THE NEAR-HORIZON GEOMETRY OF DILATON-AXION BLACK HOLES

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Static black holes of dilaton-axion gravity become singular in the extreme limit, which prevents a direct determination of their near-horizon geometry. This is addressed by first taking the near-horizon limit of extreme rotating NUT-less black holes, and then going to the static limit. The resulting four-dimensional geometry may be lifted to a Bertotti-Robinson-like solution of six-dimensional vacuum gravity, which also gives the near-horizon geometry of extreme Kaluza-Klein black holes in five dimensions.

The discovery of the AdS/CFT dualities stimulated the search for geometries containing AdS sectors, which typically arise as the near-horizon limit of BPS black holes or p-branes in various dimensions. Here we discuss the near-horizon limit of 4-dimensional black holes arising in the truncated effective theory of the heterotic string: dilaton-axion gravity with one Abelian vector field (EMDA).

The Einstein-frame metrics of the NUT-less rotating extreme black hole solutions of EMDA are given by

$$ds^2 = \frac{\Sigma}{\Gamma} dt^2 - \frac{\Gamma}{\Sigma} \sin^2 \theta \left( d\varphi - \frac{2Ma(r+a)}{\Gamma} dt \right)^2 - \Sigma \left( \frac{dr^2}{r^2} + d\theta^2 \right),$$  \hspace{1cm} (1)

$$\Sigma = h - a^2 \sin^2 \theta, \quad \Gamma = h^2 - r^2 a^2 \sin^2 \theta, \quad h = r^2 + 2M(r+a),$$

$M$ and $a$ being the mass and the rotation parameter. The horizon $r = 0$ reducing to a point in the static case $a = 0$, we carry out the near-horizon limit in the rotating case $a \neq 0$. First, we transform to a frame co-rotating with the horizon, and rescale time by $t \rightarrow (r_0^2/\lambda) t (r_0^2 \equiv 2aM)$. Then, we put $r \equiv \lambda x, \cos \theta \equiv y$, and take the limit $\lambda \rightarrow 0$, arriving at

$$ds^2 = r_0^2 \left[ (\alpha + \beta y^2) \left( x^2 dt^2 - \frac{dx^2}{x^2} - \frac{dy^2}{1-y^2} \right) - \frac{1-y^2}{\alpha + \beta y^2} (d\varphi + x dt)^2 \right],$$  \hspace{1cm} (2)

with $\beta = a/2M, \alpha = 1 - \beta$. This is similar in form to the extreme Kerr-Newman near-horizon metric, both having the symmetry group $SL(2,R) \times U(1)$, and coinciding in the extreme Kerr case $M = a$. Now take the static limit $a \rightarrow 0$ in (2), keeping $r_0^2 = 2aM$ fixed. This yields (for $r_0^2 = 1$) the metric

$$ds^2 = x^2 dt^2 - \frac{dx^2}{x^2} - \frac{dy^2}{1-y^2} - (1 - y^2)(d\varphi + x dt)^2.$$  \hspace{1cm} (3)

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This metric, together with the associated near-horizon dilaton $\phi$, axion $\kappa$ and gauge potential in the static limit (independent of the original parameters of the black hole solution)

$$\phi = 0, \quad \kappa = -y, \quad A = -y(d\varphi + xdt)/\sqrt{2}, \quad (4)$$

constitute a new Bertotti-Robinson-like solution of EMDA, the BREMDA solution.

This solution has properties remarkably similar to those of $AdS_2 \times S^2$: all timelike geodesics are confined, and the Klein-Gordon equation is separable in terms of the usual spherical harmonics. The isometry group $SL(2, R) \times U(1)$, with $U(1)$ being the remnant of the $SO(3)$ symmetry of $S^2$, is generated by the Killing vectors $L_0, L_1$ and $L_\phi = \partial_\phi$. The first three of these constitute the $sl(2, R)$ subalgebra of an infinite-dimensional algebra of asymptotic symmetries of (3), with generators

$$L_n = t^{-n} \left[ \left( t + \frac{n(n-1)}{2x^2t} \right) \partial_t + x(n-1)\partial_x - \frac{n(n-1)}{xt} \partial_\phi \right] \quad (5)$$

(where $n \in Z$) satisfying the Witt algebra $[L_n, L_m] = (n-m)L_{n+m}$ up to terms $O(x^{-4})$. It can be expected that a representation in terms of asymptotic metric variations will lead to the Virasoro extension of this algebra with a classical central charge, opening the way for microscopic counting of the horizon microstates of dilaton-axion black holes.

The BREMDA solution does have a close connection with $AdS_2 \times S^2$, which is discovered by lifting it to 6 dimensions. EMDA in 4 dimensions may be shown to derive from 6-dimensional vacuum gravity with 2 commuting spacelike Killing vectors $\partial_\eta, \partial_\chi$ via a two-step Kaluza-Klein reduction together with the assumption of a special relation between the Kaluza-Klein gauge fields, according to the ansatz

$$ds_6^2 = ds_4^2 - e^{-\phi} \theta^2 - e^\phi (\zeta + \kappa \theta)^2, \quad (6)$$

$$\theta = d\chi + A_\mu dx^\mu, \quad \zeta = d\eta + B_\mu dx^\mu, \quad G_{\mu\nu} = e^{-\phi} \tilde{F}_{\mu\nu} - \kappa F_{\mu\nu} \quad (F = dA, \ G = dB). \text{ Using this ansatz, the BREMDA solution (3)-(4) may be lifted and rearranged, yielding the vacuum solution BR6 of 6-dimensional gravity}$$

$$ds_6^2 = x^2 dt^2 - \frac{dx^2}{1 - y^2} - \frac{dy^2}{(1 - y^2)(d\varphi - (d\chi + \sqrt{2} x dt + \sqrt{2} y d\varphi)^2 - d\chi^2).} \quad (7)$$

This is the trivial 6-dimensional embedding of a 5-dimensional metric BR5, which can be shown to be the common near-horizon limit of all static, NUT-less black holes of 5-dimensional Kaluza-Klein theory. It enjoys the higher symmetry group $SL(2, R) \times SO(3) \times U(1) \times U(1)$, but breaks all fermionic symmetries: no covariantly constant spinors exist.

References