The polarization of M5 branes and little string theories with reduced supersymmetry

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Abstract

We construct an M-theory dual of a 6 dimensional little string theory with reduced supersymmetry, along the lines of Polchinski and Strassler. We find that upon perturbing the (2,0) theory with an R-current, the M5 branes polarize into a wrapped Kaluza Klein monopole, whose isometry direction is along the R current. We investigate the properties of this theory.

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1 Introduction

A while ago Polchinski and Strassler [1] found a string theory dual of a four dimensional SYM theory with reduced supersymmetry. They perturbed the $\mathcal{N} = 4$ theory by giving mass to the hypermultiplets. In the dual bulk theory this corresponded to turning on transverse RR and NS 3 form field strengths which polarized [2] the D3 branes into D5 or NS5 branes. This extension of the original AdS/CFT duality [3] had implications in both directions. It allowed for a detailed string theory interpretation of domain walls, baryon vertices, confinement, etc., and it illustrated a way in which string theory can cure some singularities in supergravity.

Since then, papers have appeared generalizing the PS construction to theories in other dimensions (M2 branes [4], D2 branes [5, 6]), as well as to 4 dimensional theories on orbifolds [7], with temperature [8], even less susy [9, 10], etc.

One of the possible extensions of [1] left unexplored has been to the $\text{AdS}_7 \times S^4$ geometry. In the same line of thought from [1, 4, 5] one would expect the dual of a little string theory with reduced supersymmetry to be given in terms of an M theory bulk containing polarized M5 branes. It is the purpose of this paper to explore this possibility.

Since the degrees of freedom of a large number of M5 branes are not known, giving a matrix description to this polarization (like in the D-brane $\rightarrow$ D-brane polarization) is not possible. The only way to describe this polarization is thus to find the action of the object in which the M5 branes are polarized, and to show that there exists a configuration with nonzero radius which is energetically stable.

In order to understand which operators one needs to turn on to obtain polarized M5-branes, and find what they might polarize into, it is worth thinking about the polarization of a large number of D4 branes into D6 branes. These configurations exist and are understood very well (they are T-dual to the D3 $\rightarrow$ D5 system discussed in [1]). They are produced by turning on a fermion scalar bilinear on the worldvolume of the D4 branes, which corresponds to a combination of RR 1 form and NS-NS 2 form on the transverse space $^\dagger$.

One expects upon lifting this polarized configuration to M theory to obtain a configuration in which M5 branes polarize into Kaluza-Klein monopoles (KKM). One of the directions of the M5 brane is special in that it is along the isometry direction of the monopole. The fermion scalar bilinear on the D4 brane can come from the dimensional reduction of a fermion scalar or a fermion vector bilinear oriented in the M-theory direction. The bulk RR 1 form and NS-NS 2 form come from a graviton with one leg on the 11'th direction (the graviphoton) and respectively from a 3 form potential with one leg on the 11'th direction. Since these bulk fields transform as vectors under the Lorentz $\text{SO}(5,1)$ group, we expect them to correspond to a fermion bilinear which is

$^\dagger$We should note, however, that the naive expectation that the D4-D6 polarization configuration is still supersymmetric fails.
a vector oriented in the 11'th direction. This same polarization configuration can be obtained by T-dualizing the D3 → NS5 polarization of [1] and obtaining a D4 brane polarized into a IIA KK monopole. The D4 → IIA KKM configuration lifts to the same configuration as above, with the M-theory direction identified differently.

One might wonder whether there are other possible polarizations for M5 branes. It appears possible to polarize it into a codimension 4 higher object, namely an M9-brane. However, since we are interested in M-theory configurations only, and the M9 is a brane solution of massive 11d supergravity (sugra), we shall not explore this possibility.

Nevertheless, it does not appear possible to polarize the M5's into a KKM which does not have its isometry direction along the brane, simply because the original M5 branes do not have that isometry.

There are however a few problems which an astute reader might already be concerned with. The first one has to do with the tension of the KK monopole. As we shall discuss, the tension of the KKM is proportional to the square of the length of the 11'th dimension. Thus, as we open up the 11'th dimension the monopole becomes more and more massive. If for example in IIA we had \( N \) D4 branes polarizing into one D6 brane, as we open the 11'the dimension the ratio of the charges changes, and at one point the KKM in which the M5 branes polarize will be heavier than the 5-branes, in which case it will not be treatable as a probe. We will address these problems in due time.

While this paper was being completed, [11] appeared, in which the polarization of M5 branes into KKM's was also suggested.

2 Perturbing the \( \text{AdS}_7 \times S_4 \) background

2.1 The sugra set-up

The 11 dimensional supergravity bosonic Lagrangian is

\[
\mathcal{L} = R * 1 - \frac{1}{2} F_4 \wedge * F_4 + \frac{1}{6} F_4 \wedge F_4 \wedge C_3
\]  

(2.1)

where \( F_4 = dC_3 \), and * is the 11-dimensional Hodge star. One can define the 6 form potential \( C_6 \) as the Poincaré dual of \( C_3 \)

\[
F_7 \equiv dC_6 = * F_4 + \frac{1}{2} C_3 \wedge F_4
\]  

(2.2)

The M5 brane is a soliton solution of 11d sugra

\[
(ds^2)^0 = Z^{-1/3} \eta_{\mu\nu} dx^\mu dx^\nu + Z^{-1/3} dx_1^2 + Z^{2/3} dx^i dx^j \delta_{ij}
\]  

(2.3)

\[
F_7 = \partial_i Z^{-1} dx^0 \wedge \ldots \wedge dx^4 \wedge dx^{11} \wedge dx^i
\]  

(2.4)

where the brane is aligned along \( \mu = 0, 1, 2, 3, 4, 11 \), the transverse coordinates are \( i = 5 \ldots 9 \), and \( Z \) is the harmonic function on the transverse space.

It is well known that the background geometry of a large number of coinciding M5 branes is \( \text{AdS}_7 \times S_4 \), and that 11d sugra compactified on \( \text{AdS}_7 \times S_4 \) gives a dual
description to the 6d (2,0) CFT theory living on the worldvolume of the M5 branes. For \( N \) coincident M5 branes the harmonic function is

\[
Z = \frac{R^3}{r^3}, \quad r^2 = x^i x^i, \quad R^3 \equiv \pi N l_p^3
\]  

(2.5)

The \( AdS_7 \times S_4 \) geometry becomes transparent if we redefine \( r = 4R/z_0^2 \). The \( AdS_7 \) radius is thus twice the \( S_4 \) radius. For later need, we give the Poincaré dual of (2.4)

\[
F_4^0 = \frac{1}{4!} \epsilon_{ijklm} \partial_m Z dx^i \wedge dx^j \wedge dx^k \wedge dx^l
\]  

(2.6)

where \( \epsilon_{ijklm} \) is the flat antisymmetric tensor restricted to the transverse coordinates.

We want to perturb this background with a non-normalizable mode of the field which is dual to a fermion mass operator in the CFT. The fermions of (2,0) CFT transform in the 4 \( \text{spinor} \) representation of the R-symmetry group \( SO(5) \). Thus a fermion bilinear which is a 10 \( \text{of} \) \( SO(5) \) corresponds to a 2-form (or by Poincaré duality a 3-form) field strength perturbation in the space transverse to the branes. Since 11d SUGRA has only a 4-form field strength and a graviton, the only way to obtain a transverse 3-form is to consider a 4 form with a leg along one of the longitudinal directions. Similarly the transverse 2-form is the field strength of the graviphoton.

