A uniformly accelerated system will get thermally excited due to interactions with the vacuum fluctuations of the quantum fields. This is the Unruh effect. Also a system accelerated in a circular orbit will be heated, but in this case complications arise relative to the linear case. An interesting question is in what sense the real quantum effects for orbital and spin motion of a circulating electron can be viewed as a demonstration of the Unruh effect. This question has been studied and debated. I review some of the basic points concerning the relation to the Unruh effect, and in particular look at how the electron can be viewed as a thermometer or detector that probes thermal and other properties of the vacuum state in the accelerated frame.

1 Introduction

A uniformly accelerated observer will – at least in theory – see Minkowski vacuum as a thermally excited state, with a temperature determined by the acceleration. Thus, in the accelerated frame a natural definition of the vacuum state is the Rindler vacuum, which is different from the Minkowski vacuum. When expressed in terms of the Rindler field quanta, Minkowski space is thermally excited. A quantum system that is uniformly accelerated will act as a detector or thermometer that probes the temperature of the Minkowski vacuum state in the accelerated Rindler frame. When coupled weakly to the radiation field, it will end up in a stationary state with a thermal probability distribution over energy levels.

There is a close relation between this effect – the Unruh effect – and the Hawking effect, i.e. the effect that a black hole emits thermal radiation. The accelerated observer in Minkowski space then corresponds to a stationary observer with a fixed distance from the event horizon of the black hole. This observer will detect the local effects of the Hawking temperature.

An interesting question is whether the Unruh effect can be seen in any real experiment. There have been several suggestions, but there are obvious problems with the implementation of such experiments. A main problem is
that the acceleration has to be extremely high to give even a very modest
temperature. Thus, a temperature of $1K$ corresponds to an acceleration of
$2.5 \cdot 10^{20} m/s^2$. A detector indeed has to be very robust to take such an
acceleration without being destroyed.

Some years ago it was suggested by John Bell and myself that that an
electron could be viewed as such a detector\footnote{5}. It certainly is robust at this level
of acceleration and in an external magnetic field the measured occupation of the
spin energy levels would give a way to determine the temperature. However, for
linearly accelerated electrons the obtainable temperature and the time scale for
reaching equilibrium, for realistic values of the accelerating fields, makes it clear
that such effects cannot not be seen in existing particle accelerators. Recently
there have been interesting suggestions of how to obtain much more violent
acceleration of electrons by use of laser techniques\footnote{6,7}, but even in this case the
time scale for spin excitations is too long. (There are however suggestions of
other ways to see the Unruh effect in these cases\footnote{6}.)

For electrons circulating in a storage ring the situation is different. The
acceleration is larger than in linear accelerators and the time available is suffi-
cient to reach equilibrium. In the discussion with John Bell the question came
up: Could the the Sokolov-Ternov effect\footnote{8,9,10}, which predicts a equilibrium
polarization lower than 100% be related to the Unruh effect in the sense that
the upper spin energy level, measured in the rest frame, is partly occupied due
to the heating? In two papers this question was studied an the answer was a
qualified yes\footnote{5,11}. (See also several later papers, some of these included in the
proceedings from the ICFA meeting in Monterey two years ago\footnote{12,13,14}.) For
circular motion there are however important complications relative to the case
of linear acceleration. One point is that the simple connection to temperature
is correct only for linear acceleration. For circular motion the excitations may
be described in terms of an effective temperature, but this temperature is not
uniquely determined by the acceleration as in the linear case. However, as a
more important point, the electron does not act as a simple point detector.
The spin is also affected by oscillations in the orbit, and the Thomas precession
makes the direct coupling to the magnetic field act differently from the indirect
coupling mediated through oscillations in the particle orbit. The net effect is
that the correct expression for the electron polarization will deviate from one
derived by a naive application of the Unruh temperature formula.

In this talk I will give a brief review of the connection between the Unruh
and the Sokolov-Ternov effect and I will stress some points which I find inter-
esting concerning this connection. First I will review how the Unruh effect, in
principle, could be demonstrated as a spin effect for linearly accelerated elec-
trons. Then I will discuss the relation between linear acceleration and circular
motion and finally consider the spin effect for circulating electrons. I should stress that all the basic elements in the discussion of the Unruh effect for electrons in a storage ring have been presented in the papers referred to above. However, the discussion of the spin effect I will do in a slightly different way by treating the spin and orbital motion on equal footing.

