mu) \cos \frac{2ny}{R}, \]  
(6)

\(^5\)The exchange of \( P \) for \( P' \) is equivalent to the exchange of two walls.

\(^6\)Our symmetry reduction mechanism is different from the Hosotani mechanism. In fact, the Hosotani mechanism does not work in our case, because \( A_5^\mu(x^\mu, y) \) has odd parity, as given in (2), and its VEV should vanish.
\[
\phi_{++}(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{++}^{(2n+1)}(x^\mu) \cos \left(\frac{(2n+1)y}{R}\right),
\]
(7)
\[
\phi_{--}(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{--}^{(2n+2)}(x^\mu) \sin \left(\frac{(2n+2)y}{R}\right),
\]
(9)

where \(n\) is zero or a positive integer, and each field \(\phi_{++}^{(2n)}(x^\mu)\), \(\phi_{++}^{(2n+1)}(x^\mu)\) and \(\phi_{--}^{(2n+2)}(x^\mu)\) acquire a mass \(\frac{2n}{R}, \frac{2n+1}{R}\) and \(\frac{2n+2}{R}\) upon compactification.

Note that 4D massless fields appear only from components with even parity (+1, +1). The contribution from the potential \(V(\Phi_R)\) is not considered in the fourth column. In the low-energy spectrum, there are a pair of lepto-quark bosons \(Q^{(0)}_0\) and \(\overline{Q}^{(0)}_0\), which have both color and weak charge. The SM gauge bosons and the weak Higgs doublet are equivalent to \(A^{(0)}_\mu\) and \(\phi^{(0)}_W\) (or \(\overline{\phi}^{(0)}_W\)), respectively. The mass split of bosons is realized by the \(Z_2 \times Z'_2\) projection.

After integrating out the fifth dimension, we obtain the 4D Lagrangian density
\[
L_{\text{eff}}^{(4)} = -\frac{1}{4} \sum_a F_{\mu\nu}^{a(0)} F^{a(0) \mu\nu} + |D_\mu \phi^{(0)}_W|^2 + |D_\mu \overline{\phi}^{(0)}_W|^2
+ |D_\mu Q^{(0)}|^2 + |D_\mu \overline{Q}^{(0)}|^2 - V(\phi_W^{(0)}, \overline{\phi}_W^{(0)}; Q^{(0)}, \overline{Q}^{(0)})
+ \sum_{3\text{families}} (i \overline{\psi}_1 \gamma^\mu D_\mu \psi_1 + i \overline{\psi}_5 \gamma^\mu D_\mu \psi_5)
+ f_U \phi^{(0)}_W q\overline{u} + f_D \phi^{(0)}_W \overline{q}d + f_D \phi^{(0)}_W \overline{l}e + f_Q Q\overline{Q} + \text{h.c.}) + \cdots,
\]
(10)

where \(D_\mu \equiv \partial_\mu - ig_U A_\mu^{(0)}\), the dots represent terms including Kaluza-Klein modes, \(g_U \equiv \sqrt{2/\pi R g_{U(5)}}\) is a 4D gauge coupling constant, \(f_V \equiv \sqrt{2/\pi R f_{V(5)}}\), \(f_D \equiv \sqrt{2/\pi R f_{D(5)}}\) and \(f_Q \equiv \sqrt{2/\pi R f_{Q(5)}}\) are 4D Yukawa coupling matrices, and \(q, \overline{u}\) and \(\overline{d}\) are quarks, \(l\) and \(\overline{e}\) are leptons. With our parity assignment, we have obtained an extension of the SM with two Higgs doublets \(\phi^{(0)}_W\) and \(\overline{\phi}^{(0)}_W\) and extra lepto-quark bosons \(Q^{(0)}\) and \(\overline{Q}^{(0)}\).

The theory predicts that coupling constants are unified around the compactification scale \(M_C \equiv 1/R\), as in the ordinary \(SU(5)\) GUT,[1]
\[
g_3 = g_2 = g_1 = g_U, \quad f_d = f_e = f_D.
\]
(11)
where $f_d$ and $f_e$ are Yukawa coupling matrices on down-type quarks and electron-type leptons, respectively. As shown in Ref.[15], this type of extension of the SM can survive with the precision measurements at LEP.[3]

It is known that there is a significant contribution to the proton decay due to the $X$ and $Y$ gauge bosons in the minimal $SU(5)$ GUT.[4] In our model, we have similar diagrams as those in the minimal $SU(5)$ GUT because quark and lepton couple to the Kaluza-Klein modes of extra vector bosons at the tree level. However we expect that the proton stability guarantees if our 4D wall fluctuates flexibly. This is due to the fact that there is an exponential suppression factor in the coupling to the Kaluza-Klein excitations by the brane recoil effect.[24]

We have obtained the simplest extension of the SM compatible with $SU(5)$ grand unification. It would be possible to construct more complex models by increasing a number of extra multiplets. For reference, the pattern of split due to $Z'_2$ parity is given in Table 2 for several low dimensional representations of $SU(5)$. In the second column, we give $SU(3) \times SU(2)$ quantum numbers of split multiplets. In the third column, $P_R$ is an eigenvalue of $Z'_2$ parity, i.e., $P_R = 1$ or $-1$. The table includes components which can induce a rapid nucleon decay when they couple to quarks and leptons.