We can see that only a fermion bilinear which is a Lorentz vector is a chiral primary. Thus, to turn on a fermion bilinear which is chiral we need to break Lorentz covariance and introduce a preferred direction, or equivalently, a Killing vector. We shall see later that this breaking of Lorentz covariance is the key to the construction in which the M5 branes polarize into Kaluza-Klein monopoles.

The background geometry admits a Killing vector

\[
D^0_{[\Lambda k\Pi]} = 0, \: \Lambda = 0 \ldots 9, 11
\]  

(2.7)

with \( k_\Lambda = Z^{-1/3} \delta_{\Lambda 11} \). We note that the contravariant Killing vector is simply \( k^\Lambda = \delta^{\Lambda 11} \). From now on we will use \( ^0 \) to denote background fields and \( ^1 \) to denote perturbed fields.

We want to turn on the nonnormalizable modes of the perturbations \( \delta g_{i1i} \equiv g_{i1i}^1 \equiv h_{i1i} \) and \( \delta C_{11ij} \equiv C_{11ij}^1 \) which only depend on the transverse coordinates. The metric perturbation is related to the 3-form perturbation by

\[
\delta R_{11i} = \delta \left\{ \frac{-1}{12} (F_{11i} F_i \cdots - \frac{1}{12} g_{11i} F \cdots F) \right\}
\]  

(2.8)

The variation of the Ricci tensor

\[
\delta R_{\Lambda\Pi} = \frac{1}{2} \left( h_{\Lambda,\Omega}^\Omega + h_{\Pi,\Lambda\Omega}^\Omega - h_{\Lambda\Pi} - \Box h_{\Lambda\Pi} \right)
\]  

(2.9)

in the background (2.3) is

\[
\delta R_{11i} = \frac{1}{2} \partial_j \left( Z^{-1} \partial_j (h_{11i} Z^{1/3}) - Z^{-1} \partial_i (h_{11j} Z^{1/3}) \right) - \frac{1}{24} h_{11i} \partial_m Z \partial_m Z \partial_m Z^{-8/3}
\]  

(2.10)

Combining this with (2.8) we obtain

\[
d[Z^{-1} (\ast_5 d(hZ^{1/3}) + G_3)] = 0
\]  

(2.11)
where \(*_5^*\) denotes the flat Hodge dual in the transverse space, \(G_3 \equiv (1/3!)F_{11ijk}dx^i \wedge dx^j \wedge dx^k\), and \(h \equiv h_{11} \wedge dx^i\).

Similarly, from the first order variation of the \(F_4\) field equation:

\[
d *^0 F_1^1 + d *^1 F_4^0 = 0
\]

we can obtain after a few straightforward steps:

\[
d[Z^{-1}(d(hZ^{1/3}) + *_5^* G_3)] = 0.
\]

In order to find the exact form of the bulk 2 and 3-forms corresponding to the fermion bilinear we need to relate them to transverse tensors, and to solve (2.11) and (2.13).

### 2.2 Fermion bilinears and transverse tensors

The fermion R-current can be related to a tensor on the transverse space by analyzing their properties under the action of the R-symmetry group. We pair the 4 worldvolume fermions and 4 of the transverse space coordinates into complex variables

\[
z_1 = x^5 + i x^6 \quad z_2 = x^8 + i x^9
\]

\[
\Lambda_1 = \lambda_1 + i \lambda_3 \quad \Lambda_2 = \lambda_2 + i \lambda_4
\]

and we notice that under an SO(5) rotation \(z_i \rightarrow e^{i \phi_i} z_i\), the fermions transform as

\[
\Lambda_1 \rightarrow e^{i(\phi_1 - \phi_2)/2} \Lambda_1
\]

\[
\Lambda_2 \rightarrow e^{i(\phi_1 + \phi_2)/2} \Lambda_2
\]

Thus, a diagonal fermion “mass” term behaves in the same way under SO(5) rotations as

\[
T_2 = -\text{Re}[m_1 dz_1 \wedge d\bar{z}_2 + m_2 dz_1 \wedge dz_2]
\]

To make \(T_2\) a Lorentz vector on the worldvolume all we have to do is tensor it with \(\wedge dx^{11}\). We can also define on the transverse space the Hodge dual to \(T_2\)

\[
T_3 = *_5^* T_2 = \text{Re}[m_1 dz_1 \wedge d\bar{z}_2 + m_2 dz_1 \wedge dz_2] \wedge dx^7.
\]

Note that if we set either fermion mass to zero, there is a remaining global \(U(1)\) symmetry.

### 2.3 The first order perturbation

A general bulk 3-tensor with the SO(5) transformation properties of a fermion bilinear will be a combination of \(T_3\) and \(V_3\), multiplied by a power of \(r\), where \(V_3\)
is given in the Appendix. Thus, we can express the bulk 3-tensor $H_3$ and 2-tensor $F_2 \equiv d(hZ^{1/3})$ as

$$F_2 = \left(\frac{r}{R}\right)^p (aV_2 + bT_2)$$
$$G_3 = \left(\frac{r}{R}\right)^p (AV_3 + BT_3)$$

(2.20)

where $T_2$ and $T_3$ were found above, and $V_2$, $V_3$ together with some useful identities are given in the Appendix. Using the Bianchi identities for the field strengths $F$ and $G$ we get the constraints

$$pb = 2a,$$
$$pB = 3A.$$  

(2.21)

Substituting (2.20), (2.21) into the harmonic form eq. (2.11), (2.13) we obtain the system:

$$(p + 3)(b + pb/2 + B) - 3(pB/3 - pb/2) = 0,$$
$$(p + 3)(b + B + pB/3) - 2(pB/3 - pb/2) = 0,$$

(2.22)

which admits four real solutions: $(p = 0, b/B = -1), (p = -8, b/B = -1); (p = -5, b/B = 2/3), (p = -3, b/B = 2/3)$.

To interpret these modes, we need to know their conformal dimensions. To translate $G_3 = (1/3!) \partial_i C_{11jk} dx^i \wedge dx^j \wedge dx^k$ to an inertial frame, we multiply (2.20) with $\sqrt{G_{11}} G_{ij} G_{jk} = Z^{-5/6} \propto r^{5/2}$.

Remembering that in AdS coordinates $z_0 \sim r^{-1/2}$, we find that the modes of $G_3$ corresponding to $p = -3$ and $p = -5$ behave near the boundary as $z_0$ and $z_0^5$ respectively. Thus, they are the non-normalizable and respectively normalizable solutions corresponding to an operator of conformal dimension $\Delta = 5$, which is our fermion bilinear. By matching the $R$-symmetry representation and conformal dimension with the table of CFT operators given in [12], we find that the CFT operator is indeed a Lorentz vector, bilinear in fermions - the $R$-symmetry current. In previous situations [1, 4, 5], the CFT operator corresponding to perturbations in the bulk matching the $R$-symmetry representation of a fermion bilinear was always a Lorentz scalar. In our case, the only bilinear in fermions which is a chiral primary operator is the $R$-current. The corresponding bulk perturbations are vectors under the $SO(5,1)$ Lorentz group, as anticipated. The non-normalizable mode $p = -3$ corresponds to the fermion “mass” perturbation, while the normalizable mode $p = -5$ corresponds to the vev $<\psi \psi>$ [13]. Thus an $R$-current corresponds to the bulk perturbation:

$$Z^{-1}[*_5 d(hZ^{1/3}) + G_3] = \xi T_3$$

(2.23)

where $\xi$ is a numerical constant which relates the physical mass of the fermions to the coefficient of the bulk mode. From now on we will absorb $\xi$ in the fermion mass, and thus set it to 1 in the bulk †. We only wrote down the combination of modes

†Let us briefly discuss how one can relate the fermion “mass” term of the theory living on the boundary
which later will appear in the KKM Lagrangian. It is interesting to notice that this particular combination is a harmonic function (as one can easily see from (2.11) and (2.13)). Therefore, it is determined by its value at the boundary (2.23), and does not depend on the particular form of $Z$.