2 Linear acceleration and the Unruh effect

A pointlike object which is uniformly accelerated, i.e. which has a constant acceleration as measured in the instantaneous inertial rest frame, describes a hyperbolic path in space time. We may associate a co-moving frame with the motion, with the time like unit vector as tangent vector of the trajectory and the three space like vectors spanning the hyperplane of simultaneity as defined by the moving object. The directions of the three (orthogonal) space like vectors are determined only up to a rotation, but if a non-rotational frame is chosen (Fermi-Walker transported) this degree of freedom is eliminated. The non-rotational frame is also a stationary frame in the sense that the acceleration is fixed with respect to the unit vectors.

The local frame can be extended in a natural way to an accelerated coordinate system, the Rindler coordinate system. This is a stationary coordinate system in the sense that the metric is independent of the time coordinate. The transformation from Rindler coordinates \((x', \tau)\) to Cartesian coordinates \((x, t)\) is, with the \(x\)-axis chosen as the direction of acceleration,

\[
x = (x' + \frac{c^2}{a}) \cosh \left( \frac{a \tau}{c} \right), \quad t = \left( \frac{x'}{c} + \frac{c}{a} \right) \sinh \left( \frac{a \tau}{c} \right), \quad y = y', \quad z = z'
\]  

The Rindler time \(\tau\) is the proper time of the trajectory \(x' = 0\) \((y' = z' = 0)\), which is assumed to be the trajectory of the pointlike object. However any trajectory \(x' = \text{const}\) is equivalent to this in the sense of having a constant proper acceleration, although with a value of the acceleration that depends on the \(x'\)-coordinate.

The coordinate system \((x', \tau)\) is well behaved only in a part of space-time. At finite distance from the object (here chosen as the origin) there is a coordinate singularity. The two hyperplanes \(x = \pm ct\) which intersect there define event horizons for the accelerated object. For points with \(x < ct\) (behind the future horizon) an emitted light signal will not reach the object at any future time. A light signal emitted from the object at any past time will not be able to reach the points with \(x < -ct\) (behind the past horizon).

The Rindler coordinate system is similar to the Schwarzschild coordinate system of an eternal black hole. In fact the Rindler coordinate system can be
seen as a limit case of black hole geometry, for points at finite distance from the horizon with the mass of the black hole tending to infinity. In this limit the space time curvature vanishes and only the effect of acceleration (due to gravitation) remains.

The Rindler coordinate system can be regarded as constructed from a continuous sequence of inertial frames, the instantaneous rest frames of the accelerated object. This implies that the time evolution of a system, described in the accelerated coordinate system, can be expressed in terms of the generators of the Poincaré group, which transform between inertial frames for infinitesimal different values of $\tau$. For linear acceleration only the time translation and the boost generator (in the $x$-direction) are involved, and the Hamiltonian will have the form

$$H' = H + \frac{a}{c} K_x$$

If $H'$ is independent of $\tau$, the situation is stationary in the accelerated frame, and the ground state as well as the excited states of the accelerated system can be defined as the eigenstates of $H'$. (Such a physical system has to have a finite size to avoid problems related to the coordinate singularity.)

If a uniformly accelerated system, described by a stationary Hamiltonian $H'$, is coupled to a quantum field and this field is in the Minkowski vacuum state, the vacuum fluctuations will cause transitions in the accelerated system. The important point is, as pointed out by Unruh$^3$, that transitions to higher energy (with respect to $H'$), is caused by positive frequency fluctuations with respect to the Rindler time $\tau$. The quantum fields in the vacuum state have only negative frequency components with respect to Minkowski time $t$, but in terms of $\tau$ they have both positive and negative frequency parts.

