Geometry of Non-expanding Horizons and Their Neighborhoods

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Abstract

This is a contribution to MG9 session BHT4. Certain geometrically distinguished frame on a non-expanding horizon and in its space-time neighborhood, as well as the Bondi-like coordinates are constructed. The construction provides free degrees of freedom, invariants, and the existence conditions for a Killing vector field. The reported results come from the joint works with Ashtekar and Beetle [2].

In the quasi-local theory of black holes proposed recently by Ashtekar [1] a BH in equilibrium is described by a 3-dimensional null cylinder $\mathcal{H}$ generated in space-time by null geodesic curves intersecting orthogonally a space-like, 2-dimensional closed surface $S$. The standard stationarity of space-time requirement is replaced by the assumption that the cylinder has zero expansion, that is $\mathcal{H}$ is a non-expanding horizon. This implies, upon the week and the dominant energy conditions, that the induced on $\mathcal{H}$ (degenerate) metric tensor $q$ is Lie dragged by a null, geodesic flow tangent to $\mathcal{H}$. The geometry induced on $\mathcal{H}$ consists of the metric tensor $q$ and the induced covariant derivative $\mathcal{D}$. It is enough for the mechanics of $\mathcal{H}$ [1]. The geometry of a non-expanding horizon is characterized by local degrees of freedom. They are an arbitrary 2-geometry of the null generators space $S$, the rotation scalar, and certain tangential ‘radiation’ evolving along the horizon.
In the standard, Kerr-Newman case, the event horizon is equipped with a null Killing vector field. In our general non-expanding horizon case, however, a Killing vector field may not exit at all. Our first goal is a geometric condition which distinguishes a null vector field $\ell_0$ tangent to $\mathcal{H}$ and which is satisfied by the Killing vector field whenever it exists. We made extra assumptions about the stress energy tensor at $\mathcal{H}$ that are satisfied for the Maxwell and/or scalar and/or dylaton fields. The condition distinguishing the null vector field $\ell_0$ was obtained by making as many components of the tensor $[\ell, \mathcal{D}]_{bc}$ defined on $\mathcal{H}$ as possible zero, as we vary $\ell$. But here we give a more geometric definition of this choice. Due to the evolution equations of $\mathcal{D}$ along $\mathcal{H}$, there is a unique extension $(\dot{\mathcal{H}}, \dot{q}, \dot{\mathcal{D}})$ of $(\mathcal{H}, q, \mathcal{D})$ in an affine parameter along the null geodesics. We claim, that generically $\mathcal{H}$ admits a unique global crossection $S_0$ such that its expansion in the transversal null direction orthogonal to $S_0$ (this information is contained in $\dot{\mathcal{D}}$) is zero everywhere on $S_0$. Given the crossection $S_0$, there is a unique null vector field $\ell_0$ vanishing identically on $S_0$ and such that $\mathcal{D}_\ell \ell_0 = \kappa_0 \ell_0$, $\kappa_0 \neq 0$ being a constant. Fixing some value $\kappa_0(q, \mathcal{D})$ determines $\ell_0$ completely. The shear of $S_0$ vanishes in the null transversal direction orthogonal to $\mathcal{H}$, iff $\ell_0$ generates a symmetry of the geometry $(q, \mathcal{D})$. The commutator $[\mathcal{L}_{\ell_0}, \mathcal{D}]$ represents the tangential radiation, and $\mathcal{H}$ is not a Killing horizon unless the comutator is zero.

The rotation 1-form potential $\omega_0$ of $\ell_0$ is defined by $\mathcal{D} \ell_0 = \omega_0 \otimes \ell_0$. We define a good cut as a space-like section of $\mathcal{H}$ such that the pullback of $\omega_0$ thereon is a harmonic 1-form. The good cuts define a foliation of $\mathcal{H}$ invariant with respect to the flow of $\ell_0$, owing to $\mathcal{L}_{\ell_0} \omega_0 = 2 \delta \kappa_0 = 0$.

Given $\ell_0$ and the good cuts foliation, we determine a null frame $(m_0, \bar{m}_0, n_0, \ell_0)$ by using another null vector $n_0$ orthogonmal to the lives requiring $n_0 \mu \ell_0 \nu = -1$, and $R_{\mu \nu} K_{\mu} = 0$ where $K$ is the Gauss curvature of $\mathcal{H}$, generically non-constant.

In a neighborhood of $\mathcal{H}$, the good cuts foliation and the distinguished $\ell_0$ define a unique geodesic extension of the vector field $n_0$. It is used to extend the foliation and frame to the neighborhood.

The applications and results of this construction are $a)$ invariants of the horizon and of the neighborhood, $b)$ invariant characterization and true degrees of freedom of a horizon and of its neighborhood in the vacuum or Maxwell and/or scalar and/or dylaton case, $c)$ classification of the symmetric isolated horizons, $d)$ necessary and sufficient conditions for the existence
of a Killing vector field, and the control on the space-times not admitting a
Killing vector field.

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References

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