Similarly, the modes with $p = -8$ and $p = 0$ correspond to a boundary operator of conformal dimension $\Delta = 11$. We can identify this operator to be $H H \psi \bar{\psi}$, where $H$ is the selfdual 3-form of the 6d (2,0) tensor multiplet.

One might wonder if our perturbation (2.23) does not become stronger than the background is some regime. We will discuss this possibility in section 3.2.

3 Polarizing the M5 branes into KK Monopoles

We would like to investigate whether a large number of M5 branes can polarize into KK monopoles. If the self-interacting potential of a KKM wrapped on a 2 sphere, with a very large M5 charge has a minimum when the 2-sphere radius is nonzero, polarization occurs. In order to find this self interacting potential we first find the action for a test KKM with large M5 charge $(n)$, in the potential created by a very large number $N >> n$ of M5 branes. Anticipating, this test potential will be found to be independent of the positions of the $N$ M5 branes. The full self interacting potential can be found by bringing from infinity test KKM/M5 shells. Since the potential of these shells does not depend on the positions of the other M5 branes, the probe calculation with $n$ replaced by $N$ will give the full self-interacting potential.

3.1 The KKM action

As we explained in the introduction, a Kaluza-Klein monopole in 11d supergravity is an object which has 7 longitudinal directions, 3 transverse, as well as a special isometry direction. When one compactifies along this isometry direction one obtains a D6 brane in IIA. If we express the D6 brane tension in terms of the radius of the 11’th direction ($R_{11}$) and the 11’th dimensional Planck length $l_P$, we find that it is proportional to $R_{11}^2$:

$$T_{D6} = \frac{1}{g(2\pi)^6 \alpha'^{3/2}} = \frac{R_{11}^2}{l_P^9 (2\pi)^6}.$$  \hspace{1cm} (3.1)

of $AdS_7$ to the mass parameter of the bulk perturbations (2.23, 2.18, 2.19). We start with the action of the M5-brane in an arbitrary sugra background [14]. After fixing the $\kappa$-symmetry, half of the $\theta$ superspace coordinates are eliminated, while the other half become fermions on the worldvolume. Thus, by specifying our perturbed $AdS_7$ background, we can search for terms bilinear in fermions. For instance, a Lorentz vector “mass” term comes from the WZ term (specifically from the pull-back of $C_6(X, \theta)$, where $C_6$ is no longer zero by gauge choice). However, since the only parameter we introduced is the bulk mass (in units where the AdS radius is one), this implies that we have at most a numerical constant relating the boundary mass to the bulk mass.

Using $g = l_p^{-3/2} R_{11}^{3/2}$ and $\alpha' = l_p R_{11}^{-1}$.
Therefore, the KKM cannot be thought of as some 7 brane wrapped on the isometry direction. This direction is special, and gets special treatment in the action.

The action for a Kaluza-Klein monopole (KKM) was given by [15, 16]. As expected, it depends on the background geometry Killing vector $k^\Lambda$ corresponding to the isometry direction.

In the unperturbed $AdS_7 \times S^4$ background this Killing vector was found to be $k^\Lambda^0 = \delta^{\Lambda 11}$. In the perturbed background, the variation $\delta k^\Lambda \equiv k^\Lambda^1$ can be found by solving

$$D^0_{(\Lambda} k^1_{\Pi)} = \frac{1}{2} (h_{11\Lambda,\Pi} + h_{11\Pi,\Lambda} - h_{11\Pi,11})$$

(3.2)

It may not come as a surprise that $k^\Lambda$ is still proportional with $\delta^{\Lambda 11}$, as one can easily check. However, since this Killing vector measures the length of the isometry direction at infinity, we normalize it as

$$k^\Lambda = \frac{R_{11}}{l_P} \delta^{\Lambda 11}$$

(3.3)

The KKM action is the sum of a Born-Infeld (BI) and a Wess-Zumino (WZ) piece. The BI part (in an 11-dimensional sugra background) reads

$$S_{BI} = -\frac{1}{(2\pi)^6 l_P^6} \int d^7\xi k^2 \sqrt{|\det(D_aX^\Lambda D_bX^\Pi g_{\Lambda\Pi} + (2\pi \alpha')k^{-1} F_{ab})|}$$

(3.4)

where the covariant derivative is defined with respect to a gauge field $D_aX^\Lambda = \partial_a X^\Lambda + A_a k^\Lambda$ which is not an independent field: $A_a = k^{-2} \partial_a X^\Lambda k_\Lambda$. The field strength of the one-form field $b = b_0 d\xi^a$ living on the worldvolume is

$$F_{ab} = \partial_a b_b - \partial_b b_a + 1/(2\pi \alpha') \partial_a X^\Lambda \partial_b X^\Pi k^\Sigma C_{\Sigma\Lambda\Pi}$$

(3.5)

The WZ piece is

$$S_{WZ} = \frac{2\pi \alpha'}{(2\pi)^6 l_P^6} \int K_7$$

(3.6)

where the $K_7$ is the field strength of a non-propagating worldvolume 6-form

$$K_7 = d\omega_6 + \frac{1}{2\pi \alpha'} (i_k N_8) + (i_k C_6) \wedge \mathcal{F}$$

$$-\frac{2\pi \alpha'}{(3!)^2} D X^\Lambda \wedge D X^\Pi \wedge D X^\Sigma C_{\Lambda\Pi\Sigma} \wedge (i_k C_3) \wedge (i_k C_3)$$

$$-\frac{1}{3!} D X^\Lambda \wedge D X^\Pi \wedge D X^\Sigma C_{\Lambda\Pi\Sigma} \wedge (i_k C_3) \wedge db$$

$$-\frac{1}{2 \cdot 3!} (2\pi \alpha') D X^\Lambda \wedge D X^\Pi \wedge D X^\Sigma C_{\Lambda\Pi\Sigma} \wedge db \wedge db$$

$$-\frac{1}{3!} (2\pi \alpha')^2 A \wedge db \wedge db \wedge db$$

(3.7)

where, following [16] we wrote explicitly the pull-backs only when the covariant derivatives where involved. The 8-form $N_8$ is by definition the Poincaré dual of the Killing 1-form.

*Identifying the reduced KKM action with the D6 action, not only allows us to relate their tensions, but the BI actions as well. As a consequence, the factor $2\pi \alpha'$ appears in the KKM action, with the understanding that it is a parameter which depends on $l_P$ and $R_{11}$ as specified in (3.1).
The action of the KKM is invariant under certain gauge transformations. We list only the ones which are relevant for this paper:

\[
\begin{align*}
\delta C_3 &= d\chi_2 \\
\delta i_k C_6 &= d\chi_5 - \frac{1}{2} di_k \chi_2 \wedge C_3 + \frac{1}{2} d\chi_2 \wedge i_k C_3 \\
\delta i_k N_8 &= d\chi_5 \wedge (i_k C_3) + \frac{1}{3} d\chi_2 \wedge i_k C_3 \wedge i_k C_3 - \frac{1}{3} di_k \chi_2 \wedge C_3 \wedge i_k C_3
\end{align*}
\] (3.8)

From (3.10) we infer a gauge invariant definition of the field strength of \(i_k N_8\):

\[
\begin{align*}
di_k N_8 &= i_k d^* k - i_k C_6 \wedge i_k F_4 - \frac{1}{6} C_3 \wedge i_k C_3 \wedge i_k F_4 - \frac{1}{6} i_k C_3 \wedge i_k C_3 \wedge F_4
\end{align*}
\] (3.11)

We can convince ourselves that by varying (3.11) we get the same contributions we obtain by taking an exterior derivative in (3.10).