Let us consider the case of an accelerated electron. Then $H$ and $K_x$ are respectively the Dirac Hamiltonian and the boost operator of Dirac theory for the inertial (rest) frame at time $\tau$. If we neglect the fluctuations in the trajectory and simply constrain the particle coordinates to the classical path $x' = 0$, the Hamiltonian $H'$ is reduced to a spin Hamiltonian of the form

$$H_{spin} = \frac{1}{2} \hbar (\vec{\omega} + \delta \vec{\omega}) \cdot \vec{\sigma}$$

with $\vec{\omega}$ determined by the external magnetic field and $\delta \vec{\omega}$ by the radiation field,

$$\vec{\omega}_0 = -\frac{e}{2mc} g \vec{B}_{ext}, \quad \delta \vec{\omega} = -\frac{e}{2mc} g \vec{B}_{rad}$$

(The primed fields refer to the inertial rest frame.) The simplest situation would be to consider an external magnetic field $\vec{B}_{ext}$ in the same direction (the $x$-direction) as the accelerating (electric) field.
The coupling to the radiation field causes transitions between the spin up and spin down states of the particle in the external magnetic field. Standard first order perturbation theory gives for the transition probabilities per unit time

\[ \Gamma_{\pm} = \left( \frac{e g}{4mc} \right)^2 \int_{-\infty}^{\infty} \exp(\mp i\omega \tau) C(\tau - i\epsilon) d\tau \]  

(5)

with \( \pm \) referring to transitions up/down in spin and with \( C(\tau) \) as the vacuum correlation function of the magnetic field along the accelerated trajectory,

\[ C_B(\tau) = \langle B'_\mp(\tau/2)B'_\mp(-\tau/2) \rangle \]

\[ = \frac{ha^4}{2\pi c} \left[ \sinh \left( \frac{a}{2c} \tau \right) \right]^{-4}, \quad B'_\pm = B'_x \pm iB'_y, \]  

(6)

The correlation function \( C(\tau) \) has a periodicity property with respect to shifts in the imaginary time direction,

\[ C_B(\tau + \frac{4\pi c}{a} i) = C_B(\tau) \]  

(7)

which makes it easy to solve the integral (5) by closing the integration contour along the shifted path \( \tau + \frac{4\pi c}{a} i \). For the closed contour we find

\[ \left( 1 - \exp\left( \pm \frac{2\pi c}{a} \frac{\omega}{\bar{\hbar}} \right) \right) \Gamma_{\pm} = \frac{e^2 g^2 \bar{\hbar}}{6m^2 c^5 \omega} \left( \omega^2 + \frac{a^2}{c^2} \right) \]  

(8)

and the ratio between transitions up and down is

\[ R = \frac{\Gamma_+}{\Gamma_-} = \exp\left( -\frac{2\pi c}{a} \frac{\omega}{\bar{\hbar}} \right) \]  

(9)

In an equilibrium situation the ratio between transitions up and down defines the relative occupation probabilities of the two spin levels. The dependence of the energy shows that it has the form of a Boltzmann factor corresponding to a temperature

\[ k T_U = \frac{a\bar{\hbar}}{2\pi c} \]  

(10)

This is the Unruh temperature, which is generally associated with an accelerated system. It has exactly the same form as the temperature of a black hole,

\[ k T_{BH} = \frac{\kappa \hbar}{2\pi c} \]  

(11)
whith the acceleration $a$ corresponding to the the surface gravity, $\kappa$, of the black hole.

The spin effect discussed above is a special realization of the Unruh effect. The interesting point is that this effect has a universal character, it does not depend on details of the accelerated system or on its coupling to the radiation field. The temperature effect follows from general features of the vacuum state and special properties of the accelerated trajectory which lead to the symmetry properties (7) of the vacuum correlation functions. One should also note that the limitation to point detectors is not a necessary restriction. For the case considered here the fluctuations in the particle path can be taken into account. The Hamiltonian $H'$ will then depend on both spin and orbital coordinates and the electron in this sense has to be treated as an extended system. If the accelerating fields give rise to a time independent Hamiltonian in the accelerated frame, the probability distribution over the energy levels of $H'$ will still have a thermal form. This follows from general symmetry properties of the vacuum correlation functions in the Rindler coordinate system, and is closely related to PCT-invariance. An interesting complication is that the (local) temperature of such an extended system would vary over the extension of the system. On the other hand, such a variation is also present for a hot system in a gravitational field, where the variation in temperature is induced by the redshift which follows from differences in the gravitational potential.

