A study of possible temporal and latitudinal variations in the properties of the solar tachocline

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ABSTRACT

Temporal variations of the structure and the rotation rate of the solar tachocline region are studied using helioseismic data from the Global Oscillation Network Group (GONG) and the Michelson Doppler Imager (MDI) obtained during the period 1995–2000. We do not find any significant temporal variation in the depth of the convection zone, the position of the tachocline or the extent of overshoot below the convection zone. No systematic variation in any other properties of tachocline, like width, etc., is found either. Possibility of periodic variations in these properties is also investigated. Time-averaged results show that the tachocline is prolate with a variation by about 0.02\(R_\odot\) in its position. The depth of the convection zone or the extent of overshoot does not show any significant variation with latitude.

Key words: Sun: oscillations – Sun: interior – Sun: rotation

1 INTRODUCTION

Helioseismic data allow us to probe the structure and rotation rate of the solar interior (Gough et al. 1996; Thompson et al. 1996; Schou et al. 1998). With the accumulation of Global Oscillation Network Group (GONG) and Michelson Doppler Imager (MDI) data over the last five years, it has also become possible to study temporal variations in the rotation rate and other properties of the solar interior. It is generally believed that the solar dynamo operates in the region just below the convection zone, which is also the region where the tachocline is located (Kosovichev 1996; Basu 1997). The tachocline is defined to be the region where the rotation rate undergoes a transition from differential rotation in the convection zone to almost uniform rotation in the radiative interior (Spiegel & Zahn 1992). As a result, one would expect temporal variations associated with solar cycle to manifest in this region. However, no definite changes have been detected in these layers (Basu & Antia 2000a). Recently, Howe et al. (2000) have reported a 1.3 year periodicity in variation of equatorial rotation rate at \(r = 0.72R_\odot\). It is not clear if this period is associated with solar cycle related variations, or indeed why it should manifest only in a narrow latitude and radius range. Using similar data, Antia & Basu (2000) did not find any periodic or systematic changes in rotation rate in the tachocline region. But both these studies are based on inversions of data to obtain the rotation rate. This process is not very reliable in the tachocline region where the steep gradient in the rotation rate tends to be smoothed by regularisation applied in inversion techniques (Gough & Thompson 1991; Antia, Basu & Chitre 1998). The properties of the solar tachocline have been studied using forward modelling techniques (Kosovichev 1996; Basu 1997; Antia et al. 1998; Charbonneau et al. 1999), which are probably better suited to account for the steep variation in rotation rate. Corbard et al. (1998, 1999) have modified inversion technique to account for sharp changes in the rotation rate. Using a simple calibration technique to study the tachocline, Basu & Schou (2000) found that the magnitude of the jump in the rotation rate across the tachocline increases with solar activity. This result needs to be checked using other techniques and with more data that are now available.

In this work, we attempt to use forward modelling techniques described by Antia et al. (1998) to study whether different properties of the tachocline vary with time. We also look for possible periodic changes in the rotation rate around the tachocline. In addition, we also study possible temporal changes in solar structure in that region, in particular, changes in the depth of the convection zone and the extent of overshoot below the convection zone. Since the depth of the convection zone can be measured very accurately – with statistical error of the order of 0.0001\(R_\odot\) – it should be possible to detect even small variations in the structure of this region.

Apart from temporal variations in the properties of the tachocline, we also attempt to find latitudinal variations in the position and the thickness of the tachocline. Antia et al. (1998) did not find any significant latitudinal variation in the position or thickness of the tachocline. Although, the results did show that the tachocline shifts upwards with latitude, this shift was comparable to the error estimates. Char-
bonneau et al. (1999) found that the mean position of the tachocline moves upwards with latitude. They found a shift of $(0.024 \pm 0.004) R_\odot$ in tachocline position between the latitudes of $0^\circ$ and $60^\circ$, which is comparable to the upper limit on its variation given by Antia et al. (1998). With the accumulation of data over 5 years it may be possible to find out if this latitudinal variation is statistically significant. Similarly, there are some indications that the thickness of tachocline increases with latitude (Antia et al. 1998; Charbonneau et al. 1999) but this variation is not statistically significant, at least, in the results obtained so far. Thus it is of interest to check if this variation can be confirmed with longer data set.

If the position of the tachocline varies with latitude, then a natural question is whether the depth of the convection zone also varies with latitude. The relative position of the tachocline and the convection zone base plays a crucial role in the theory of the tachocline. Gough & Kosovichev (1995) using data from Big Bear Solar Observatory (Woodard & Libbrecht 1993) have claimed a decrease by about $0.02 R_\odot$ in depth of the convection zone between the equator and latitude of $60^\circ$. This is comparable to latitudinal variations in the tachocline as found by Charbonneau et al. (1999). However, such a large variation would yield a strong signal in the even-order splitting coefficients because of the resulting asphericity. Antia et al. (2000a) did not find any significant signal in asphericity around the base of the convection zone, although they did not specifically look for a signal from varying depth of convection zone. Monteiro & Thompson (1998) attempted to detect latitudinal dependence in depth of convection zone and extent of overshoot below the convection zone, but the results were not conclusive. Similarly, Basu (1997) and Antia et al. (2000b) attempted to look for any possible magnetic field near the base of the convection zone, using the even order splitting coefficients, but once again no signal was found in the observed data. We therefore, try to check for any latitudinal variation in the depth of convection zone.