### 3.2 The KKM probe

We begin by testing the perturbed background geometry with a KKM probe. The monopole is wrapped on a 2-sphere in the \((578)\) plane (anticipating, this will be one of the planes where polarization is most likely to occur) has 5 flat directions along \(x^0, \ldots, x^4\), and has an M5 charge \(n \ll N\). The potential for more general orientations of the two-sphere will be considered in the following sections. The isometry direction is \(k^A = (R_{11}/l_P) \delta^{A11}\). As discussed in the previous subsection this is a Killing vector of the perturbed \(AdS_7 \times S_4\) geometry.

As one can see from the WZ term (3.7), the monopole will have M5 charge \(n\) if we turn on its worldvolume 2-form field strength \(F_2\):

\[
\begin{align*}
\frac{1}{(2\pi)^6 l_P^6} \frac{R_{11}}{l_P} \int_{S_2} (2\pi \alpha') F_2 &= (2\pi R_{11}) T_{M5} \ n = \frac{2\pi R_{11}}{(2\pi)^5 l_P^6} n,
\end{align*}
\] (3.12)

which gives

\[
(2\pi \alpha') F_2 = (2\pi \alpha') F_{\theta \phi} d\theta \wedge d\phi = \frac{1}{2} 2n \pi l_P^2 \sin \theta d\theta \wedge d\phi
\] (3.13)

We work in the regime in which the M5 contribution to the BI action of the probe monopole is dominant. Since we know the form of \(F_2\), we can rewrite the BI action as

\[
\begin{align*}
- \frac{1}{(2\pi)^6 l_P^6} \int_{S_2} d\theta d\phi \ k^2 \sqrt{\det G_{||}} \sqrt{\det G_{ab} + \det \left(\frac{(2\pi \alpha') F_{ab}}{k}\right)}
\end{align*}
\] (3.14)

where \(G_{||}\) denotes the induced metric on the 5 space-time directions common to the KKM and the M5 brane, and \(a, b\) are coordinates on the 2-sphere. We have assumed the integration over \(dx^0 \ldots dx^4\) to be implicit, and we will assume this throughout this section. Using the induced metric \(G_{ab} = \{G_{\theta \theta} = Z^{2/3} r^2, G_{\phi \phi} = Z^{2/3} r^2 \sin^2 \theta\}\), we find the condition that the M5 contribution to the BI action dominates to be

\[
\begin{align*}
\det \left(\frac{(2\pi \alpha') F_{ab}}{k}\right) &= \frac{(2\pi \alpha')^2}{2k^2} \det(G_{ab}) F_{ab} \gg \det G_{ab},
\end{align*}
\] (3.15)
or

\[
\frac{(2\pi)^2 l_P^6 n^2 Z^{-4/3} r^{-4} Z^{1/3}}{4R_{11}^2} \gg 1.\tag{3.16}
\]

Under this assumption we can Taylor expand the BI action

\[
-\frac{1}{(2\pi)^6 l_P^6} \int_{S^2} d\theta d\phi k^2 \sqrt{\det G} \sqrt{\det \left( \frac{(2\pi\alpha')}{k} \mathcal{F}_{ab} \right)} \left[ 1 + \frac{k^2 \det G_{ab}}{2(2\pi\alpha')^2 \det \mathcal{F}_{ab}} \right] \tag{3.17}
\]

and find the leading contribution

\[
-\frac{R_{11}}{2(2\pi)^6 l_P^6} \int_{S^2} d\theta d\phi Z^{-1} n \sin \theta = -\frac{R_{11}}{(2\pi)^6 l_P^6} Z^{-1} n \tag{3.18}
\]

and the subleading one

\[
-\frac{R_{11}^2}{(2\pi)^6 l_P^6} \int_{S^2} d\theta d\phi Z^{-1/2} Z^{-5/6} Z^{4/3} r^4 \sin \theta = -\frac{2R_{11}^3 r^4}{(2\pi)^6 l_P^6}. \tag{3.19}
\]

The leading terms in the BI and WZ actions represent M5-M5 interaction, and cancel as expected. More explicitly, the WZ leading contribution is

\[
\frac{1}{(2\pi)^6 l_P^6} \int i_k C_0^0 \wedge (2\pi\alpha')^2 F_2 = \frac{1}{(2\pi)^6 l_P^6} \frac{R_{11}}{2} \int_{S^2} d\theta d\phi Z^{-1} n \sin \theta, \tag{3.20}
\]

and cancels against (3.18).

The subleading contribution to the WZ term comes from \( \int i_k N_8 \). When the bulk graviphoton and three form perturbation is turned on, several terms in (3.11) (\( i_k \ast^0 dk^1 + i_k \ast^1 dk^0 + i_k C_0^0 \wedge i_k F_4^1 \)) become nonzero. We find

\[
di_k N_8 = \frac{R_{11}^2}{l_P^6} \left[ Z^{-1} \left( \ast_5 d(h Z^{1/3} + G_3) \right) \right] \wedge dx^0 \ldots \wedge dx^4 \tag{3.21}
\]

which has the solution \( i_k N_8 = (1/3!) (R_{11}^2 / l_P^6) T_{mn} a^m dx^n \wedge dx^0 \ldots \wedge dx^4 \). It is interesting to notice that \( i_k N_8 \) only depends on the combination \( Z^{-1}[\ast_5 d(h Z^{1/3} + G_3)] \), which is independent of the distribution of the M5 branes, as we discussed in section 2.2.

We can see from (2.19) that the WZ subleading contribution is the largest if the 2 sphere is in the (578) or the (679) planes (depending on the signs and magnitudes of \( m_1 \) and \( m_2 \)). We will assume from now on that both \( m_1 \) and \( m_2 \) are positive, and call their sum \( m \). Thus the test brane will have a local minima in both the (578) and (679) planes when \( m_2 \approx 0 \), and only in the (578) for \( m_2 \approx m_1 \).

Thus, the WZ contribution to the action is

\[
\frac{1}{(2\pi)^6 l_P^6} \int i_k N_8 = \frac{4\pi R_{11}^2}{(2\pi)^6 l_P^6} r^3. \tag{3.22}
\]

\[\text{Note this condition is not independent of } r \text{ as one might expect in a conformal theory. The source of nonconformality is the dependence of the effective size of the 11’th dimension (which is also the KKM mass) on } r.\]
Besides the BI and the WZ contributions found above there is yet another contribution of the same order which comes from the second order perturbation to the fields, and which will be discussed in the next chapter. The polarization radius $r_0$ is of the same order as the radius where $V_{BI} + V_{WZ}$ is minimized. Thus

$$r_0 \sim \frac{nm^3}{R_{11}}.$$  \hspace{1cm} (3.23)

Substituting in (3.16) we find that we can treat the KKM as a probe if

$$n \gg R_{11}mN.$$  \hspace{1cm} (3.24)

We can also ask if the strength of the perturbation is anywhere larger than the energy density in the unperturbed AdS background

$$\frac{|F_4^1|^2}{|F_4^0|^2} \sim \frac{r^{-1}Rm^2}{R^{-2}} \sim \frac{m^2l_p^4N}{r}.$$  \hspace{1cm} (3.25)

The fraction (3.25) grows as $r$ decreases towards $r_0$. Nevertheless, inside the shell we expect this ratio to be of the same order as at $r_0$. By estimating (3.25) at the polarization radius we can see that the perturbation is always small if (3.24) holds.