A system of linearly accelerated electrons could in principle be used to demonstrate the Unruh effect, in the way discussed above. Thus, the spin polarization would depend on the spin precession frequency as

$$P(\omega) = \frac{1 - R}{1 + R} = \tanh \left( \frac{\pi c \omega}{a} \right)$$

and by varying the strength of the external magnetic field the functional form of $P$ could be demonstrated. Unfortunately, this is not a realistic situation for real particle accelerators. If we use a value for the electric field $E = 10 \text{MV/m}$ we find a rather low corresponding temperature $T = 0.7 \cdot 10^{-3} \text{K}$. However, the main complication is the long time for reaching equilibrium. From the expression for the transition probabilities we find a typical time of $\tau = 3 \cdot 10^{18} \text{s}$. In the lab frame this would be enhanced even further by the time dilatation effect. So this is not very promising. If much larger accelerations can be obtained the thermalization time $\tau$ would be strongly reduced (it varies with $a$ as $a^{-3}$), but the limited time available in a linear accelerator is nevertheless a serious problem.

The limitations present for linear accelerators are not there for circular
accelerators. Thus, much higher values for the proper acceleration are obtained, mainly due to the relativistic $\gamma$ factor for the transformation from the lab frame magnetic field to the rest frame electric field. A typical acceleration (limited by synchrotron radiation effects) is $a = 3 \cdot 10^{23} \, \text{m/s}^2$ corresponding to a temperature of $T = 1200 \, \text{K}$. And more importantly, for electrons in a storage ring the time needed to reach equilibrium is available, typically of the order of minutes to hours.

In the magnetic field of a circular accelerator, the electrons will gradually build up a transverse polarization due to spin flip radiation. This is well-known from calculations by Sokolov and Ternov\textsuperscript{8}, and later by others\textsuperscript{9,19} and the effect has been seen in real accelerators. The polarization will under ideal conditions reach the equilibrium value of 92\%.

In the figure the theoretical curve for the polarization as a function of the $g$-factor is shown. In the same figure also the corresponding curve, based on the simple formula (12) obtained in the case of linear acceleration is shown. Much of the discussion of the Unruh effect in storage rings has been based on comparison of these curves. (For some critical remarks see Jackson\textsuperscript{20}.) In the following I will examine this question again. There are two important points involved in understanding the similarity and difference between the two curves. The first is the question of the difference between linear acceleration and acceleration in a circular orbit. The other question concerns the approximation
where we treat the electron as a point detector.

3 Stationary world lines

The world line of a uniformly accelerated particle and the world line of a particle moving with constant speed in a circular orbit are special cases of what has been referred to as stationary world lines\textsuperscript{21}. These space-time curves are self similar in the sense that there is no geometric difference between two points on the trajectory. Such a curve can be generated by a time-independent Poincaré transformation which in general will involve rotation in addition to boost:\textsuperscript{b}

\[ H' = H + \frac{a}{c} \cdot \vec{K} - \vec{\omega} \cdot \vec{J} \]  

(13)

In the same way as for linear acceleration \( H' \) can be seen as a time evolution operator which jumps between inertial frames, the instantaneous rest frames of the particle. It therefore generates a full (accelerated) coordinate system in Minkowski space, a system where the particle sits at rest. This is a stationary system in the sense that the space-time metric is independent of the time parameter. The operator \( H' \) can also be interpreted as the Hamiltonian of a quantum system described in the accelerated frame. For the accelerated particle \( H' \) is time independent when the accelerating fields are stationary in this frame.

By making use of the freedom to choose orientation of coordinate axis \( H' \) can be brought into the form

\[ H' = H + \frac{a}{c} K_x - \omega_z J_z - \omega_x J_x \]  

(14)

The physical interpretation of the parameters is that they correspond to acceleration and angular velocity of a stationary frame moving with particle, as measured relative to an inertial rest frame, but they also have a geometric interpretation as curvature, torsion and hypertorsion of the world line of the accelerated particle\textsuperscript{21}.

The trajectory of uniform linear acceleration corresponds to \( \omega_x = \omega_z = 0 \) and the circular orbit with constant acceleration and velocity corresponds to \( \omega_x = 0, \omega_z = a/v \) with \( v \) as the velocity of the particle. It is interesting to note that a continuous interpolation can be made between the two cases by changing \( \omega_z \) while \( \omega_x = 0 \). This looks as a purely formal interpolation, since \( v \) has to

\textsuperscript{b}Such motion has been referred to as \textit{group motion} and has been examined in the context of relativistic \textit{Born rigid motion}\textsuperscript{22}.  