Our grand unification scenario is phenomenologically interesting because it suggests the existence of extra split multiplets at the weak scale. However there is a problem of how to break the electro-weak symmetry naturally and how to stabilize the weak scale, that is, our model suffers from naturalness problem.[25] There would be an alternative that the extra space has a large radius.[26] In this case, the low-energy gauge coupling unification is expected to be realized by a power-law correction.[27]

References


Table 1: Parity and Mass spectrum.

<table>
<thead>
<tr>
<th>4D fields</th>
<th>Quantum numbers</th>
<th>$Z_2 \times Z_2'$ parity</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A^{(2n)}_{\mu}$</td>
<td>$(8, 1) + (1, 3) + (1, 1)$</td>
<td>$(+, +)$</td>
<td>$\frac{2n}{R}$</td>
</tr>
<tr>
<td>$A^{(2n+1)}_{\mu}$</td>
<td>$(3, 2) + (\overline{3}, 2)$</td>
<td>$(+,-)$</td>
<td>$\frac{R}{R}$</td>
</tr>
<tr>
<td>$A^{(2n+2)}_{5}$</td>
<td>$(8, 1) + (1, 3) + (1, 1)$</td>
<td>$(-,-)$</td>
<td>$\frac{2n + 2}{2n + 1}$</td>
</tr>
<tr>
<td>$A^{(2n+1)}_{5}$</td>
<td>$(3, 2) + (\overline{3}, 2)$</td>
<td>$(-,+)$</td>
<td>$\frac{R}{R}$</td>
</tr>
<tr>
<td>$\phi^{(2n+1)}_{C}$</td>
<td>$(3, 1)$</td>
<td>$(+,-)$</td>
<td>$\frac{2n + 1}{R}$</td>
</tr>
<tr>
<td>$\phi^{(2n)}_{W}$</td>
<td>$(1, 2)$</td>
<td>$(+,+)$</td>
<td>$\frac{R}{R}$</td>
</tr>
<tr>
<td>$\phi^{(2n+1)}_{C}$</td>
<td>$(3, 1)$</td>
<td>$(+,-)$</td>
<td>$\frac{2n + 1}{R}$</td>
</tr>
<tr>
<td>$\phi^{(2n)}_{W}$</td>
<td>$(1, 2)$</td>
<td>$(+,+)$</td>
<td>$\frac{R}{R}$</td>
</tr>
<tr>
<td>$Q^{(2n)}$</td>
<td>$(3, 2)$</td>
<td>$(+,+)$</td>
<td>$\frac{R}{2n + 1}$</td>
</tr>
<tr>
<td>$\overline{U}^{(2n+1)}$</td>
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<td>$(+,-)$</td>
<td>$\frac{2n}{2n + 1}$</td>
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<tr>
<td>$\overline{E}^{(2n+1)}$</td>
<td>$(1, 1)$</td>
<td>$(+,-)$</td>
<td>$\frac{R}{R}$</td>
</tr>
<tr>
<td>$\overline{Q}^{(2n)}$</td>
<td>$(\overline{3}, 2)$</td>
<td>$(+,+)$</td>
<td>$\frac{2n}{R}$</td>
</tr>
<tr>
<td>$U^{(2n+1)}$</td>
<td>$(3, 1)$</td>
<td>$(+,-)$</td>
<td>$\frac{R}{2n + 1}$</td>
</tr>
<tr>
<td>$E^{(2n+1)}$</td>
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<td>$(+,-)$</td>
<td>$\frac{R}{R}$</td>
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</table>
Table 2: Split due to $Z'_2$ parity.

<table>
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<tr>
<th>$R$</th>
<th>Quantum numbers</th>
<th>$Z'_2$ parity</th>
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<tr>
<td>5</td>
<td>(3, 1)</td>
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<td>10</td>
<td>(3, 2)</td>
<td>$P_{10}$</td>
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<tr>
<td>15</td>
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<tr>
<td>75</td>
<td>(6, 2) + (6, 2) + (3, 2) + (3, 2)</td>
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<tr>
<td></td>
<td>(3, 1) + (3, 1) + (1, 1) + (8, 3) + (8, 1)</td>
<td>$-P_{75}$</td>
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