We would like to examine more the physical implications of (3.24), which is necessary for this polarization picture to be correct. Since in the final computation $n$ will be replaced by $N$, (3.24) is equivalent to $R_{11}m \ll 1$, which seems to imply that we are in fact describing only 5-dimensional physics. What we are describing is in fact the perturbed (2,0) theory compactified on a circle of radius $R_{11}$.

We can estimate the effective radius of the 11th dimension $R_{11}^{\text{physical}} = R_{11}Z^{-1/6}$ we find its lowest bulk value to be $N^{1/3}(mR_{11})^{1/2}l_p$ (except very near the KKM shell, when physics is described by the theory living on the KKM). In the spirit of [17], this implies that for $N \gg (mR_{11})^{-3/2}$ (which is the regime we will be considering), the 10 dimensional string coupling constant is large everywhere in the bulk, and thus 11 dimensional supergravity is the appropriate perturbative description. For smaller energy scales the perturbative description of our theory will be described by a IIA supergravity in the bulk, and for even smaller energy scales $R_{11}m \ll 1/N$ the super-Yang Mills theory on the D4 branes. We will also find that the bulk describes the properties of the objects living in this theory, which are the objects living on coincident M5 branes.

The fact that we can only have a polarization state in the regime $mR_{11} < 1$ seems puzzling if the theory we are dealing with is supersymmetric. Indeed, one does not expect a vacuum to just go away, as we increase adiabatically $m$ or $R_{11}$. Nevertheless, this is exactly what happens. The answer to this puzzle will be found in the next chapter - in order to polarize the branes one needs to break all supersymmetry.

### 3.3 The full problem

We use the results of the previous section to solve the full problem of a self-interacting KKM with large M5 charge. The self-interaction potential can be found by
bringing from infinity probe KKM’s with M5 charge in the background of polarized M5 branes. As we will immediately show, the potential these probes feel is unaffected by the distribution of the M5 branes. Therefore, the total self-interaction potential will be the same as the potential of a single shell, with \( n \) replaced by \( N \).

The geometry of a distribution of M5 will still be given by (2.4), but the warp factor \( Z \) is a superposition of harmonic functions. For the case of \( N \) M5 branes uniformly distributed on a 2-sphere of radius \( r_0 \) in the \((578)\) hyperplane, the warp factor is

\[
Z = \frac{R^3}{2r_0r_1} \left( \frac{1}{\sqrt{(r_0 - r_1)^2 + r_1^2}} - \frac{1}{\sqrt{(r_0 + r_1)^2 + r_1^2}} \right)
\]

where \( r_1 \) is the radius in the \((578)\) hyperplane and \( r_2 \) is the radius in the \((69)\) plane. For large \( r_1, r_2 \) we recover the \( AdS_7 \) warp factor. If the M5 branes are distributed on several shells, then the total warp factor will be the sum of each shell \( Z \) factor.

As we explained in Section 2, equations (2.11) and (2.13) imply that \( Z^{-1}[g_5 d(hZ^{1/3}) + G_3] \) is a harmonic function, and therefore it is given by its value at infinity, regardless of the value of \( Z \). This is exactly the combination that enters the WZ term of the KKM. Also, in the large \( N \) limit, the BI term can be expanded as in (3.17), and the cancellation of the leading WZ and BI takes place as before. The subleading terms are unchanged, and have no \( Z \) dependence. We thus conclude, that for each M5 shell the potential is the same as the probe potential.

The most general polarized configuration consistent with the symmetries is a KKM wrapped on a two-ellipsoid:

\[
x_7 = x_7 \cos \alpha, \quad z_1 = z_1 \sin \alpha \cos \beta, \quad z_2 = z_2 \sin \alpha \sin \beta,
\]

where \( z_1 \) and \( z_2 \) are defined in (2.15), \( \alpha \) and \( \beta \) parameterize the embedding, and \( x_7, z_1 \) and \( z_2 \) give the length and orientation the semiaxes. It is a straightforward exercise to express the contribution of the BI and WZ potentials in terms of the semiaxes of the probe ellipsoid

\[
V_{BI+WZ} = \frac{2R_1^3}{3n(2\pi)^6l_P^2} (|z_1|^2 x_7^2 + |z_2|^2 x_7^2 + |z_1|^2 |z_2|^2)
\]

\[
- \frac{4\pi R_1^2}{(2\pi)^6l_P^9} \text{Re}(m_1 z_1 \bar{z}_2 x_7 + m_2 z_1 z_2 x_7),
\]

where we have dropped the subscripts on the semiaxes to keep notation simple. From now and throughout this chapter we will denote the semiaxes by \( x_7, z_1 \) and \( z_2 \) without an underline.

There is another term in this potential which comes from a second order correction to the background. This term has the same relevance as the first two, and is hard to compute explicitly (it has been done in [8]). We have already discussed that by giving masses to either of the two fermions we break half of the original amount of supersymmetry \(((2,0) \text{ in 6 dimensions})\). Thus, when one of the fermion masses \( (m_2 \text{ in} \)
this case) is zero, we can use supersymmetry to complete the squares in (3.28). This gives the complete effective potential

$$V_{\text{total}} = \frac{2R_{11}^3}{3n(2\pi)^6 l_P^2} \left[ (|z_1|^2 x_7^2 + |z_2|^2 x_7^2 + |z_1|^2 |z_2|^2) - \frac{6\pi nl_P^3}{R_{11}} \text{Re}(m_1 z_1 \bar{z}_2 x_7) \right] + \left( \frac{3\pi nl_P^3}{2R_{11}} \right)^2 m_1^2 (|z_1|^2 + |z_2|^2),$$

(3.29)

is derived in the usual way from the superpotential

$$W = \sqrt{\frac{2R_{11}^3}{3n(2\pi)^6 l_P^2}} \left[ z_1 z_2 x_7 - \frac{3\pi nl_P^3}{4R_{11}} m_1 (z_1^2 + z_2^2) \right]$$

(3.30)

The potential (3.29) has only one minimum at $x_7 = z_1 = z_2 = 0$. We conclude that the polarization of M5 branes into a wrapped KKM is not possible if we have any supersymmetry left.

We remember that in [1], one had to give mass to at least 3 of the 4 complex Weyl fermions in order for polarization to occur. In our case (as well as in the D4-D6 case), we only have two fermions, so one can either give mass to one (1/2 of the fermions) which preserves $\mathcal{N} = 1$ supersymmetry but is not enough for polarization, or give mass to both fermions, break supersymmetry completely, and polarize the branes **.

There are two ways one can bypass this problem ††. The first one is to consider an “almost supersymmetric” situation, when the susy breaking mass $m_2$ is far smaller than $m_1$. One can still use supersymmetry to complete the squares in (3.28), and find the potential to be

$$V = \frac{2R_{11}^3}{3n(2\pi)^6 l_P^2} \left[ (|z_1|^2 x_7^2 + |z_2|^2 x_7^2 + |z_1|^2 |z_2|^2) - \frac{6\pi nl_P^3}{R_{11}} \text{Re}(m_1 z_1 \bar{z}_2 x_7 + m_2 z_1 z_2 x_7) \right] + \left( \frac{3\pi nl_P^3}{2R_{11}} \right)^2 (m_1^2 |z_1|^2 + m_2^2 |z_2|^2 + 4m_2^2 x_7^2)$$

(3.31)

In this case, the M5 branes will polarize into a monopole wrapped on an ellipsoid. The new potential has a minimum at

$$z_1 = z_2 = \sqrt{\frac{m_1 m_2}{2} \frac{3\pi nl_P^3}{R_{11}}} = x_5 = x_8, \quad x_7 = \frac{3\pi nl_P^3}{2R_{11}} m_1$$

(3.32)

Clearly, the ellipsoid is very elongated in the $x_7$ direction, and in the supersymmetric case it degenerates into a line.