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Figure 2: Stationary world lines projected on the $x, y$-plane for varying values of the angular velocity $\omega$ with fixed proper acceleration $a$.

Exceed the speed of light, but that is not really the case. If the circular motion is not described in the rest frame of the center of the orbit, but rather in the rest frame of the circulating particle at one of the points of the orbit, then there is a smooth interpolation between physically realizable trajectories. For $\omega_z > a/c$, the trajectories in the $(x, y)$-plane are cycloids, i.e., they are periodic orbits. For $\omega_z \leq a/c$ they are non-periodic (see figure.) The limit case $\omega_z = a/c$ can be interpreted as the limit of circular motion where the radius tends to infinity (and the velocity $v \to c$) while the proper acceleration is fixed.

In the same way as for linear acceleration, all the acceleration dominated trajectories $\omega_z < a/c$ have an event horizon. This horizon disappears in the limit $\omega_z = a/c$ and is not present for the rotation dominated trajectories $\omega_z > a/c$, i.e., for circular motion. It is interesting to note the similarity with the situation of a rotating (Kerr) black hole. The black hole is characterized by two parameters, mass and spin (or surface gravity and angular velocity). For fixed mass there is an upper limit to the angular velocity for which the event horizon exists. For angular velocities beyond this one has the (unphysical) situation with a space-time singularity but no event horizon. In addition to the event horizon a rotating black hole is characterized by the presence of a static limit, a limiting distance from the black hole, within which no physical object can be stationary with respect to a distant observer. The circular motion in Minkowski space corresponds to a situation with no event horizon, but there is in this case a static limit outside which no object can be stationary in the accelerated coordinate system. The presence of this limit is easy to understand;
due to the rotation points with fixed space coordinates will at some distance from the center of rotation move with a velocity larger than the velocity of light. At such a point a physical body cannot stay fixed.

As previously mentioned a detector which is uniformly accelerated through Minkowski vacuum is related to a detector which sits at rest in a stationary coordinate frame outside a (large) black hole. The black hole can be viewed as a thermodynamic system, and in a stationary state the detector will be in thermal equilibrium with the black hole, with a (locally determined) temperature that depends on its position due to the redshift effect. This suggests that in a similar way a detector following a trajectory with \( \omega_z \neq 0 \) is related to a stationary detector outside a rotating black hole. Also a rotating black hole can be viewed as a thermodynamic system, with the angular velocity now acting as a chemical potential for the conserved angular momentum. However, in this case we do not expect the occupation probabilities over the detectors energy levels to be determined simply by the temperature and angular velocity. This is because the detector will not be coupled to the black hole in a rotationally invariant way. In a similar way rotational invariance will be broken in the accelerated frame for any trajectory with \( \omega_z \neq 0 \), and there is no reason why the probability distribution in this case should have a thermal form. Only for \( \omega_z = a/c \) that will be the case. The deviation from thermal form is well-known for circular motion and also for the case \( \omega_z < a/c \) this has been discussed\textsuperscript{21}.

However, even though there is no exact temperature associated with the motion where \( \omega_z \neq 0 \), the notion of an effective temperature is meaningful, as has been discussed before. The vacuum state, in some approximate meaning seems hot in the accelerated frame, and in addition there is a relative rotation between the stationary detector and the vacuum. In the following I will re-examine the case of the circulating electron from this point of view. The intention is to show that such a picture of the vacuum state is relevant and to demonstrate that much of the discrepancy between the polarization curve and the curve derived from the assumption of thermal excitations is due to the way the electron works as a detector.

4 Electrons in a storage ring

When we consider electrons in a storage ring under ideal conditions, where they move in a rotationally symmetric magnetic field and with the correction due to radiation loss neglected, they can be described by a time-independent Hamiltonian \( H' \) of the form previously discussed,

\[
H' = H + \frac{a}{c} K_x - \frac{a}{v} J_z
\]  

(15)
This Hamiltonian refers to an accelerated (rotating) coordinate system which rotates with the frequency of electrons moving along a (classical) reference trajectory in the magnetic field. The transformation between the accelerated and the lab frame coordinates can be written as

\[
x = (x' + R) \cos (a \tau/v \gamma) - \gamma y' \sin (a \tau/v \gamma)
\]
\[
y = \gamma y' \cos (a \tau/v \gamma) + (x' + R) \sin (a \tau/v \gamma)
\]
\[
z = z'
\]
\[
t = \gamma \tau + \gamma v y' / c^2
\]

(16)

with \( R \) as the bending radius of orbit and \( \gamma \) as the relativistic gamma factor.

The electrons described by the Hamiltonian are not restricted to the reference trajectory \( x' = y' = 0 \). Also quantum fluctuations in the orbital motion are included. However, we may assume deviation from this orbit both in position and velocity to be small, which makes it possible to linearize in the deviation and to consider a non-relativistic approximation.

The operators \( H, K_x \) and \( J_z \) are Dirac operators in the inertial rest frame of the reference trajectory at time \( \tau = 0 \),

\[
H = c \alpha \cdot \pi + \beta mc^2 + e \phi + (g - 2) \frac{e \hbar}{4mc} (i \beta \alpha \cdot E - \beta \sigma \cdot B)
\]
\[
K_x = -\frac{1}{2c} (xH + Hx)
\]
\[
J_z = xp_y - yp_x + \frac{1}{2} \hbar \sigma_z
\]

(17)

A term for the anomalous magnetic moment \((g - 2)\) has been introduced in the expression for \( H \). All the coordinates and fields refer to the inertial rest frame of the reference trajectory, but here and in the following I will omit the primes on these coordinates and fields. \( \pi = \vec{p} - \frac{e}{c} \vec{A} \) is the mechanical moment of the electron, and \( \alpha \) and \( \beta \) are the standard Dirac matrices.

When a Foldy-Wouthuysen transformation is performed and the equations are linearized in the orbital fluctuations, the Hamiltonian can be reduced to a non-relativistic form which involves only spin and vertical oscillations. The coupling between the horizontal oscillations and the spin can be considered as a higher order effect when we primarily are interested in the spin degree of freedom. However the coupling to vertical oscillations cannot be neglected, as has been discussed in earlier papers. When we take into account only the external, accelerating fields the resulting electron Hamiltonian gets the simple form

\[
H_e = \frac{p^2}{2m} + \frac{1}{2} m \Omega^2 z^2 + \frac{1}{2} \omega \hbar \sigma_z - \frac{1}{mc} \omega \hbar p \sigma_y - \frac{\hbar}{4c} g \Omega^2 z \sigma_x
\]

(18)
where $p$ is the momentum in the $z$-direction $\omega = \frac{e}{2m}(g - 2)$ is the spin precession frequency. A confining harmonic oscillator potential has been introduced for the vertical motion. In a weak focusing machine this is introduced by a gradient in the magnetic field, $\Omega^2 = -\frac{e}{m}\frac{\partial B}{\partial r}$. The coupling to the radiation field gives an additional term,

$$H_1 = -\frac{e\hbar}{4mc} \vec{B} \cdot \vec{\sigma} - ezE_z$$

which we treat as a perturbation. The fields $\vec{B}$ and $E_z$ are rest frame fields.

Thus, the transformation to the accelerated frame gives us a fairly simple and straightforward way to calculate the equilibrium properties of the electron beam. We first determine the eigenvectors and eigenvalues of $H_e$, which is a two-level system coupled to a harmonic oscillator, and we then find the occupation of the levels by calculating the transition probabilities induced by $H_1$.

The coupling terms between spin and orbital motion in (18) are normally quite small. If they are treated perturbatively we find to first order the following expressions for the eigenvectors of $H_e$

$$|\chi_{n,\pm}\rangle = |n,\pm\rangle - i\sqrt{\frac{\hbar\omega}{32mc^2}} \left\{ (2\omega \mp g\Omega) \frac{\sqrt{n+1}}{\omega \mp \Omega} |n+1,\mp\rangle + (2\omega \pm g\Omega) \frac{\sqrt{n}}{\omega \pm \Omega} |n-1,\mp\rangle \right\}$$

(20)

$|n,\pm\rangle$ is the eigenvector of the uncoupled system, with $n$ referring to the harmonic oscillator and $\pm$ to the spin levels. Note the resonance between spin and orbital motion for $\Omega = \omega$. The energy levels then are degenerate, and improved expressions close to resonance could be found by use of degenerate perturbation theory, but I will not do that here. Also note that to higher order in perturbation theory also higher order resonances, for $\Omega = n\omega$, will be present.