The second way to find the last term of the potential (3.28) is to consider an SO(3) invariant nonsupersymmetric configuration. The last term of (3.29) represents a mass

**Naively one might think that by T-dualizing the D3-D5 polarization of [1] to a D4-D6 configuration we do not lose supersymmetry. We can see that this is not the case, both by investigating the theory on the brane (one needs to pair the D3 fermions two by two to obtain D4 brane fermions, and thus cannot consistently give mass to one fermion and a half), or by realizing that the bulk in [1] has no isometry along the T-duality direction.

††Similar methods and a more thorough discussion can be found in [5].
given to 4 of the 5 scalars in our theory. The most general scalar mass term can be written as:

$$m^2(\Phi_5^2 + \Phi_6^2 + \Phi_8^2 + \Phi_9^2 + \Phi_7^2) + \mu_{ij}\Phi_i\Phi_j,$$  \hfill (3.33)

where the first piece is an $L = 0$ combination and the second one is a combination of various $L = 2$ modes. In the supersymmetric case $\Phi_7$ is part of the same multiplet as the self-dual 3-form and half of the fermions, and remains massless, while the other 4 scalars receive identical masses. Thus, supersymmetry constrains the last term of (3.33) to be

$$\mu_{ij}\Phi_i\Phi_j = m^2(\Phi_5^2 + \Phi_6^2 + \Phi_8^2 + \Phi_9^2 - 4\Phi_7^2)$$ \hfill (3.34)

In the nonsupersymmetric case only the $L = 0$ mode is determined by the back reaction of the first order fields. The $L = 2$ modes can be specified independently in the boundary theory (this issue is discussed very briefly here, for a more thorough treatment we refer the reader to [1, 5, 6, 9]). Let us consider the case when both fermions have equal masses $m_1 = m_2 = m$. We can turn off (3.34) to restore the SO(5) symmetry between $x_5, x_6, x_8, x_9$ and $x_7$, and turn on an $L = 2$ mode which preserves the SO(3) symmetry in the polarization plane spanned by $x_5, x_8, x_7$.

$$-\Lambda(\Phi_5^2 + \Phi_6^2 + \Phi_8^2 - \frac{3}{2}\Phi_6^2 - \frac{3}{2}\Phi_8^2)$$ \hfill (3.35)

For $\Lambda$ of order $m^2$ or higher, a state in which the KKM is wrapped on a sphere of radius of order $r_0$ will have energy less than the vacuum energy. Thus, the M5 branes will polarize into a KKM wrapped on a 2-sphere of radius of order $r_0 \sim (nml^3_p)/R_{11}$, in the 578 hyperplane.

When there is more than one shell, the potential is the sum of individual pieces of the form (3.28). One can also estimate the polarization potential for $Q$ coincident KKM shells and find that $r_{polarization} \sim r_0/Q$. This configuration should also be part of the many vacua the perturbed theory should have.

Thus, the vacua of the theory will consist of multiple M5-KKM (ellipsoidal or spherical) shells. We will concentrate in the rest of this paper on the SO(3) invariant nonsupersymmetric theories which correspond to polarization into a 2-sphere. The generalization to the almost supersymmetric theories can be easily done.

### 3.4 The near shell solution

The KK monopole solution without M5 charge, is given by the $R^7 \times$ Taub-NUT space,

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + V(r)(dx^{11} + 4M(1 - \cos \theta)d\phi)^2$$

$$+ V(r)^{-1}(dr^2 + r^2d\theta^2 + r^2\sin^2 \theta d\phi^2)$$

$$V(r)^{-1} = 1 + \frac{4M}{r}.$$ \hfill (3.36)

Near $r = 0$ the metric $ds^2 \sim (dr^2 + r^2d\theta^2 + r^2\sin^2 \theta d\phi^2 + r^2(dx^{11}/4M)^2)$ has a nut singularity which is removed only if $x^{11}$ has period $16\pi M = 2\pi R_{11}$. Thus the charge of
the monopole is proportional to the length of the periodic isometry coordinate. This explains why the effective tension of the monopole scales with $R_{11}^2$. The requirement for the periodicity of $x^{11}$ can also be extracted from the requirement that there is no Dirac string singularity (this is explained in more detail in [18]).

One might be worried that upon adding a very large M5 charge to the monopole, the metric would be changed and the condition for not having a Dirac string would change from $R_{11} \sim M$, to some formula which also includes the M5 charge. We thus need to examine the metric of a flat KKM with large M5 charge. This metric can be easily found [19] by lifting the D4-D6 metric [20] obtained by T-dualizing smeared tilted D5 branes. If we call $\alpha$ the original D5 tilting angle, and $\tilde{N}$ the original number of D5 branes, we obtain the flat KKM/M5 metric to be:

$$ds^2 = h^{-1/3}dx_{\parallel}^2 + h^{2/3}(dx_7^2 + dx_8^2) + Z h^{-1/3}(dr^2 + r^2d\theta^2 + r^2\sin^2 \theta d\phi^2)$$

$$+ h^{2/3}Z^{-1}(dx^{11} + \tilde{N}l_P \cos(1 - \cos \theta)d\phi)^2,$$  \hspace{1cm} (3.37)

where

$$Z = 1 + \frac{\tilde{N}l_P}{r},$$  \hspace{1cm} (3.38)

$$h = \frac{1 + \tilde{N}l_P/r}{1 + \tilde{N}l_P \cos^2 \alpha/r}$$  \hspace{1cm} (3.39)

We can see that when $\cos \alpha = 0$, $h$ and $Z$ are equal and we recover the metric of multiple M5 branes smeared on a 2-plane. When $\cos \alpha = 1$, $h = 1$ and we recover the plain KKM solution (3.36). By comparing (3.37) with (3.26) near the shell we can identify the coordinates $r$ and $\rho = \sqrt{(r_0 - r_1)^2 + r_2^2}$. We can also express the dummy coefficients $\tilde{N}$ and $\alpha$ in (3.39) in terms of our physical near shell parameters by comparing (3.37) with (3.26) and (3.36). We find the near horizon limits of $Z$ and $h$:

$$Z = \frac{\pi l_p^3 N}{2r_0^2 \rho}, \quad h = \frac{\pi l_p^3 N}{2r_0^2 \left( \rho + \frac{R_{11}^2}{2\pi l_P^2 N} \right)}$$  \hspace{1cm} (3.40)

We can also express the factor $\tilde{N} \cos \alpha$ from (3.37) as:

$$\tilde{N}l_P \cos \alpha = R_{11}/2.$$  \hspace{1cm} (3.41)

Thus, $dx^{11}$ and $d\phi$ appear with the same relative coefficient as in in (3.36) and therefore the condition for no Dirac string is unaffected by the presence of M5 charge.

The KKM contribution to the metric becomes important only very close to the shell, at $\rho \sim \frac{R_{11}^2}{2\pi l_P^2 N} \sim N m^2 l_P^3 \equiv \rho_{\text{crossover}}$. We can see from (3.23) that $\rho_{\text{crossover}} \sim mR_{11} \ll 1$, and thus the picture of the KKM wrapped on a 2-sphere is consistent.

We can find the range of validity of the supergravity approximation by comparing $l_p$ to the radius of the transverse sphere in (3.37). We find supergravity valid if

$$\left( \frac{\rho}{\rho_{\text{crossover}}} \right)^{3/2} N m^2 R_{11}^2 \gg 1,$$  \hspace{1cm} (3.42)

which is easily satisfied at large $N$, even for $\rho < \rho_{\text{crossover}}$. We also know that for $\rho < \rho_{\text{crossover}}$ the geometry becomes that of a KKM, which is regular everywhere. Thus, if supergravity is valid at $\rho \sim \rho_{\text{crossover}}$, it is valid everywhere.
4 The properties of the vacua of the perturbed theory

We are now at the stage to begin exploring the properties of the M5-KKM vacua we found above. We know that the original (2,0) theory has little strings, and “baryon vertices”. We expect to find these objects in the perturbed theory as well. Moreover, since now we have many vacua we expect to have domain walls separating them. We would like to find the bulk M-theory configurations corresponding to these objects.

4.1 Little strings

The nonperturbed (2,0) theory has two types of little strings. The “quark” little strings are massive, and correspond to M2 branes lowered from infinity and ending on the stack of M5 branes. The other little strings are M2 branes going from one M5 brane to another, and are massless when the M5 branes are coincident.

The interaction energy between two “quark” little strings can be found by estimating the energy of the bulk M2 brane whose boundary the strings are [21]. In the UV, the M2 brane stays very near the AdS boundary, and thus does not know that the M5 branes sourcing the geometry are polarized. This is expected - giving a small mass to massless fields should not affect UV physics. In the infrared, the M2 brane gets near the M5-KKM shell and interacts with it, and thus the physics will change.

Let us consider two largely separated parallel strings which are the boundary of an M2 brane which comes very near the KKM-M5 shell. In the limit when the strings are very far apart, the largest contribution to the interaction energy comes from the part of the M2 brane which is parallel to the shell.

Since we have broken Lorentz invariance by turning on an R-current, the interaction energy will depend on the orientation as well as on the direction of separation of the little strings.

For the sake of clarity, let us explain in a bit more detail what the setup is. The polarized M5’s span the 0, 1, 2, 3, 4, 11 directions, with the 11th direction compactified on a circle of radius $R_{11}$. The KKM in which the M5’s polarize has the topology $R^4 \times S_2$, with the 11th direction being the isometry direction. If we use polar coordinates in the plane of the $S_2$, the KKM is aligned along 0, 1, 2, 3, 4, $\theta, \phi$. The geometry at infinity is still $AdS_7 \times S_4$, with the boundary of $AdS_7$ spanned by 0, 1, 2, 3, 4, 11. The radial coordinate of $AdS_7$ is $r$.

We distinguish two possible situations, depending on the orientation and separation of the “quark” little strings:

i) None of the directions of the M2 brane is 11. This corresponds to non-11 strings separated along any direction but 11. The action of the M2 brane is proportional to the square root of the determinant of the induced metric. From (3.37) we find this to be $\sqrt{\det(G_{\text{induced}})} = h^{-1/2}$. Since $h$ remains finite as we approach the shell ($\rho \to 0$), the interaction energy of two little strings is proportional to their separation. Thus the strings are confined.
One can easily estimate the tension of the confining “flux tube”, However, there is a caveat - the supergravity approximation is valid at $\rho \sim \rho_{\text{crossover}}$ only for $Nm^2R_{11}$ large enough (3.42). Unless this condition is satisfied, we cannot use (3.37) at $r \approx \rho_{\text{crossover}}$ to reliably compute this tension. Thus,

$$T = T_{M2}h^{-1/2}\big|_{\text{min}} \sim \frac{r_0^2R_{11}}{l_p^6N} \sim \frac{m^2N}{R_{11}}. \quad (4.1)$$

ii) One of the direction of the M2 brane is 11. This corresponds to either non-11 strings separated along the 11'th directions, or to (0,11) strings separated along any other direction. In this case $\text{det}(G_{\text{induced}}) = Z^{-1/2}$. Since $Z^{-1}$ goes to zero as $r \to 0$, it does not cost any energy to take the strings further apart. Thus the strings are “screened”. The M-theory explanation of this phenomenon is that the 2 sides of the M2 brane attach to the shell, and they are free to move apart.

We also expect the tensionless strings to get an effective tension as we turn on $m$. We can use a naïve argument to estimate this tension. The original tensionless strings come from M2 branes stretched between coincident M5 branes. When the $N$ M5 branes are spread on a 2-sphere of radius $r_0$, the average separation between them will be of order $r_0N^{-1/2}$, which will give an effective tension of order

$$T_{\text{effective}} \sim r_0N^{-1/2}(G_{55}G_{00}G_{ii})^{1/2} = r_0N^{-1/2} \quad (4.2)$$

for (0,i) little strings, and

$$T_{\text{effective}} \sim r_0N^{-1/2}(G_{55}G_{00}G_{11,11})^{1/2} = r_0N^{-1/2}hZ^{-1} \quad (4.3)$$

for (0,11) little strings. It appears that the (0,11) strings remain massless. However, more is happening. At the KKM core the size of the 11'th dimension is zero, and thus a string originally winding on the 11'th direction disappears from the spectrum.

### 4.2 Domain walls

The perturbed theory has many isolated vacua, which must be separated by domain walls. If the two vacua have the same number of KKM’s but different distributions of M5 charge, the KKM’s will generically intersect on a 2 sphere on which they will exchange M5 charge. This bulk configuration is dual to the boundary theory domain wall. The tension of the domain wall is given by the bending tension of the configuration.

If the vacua have different numbers of KKM’s, $Q_1 \neq Q_2$, the monopoles intersect again on a 2-sphere where they exchange M5 charge. By KKM charge conservation, a KKM with charge $Q_2 - Q_1$ will fill the 3-ball whose boundary the 2-sphere is. The tension will have an extra piece coming from the energy of the KKM’s which fill the 2-sphere.

If we do not have any supersymmetry, the two vacua separated by the domain wall will have generically different energies, and thus the domain wall will be moving. In the limit in which supersymmetry is restored, the energies of the vacua are almost the same, and the domain walls will be almost stationary.
4.3 Baryon vertices

In the unperturbed $AdS_7 \times S^4$ background the string baryon vertex is an M5 brane wrapped on $S_4$, with $N$ M2 branes lowered from the boundary ending on it. This configuration has been discussed thoroughly in [22, 23].

Let us analyze what happens to the baryon vertex in the background of polarized M5’s. In the UV the physics is not affected by the perturbation, and thus the baryon vertex corresponds to an M5 brane wrapped on $S_4$. As we flow to the IR, the wrapped M5 eventually reaches the KKM shell and crosses it. In order to see if any objects are created via the Hanany-Witten effect [24], let us “zoom in” on a region near the shell where the branes look almost flat and $\hat{r} \parallel \hat{x}_5$.

Since the 11’th direction was made special by turning on an R-current, we will first analyze a baryon vertex which is not extended along the 11’th direction, and then analyze one which is. Let us consider a baryon vertex M5 brane locally extended along the 0, 1, 6, 7, 8, 9 directions, and a KKM locally extended along 0, 1, 2, 3, 4, 6, 7. We claim that when we bring the M5 brane along $x_5$ and cross the KKM another M5 brane is created, extended in the 0, 1, 5, 6, 7, 11 directions. It appears puzzling that the new brane is extended along the 11’th direction. Nevertheless, we believe that this is indeed the case. There are three arguments one can give to support this. The first is that one obtains this configuration by lifting to 11 dimensions the Hanany-Witten effect at the crossing of an NS5-brane and a D6-brane, when a D4-brane is created transverse to the other two. The second is that, the KKM end of the created brane is magnetic with respect to the 2-form field strength on the monopole, as it should be. The third one will follow when we investigate the properties of the new baryon vertex.