Let us first consider the transition matrix elements for non-spin flip transitions. It is clear from the form of $H_1$ that the small terms of (20) will only give small contributions to the matrix elements. If they are neglected the result is

$$\langle \chi_{n+1,\pm} | H_1 | \chi_{n,\pm}\rangle = ie\sqrt{\frac{\hbar(n+1)}{2m\Omega}} E_z$$

(21)

For spin transitions the situation is different. Even if the coupling terms are small they give significant contributions since they are influenced by the coupling of the charge to the radiation field, which is much stronger than the spin
coupling. The result for spin flips is
\[
\langle \chi_{n+1, \pm} | H_1 | \chi_{n, \pm} \rangle = -\frac{\hbar}{4mc} \left( gB_+ + \left( 2 - (g - 2) \frac{\Omega^2}{\omega^2 - \Omega^2} \right) E_z \right)
\] (22)

The equilibrium populations \( p_{n, \pm} \) of the energy levels can be found be assuming detailed balance. Thus, the transition probabilities can be expressed in terms of the electromagnetic fields, as for linear acceleration, and the relative populations can be determined as the ratio between the probabilities for transition between the levels one way and the other. Thus, for fixed spin the ratio is determined by the matrix element (21), and thereby by the correlation function of the (rest frame) electric field,
\[
R(\Omega) = \frac{p_{n+1, \pm}}{p_{n, \pm}} = \frac{C_E(\Omega)}{C_E(-\Omega)}
\] (23)
\[
C_E = \int d\tau \exp(-i\Omega \tau) \langle E_z(\tau) E_z(0) \rangle
\] (24)

Note that the ratio \( R(\Omega) \) is independent of the state \((n, \pm)\). This implies that the excitation spectrum has a thermal form,
\[
p_{n, \pm} = N_\pm \exp(-n \ln R)
\] (25)
with a frequency dependent temperature\(^{12}\)
\[
eT_{eff} = \Omega \hbar \ln R
\] (26)

The relative population of spin up and down for the same \( n \) is determined by ratios between spin flip transitions. It has a similar form,
\[
R'(\omega) = \frac{p_{n, \pm}}{p_{n, -}} = \frac{D(\omega)}{D(-\omega)}
\] (27)
but now with a composite correlation function
\[
D(\omega) = g^2 C_B(\tau) + 2g C_{EB}(\tau) + 4 C_E(\tau)
\] (28)
where \( C_B \) is the correlation function of the magnetic fields \( B_+ \) and \( B_- \) and \( C_{EB} \) is the mixed correlation function of \( B_\pm \) and \( E_z \).

The resonance term of (22) is small except close to the resonance. If it is neglected, the expression (27) for the relative populations of the spin levels will reproduce the standard result for the equilibrium polarization \( P = \)
Figure 3: Evaluated polarization curves, based on Eq.(27) in the text. "Total" refers to the full expression, and agrees with conventional result displayed in Fig.1. "Direct" refers to the curve when fluctuations in the orbit are suppressed, and "indirect" refers to the curve where only the spin excitations due to fluctuations in the orbit are retained.

\[
\frac{1 - R'}{1 + R'},
\]

earlier shown in Fig.1. It is interesting to see how this is built up from contributions from the direct coupling between the spin and magnetic field and the indirect one transmitted through fluctuations in the orbit. The effect of the fluctuations is demonstrated by not including the two terms \(C_{EB}\) and \(C_E\) in (28). The fluctuations in the orbit are then effectively suppressed and only the coupling to the magnetic moment of the electron is retained. The result is a changed curve \(P(g)\), denoted "direct" in Fig.4. It indeed has a form very similar to the one derived from the temperature formula, denoted "thermal" in Fig.1. The effective temperature indicated by the curve is however somewhat higher than the Unruh temperature \(T_U\) for the same acceleration.