As one brings the wrapped M5 through the KKM shell, the M2 branes hanging from the boundary will end on the intersection of the created M5 with the KKM, and there will be nothing preventing the original wrapped M5 to shrink to zero size.

Thus, the bulk dual of the new baryon vertex is an M5 brane which fills the 3-ball whose boundary the KKM 2-sphere is, and which has one direction along the isometry direction. To justify the baryon vertex name, we look at the KKM action (3.7) term

$$\int d^7 \xi D x^\Lambda \wedge D x^\Pi \wedge D x^\Sigma \wedge C_{\Lambda \Pi \Sigma} \wedge d b \wedge d b$$

(4.4)

where $b$ is the one-form on the worldvolume of the monopole. Let us denote $d b$ by $F_2$. Due to the magnetic ending of the created M5, the Bianchi identity gets modified and this term will no longer be gauge invariant under the background gauge transformation $\delta C_3 = d \chi_2$:

$$\delta \int F_2(\text{monopole}) \wedge F_2(\text{dissolved}) \wedge C_3 = - \int d F_2(\text{monopole}) \wedge F_2(\text{dissolved}) \wedge \chi_2$$

(4.5)

unless $N$ M2’s descend from the boundary to the M5-KKM junction.

One might also be worried that the new baryon vertex is extended along three directions (0,1,11), and might have a higher energy than the original baryon vertex which is only extended along (0,1). We can calculate this energy using the induced
metric (3.26)

\[
E = \frac{1}{l_p^6} R_{11} 4\pi \int dx^0 dx^1 dr_1 r_1^2 Z^{1/2}
\]

\[
= \frac{4\pi R_{11}}{l_p^6} \int dx^0 dx^1 \int_0^{r_0} dr_1 r_1^2 \sqrt{\frac{R^3}{2r_0 r_1} \left( \frac{1}{r_0 - r_1} - \frac{1}{r_0 + r_1} \right)}
\]

\[
= \int dx^0 dx^1 \sqrt{2\pi R_{11} (R r_0)^{3/2}}
\]

Substituting \( r_0 \) we obtain the new baryon vertex energy:

\[
E \sim \int dx^0 dx^1 \sqrt{N} m^{3/2} R_{11}^{-1/2}
\]

(4.6)

The energy of an M5 brane wrapped on \( S^4 \) of radius \( \rho_{crossover} \), namely the old baryon vertex at the point where M-theory phenomena like the Hanany-Witten effect described above appear, is

\[
E \sim T_{M5} \int dx^0 dx^1 Z(\rho) \rho_{crossover}^4 \sim \int dx^0 dx^1 N^2 m^4 R_{11}^2
\]

where we used the near shell metric to compute the induced metric on \( S^4 \).

Thus the condition that the new configuration has a lower energy is

\[
1 \ll N^{3/2} (m R_{11})^{5/2}
\]

(4.9)

which is easily satisfied for sufficiently large \( N \).

When the UV baryon vertex is extended along \( x_{11} \), there does not seem to be any object created when it passes through the KKM shell. This is consistent with the fact that strings extended along \( x_{11} \) are screened. Once the baryon vertex gets near the KKM shell these strings can attach to the it, and there is nothing to prevent the M5 brane from collapsing. Thus the baryon vertex extended along \( x_{11} \) disappears in the IR.

5 Conclusions and future directions

We investigated the effect of a fermion R-current perturbation on the (2,0) theory living on the worldvolume of \( N \) coincident M5 branes. We found the dual M-theory background to contain the M5-branes polarized into Kaluza-Klein monopoles, with the isometry direction of the monopoles on the worldvolume of the M5-branes. However, we showed that polarization is possible only when all the supersymmetry is broken.

We examined the properties of the vacua of the perturbed theory. We discovered that (0,i) tensionless little strings acquire a tension, while the (0,11) tensionless little strings disappear from the spectrum.

The “quark” little strings extended along the isometry directions are screened, and the strings which are not extended along \( x^{11} \) are confined. We also found the tension of the confining flux tube

19
We also found the new IR description of the string baryon vertex, by flowing the UV baryon vertex (which is an M5-brane wrapped on $S_4$) to the IR. A new M5-brane is formed when the UV vertex crosses the M5-KKM shell. This new baryon vertex fills the 3-ball whose boundary the KKM is, and is extended along the isometry direction. The remaining confined little strings will have precisely this baryon vertex. We checked that the new baryon configuration is energetically preferred to the old one. We have also given an M-theory description to the domain walls in the perturbed theory.

This paper completes a missing piece in the understanding of brane polarization, by showing that M5 branes can and do polarize into Kaluza-Klein monopoles. The new configuration might be a good setup for an entropy computation in the spirit of [8] to see if the correction to the entropy has $N^3$ behavior.

It is also interesting to ask if the $M_5 \rightarrow$ KKM polarization exhausts the series of possible brane polarizations. Besides the $D_p \rightarrow D(p+2)$, the $D_2,D_3 \rightarrow NS5$ polarizations explored in [1, 5], and the $F_1 \rightarrow D_4$ polarization which is the reduction of the $M_2 \rightarrow M_5$ polarization studied in [4], it looks like type IIA $F_1$ strings and $D_4$ branes might polarize into $D_2$ branes and respectively NS5 branes, wrapped on a one-sphere. This is supported by the fact that a $D_2/NS5$ brane wrapped on a one-sphere can have quantized $F_1/D_4$ charge. These possible polarizations would be however different from the ones studied so far, for two reasons.

First, the $F_1$ string and the $D_2$ brane come from the same object in M-theory. Lifting a $D_2$ wrapped on a one sphere with a large $F_1$ charge to 11 dimensions one obtains a helix shaped M2 brane wrapped on the 11'th direction. Even if this configuration might be stable in a perturbed theory, it is quite unlikely that it will be an isolated vacuum like before.

Second, one can also find the powers of $r$ which appear in the polarization potential (the equivalent of 3.29) ‡‡. The potential will contain three $r^2$ terms. Thus, in the supersymmetric case this potential will either vanish (in which case we expect to have a moduli space of possible polarization vacua) or have only one minimum at $r = 0$ (which means no polarization). In the nonsupersymmetric case a static configuration might exist at a nonzero radius (determined by the balance of potential terms from other sources). We believe it is worth investigating whether these polarization states exist at all, and what are their properties.

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6 Appendix

We list several properties of the antisymmetric 2- and 3-tensors which form a basis for forms on the transverse space.

‡‡ We thank W. Taylor for bringing this to our attention.
\[
V_3 = \frac{1}{3!} (x^q x^n x^p T_{qnp} + 2 \text{ more}) \, dx^m \wedge dx^n \wedge dx^p
\] (6.1)
\[
V_2 = \frac{1}{2!} (x^q x^i T_{qj} + 1 \text{ more}) \, dx^i \wedge dx^j
\] (6.2)
\[
T_2 - V_2 = *_5 V_3
\] (6.3)
\[
d(\ln r) \wedge V_3 = 0
\] (6.4)
\[
d(\ln r) \wedge *_5 V_3 = d(\ln r) \wedge *_5 T_3
\] (6.5)
\[
d(V_3) = -3d(\ln r) \wedge T_3
\] (6.6)
\[
d(*_5 V_3) = 2 \ln r \wedge *_5 T_3
\] (6.7)

References