It is instructive also to consider the curve obtained if the contributions to the spin transitions caused by the direct coupling is suppressed and only the indirect one, due to fluctuations in the orbit, are retained. (That means including only \(C_E\) in Eq.(28).) The result is referred to as "indirect" in Fig.4. It is very similar to the one obtained from the direct coupling to the radiation field, although differing by a shift of 2 units along the g-axis.

The relative shift of the curves obtained from the direct and indirect coupling of the spin to the radiation field is fairly easy to understand. The direct coupling of the radiation field to the spin is rotationally invariant and the corresponding occupation probabilities have (approximately) a thermal form in the frame which is non-rotational (with respect to the vacuum). This is
the Fermi-Walker transported frame, where the direct coupling term is proportional to $g$. The vertical fluctuations are insensitive to rotations in the $x, y$-plane, but the coupling between the orbital motion and the spin is time independent in the stationary frame, which is rotating relative to the Fermi-Walker transported frame. This gives rise to occupation probabilities which are (approximately) thermal in the stationary frame. The relative rotation between these two frames is represented by the shift along the $g$-axis.

Based on this understanding we may conclude that much of the difference between the two curves in Fig.1 is due to the way the electron acts as a detector. (See also the related discussion by Unruh.) As a final point I will illustrate this by considering a hypothetical situation where the detector, defined by the electron Hamiltonian (18) and the coupling term (19), is put in contact with an electromagnetic heat bath and where the detector is rotating relative to the thermal state. Thus, the true correlation functions along the orbit are replaced by thermal correlation functions in the evaluation of the polarization. In Fig.4 the correct polarization curve is compared with the modified curve obtained in this way. The temperature of the thermal state as well as the angular velocity are used as fitting parameter. We note that a good approximation is obtained for an effective temperature $T_{\text{eff}} \approx 1.77 U$ and an angular velocity only slightly different from true one, $\omega \approx 1.1 a/c$. There are some details, though, which are different in the two cases, and which show that the correct curve is not truly thermal.
5 Concluding remarks

An interesting observation made by Fulling\textsuperscript{1} and others is that the natural definition of a vacuum state is related to the choice of the space-time co-ordinates. In a curved space-time this makes the notion of a vacuum state highly non-trivial and physical effects, like the Hawking radiation, may be related to the question of what is the correct vacuum state.

Even in flat space, there are non-trivial vacuum effects associated with accelerated co-ordinate frames. For a linearly accelerated system, the Rindler coordinate system, Minkowski vacuum appears as a thermally excited state. Also for other stationary coordinate systems, which include rotation and not only acceleration, Minkowski vacuum appears as an excited state, although not characterized by a uniquely defined temperature.

It is highly interesting that these vacuum effects, that usually are considered to be detectable only under extreme situations, can be related to measurable polarization effects of electrons in a storage ring. The natural way to see this connection is to describe the electron beam in the accelerated frame of an orbiting reference particle. The electron can be described as simple quantum mechanical system, which includes the vertical motion coupled to the spin, and with transitions between the states of this system induced by the radiation field. Since the Minkowski vacuum state appears excited in this frame transitions both up and down in energy are induced, and equilibrium is produced as a balance between these two processes.

Due to the coupling between spin and orbital motion, the electron acts as a non-trivial detector. Thus, the system is not rotationally invariant, and this complicates the detection of the vacuum effects. The vacuum can be seen as being hot in the accelerated frame, not in the stationary frame of the detector, but rather the non-rotational (Fermi-Walker transported) frame. In fact, up to minor details the correct polarization curve can be reproduced if the (rotating) detector, defined by the electron Hamiltonian, is excited by an electromagnetic heat bath, rather than the Minkowski vacuum state.

The description of the orbiting electrons in the accelerated frame is natural for the discussion of how the spin effects are related to the Unruh effect. But I would also like to stress that it gives a conceptually simple way to study the quantum effects of electrons in a storage rings. Under the ideal conditions considered here, the electron is described as a harmonic oscillator coupled to a two-level system with transition probabilities determined by correlation functions of the radiation field. It should be of interest to examine this approach further, also beyond the simplest approximation used here.
References

7. R. Chiao, private communication.