2 THE DATA

We use data for GONG months 1–46 to determine the rotation rate and the spherically symmetric structure in the solar interior. Each of these data sets covers a period of 108 days. We have used only the non-overlapping sets of data for most of the work to ensure that each data set is independent. Each GONG month covers a period of 36 days with month 1, starting on 1995 May 7 and month 46 ending on 1999 November 17. There are 15 non-overlapping data sets covering this period, and these have been used to study the temporal variations in rotation rate and structure. However, for studying the oscillatory changes in rotation rate we have also used all data sets even though they overlap in time. There are 44 data sets centered on GONG months 2–45. The data were obtained from the GONG Data Storage and Distribution System. In order to provide an independent test of the results, we also use the data from MDI. Each of these 18 data sets was obtained from 72 non-overlapping days of observations covering a period from 1996 May 1 to 2000 April 9 with some gaps corresponding to period when the Solar and Heliospheric Observatory (SOHO), the space-craft on which MDI is located, was not operational.

All these data sets (GONG and MDI) contain both the mean frequencies and splitting coefficients for the observed p-modes. The GONG data are described by Hill et al. (1996), while the MDI data are described by Schou (1999). The frequency of an eigenmode of a given degree $\ell$, a given radial order $n$, and a given azimuthal order $m$ can be expressed in terms of these splitting coefficients using the expansion

$$\nu_{nlm} = \nu_{nl} + \sum_{j=1}^{\infty} a_j(n, \ell) S_j^{(0)}(m),$$

where $\nu_{nl}$ is the mean frequency of the $(n, \ell)$ multiplet, $a_j(n, \ell)$ are the splitting coefficients and $S_j^{(0)}(m)$ are orthogonal polynomials in $m$ (Ritzwoller & Lavelle 1991; Schou, Christensen-Dalsgaard & Thompson 1994). There is some ambiguity in the normalisation of the polynomials $S_j^{(0)}(m)$. We have used the definition given by Schou et al. (1994). The odd splitting coefficients, $a_1, a_3, \ldots$, are determined by rotation rate in solar interior and have been used to infer the rotation rate as a function of depth and latitude. While the even splitting coefficients $a_2, a_4, \ldots$ are determined by magnetic fields and other aspherical perturbations in solar interior, as well as by second order effects of rotation (Gough & Thompson 1990). The even splitting coefficients can be used to study the latitudinal variations in the solar structure. For the purpose of this work we have used only modes with frequencies between 1500 and 3500 $\mu$Hz.

3 THE TECHNIQUE

3.1 Depth of the convection zone

To detect possible changes in solar structure around the base of the convection zone, we determine the depth of the convection zone using the mean frequencies of the p-modes. In lower part of the convection zone the temperature gradient equals the adiabatic temperature gradient, while below the convection zone base the temperature gradient is radiative. The difference in the temperature gradient below and above the base of the convection zone introduces a characteristic feature in the sound speed difference between two models (or between a model and the Sun) if they have different convection zone depths. This signal can be used to determine this depth and we use the method described by Basu and Antia (1997) for this purpose.

We use the even-order splitting coefficients, which can be used to analyse departures from spherical symmetry (Gough 1993), to determine possible latitude-dependence of the depth of the convection zone. For simplicity, we only consider axisymmetric perturbations (with the symmetry axis coinciding with the rotation axis) that are symmetric about the equator. In this case, using the variational principle, the difference in frequency between the Sun and a solar model for a mode of a given order, degree and azimuthal order $(n, \ell, m)$ can be written as:

$$\frac{\delta \nu_{nlm}}{\nu_{nlm}} = \int_0^R dr \int_0^{2\pi} d\phi \int_0^\pi \sin \theta \, d\theta \left( K_{nl, \ell, 0}^n(r) \frac{\partial^2 \nu}{\partial \rho^2} + K_{nl, \ell, 2}^n(r) \frac{\partial \nu}{\partial \rho} \right) Y^m_\ell(Y^m_\ell)^*,$$

where $r$ is radius, $\theta$ is colatitude, $K_{nl, \ell, 0}^n(r)$ and $K_{nl, \ell, 2}^n(r)$ are the kernels for spherically symmetric perturbations (Antia
independent of and (frequencies that are functions of difference given by:

\[
\delta c^2(r, \theta) = \sum_k c_k(r) P_{2k}(\cos \theta),
\]

\[
\delta \rho r, \theta = \sum_k \rho_k(r) P_{2k}(\cos \theta),
\]

where \(c_k(r)\) and \(\rho_k(r)\) are shorthand notations for \((\delta c^2/\rho^2)_{k}(r)\) and \((\delta \rho/\rho)_{k}(r)\), respectively. The spherically symmetric component \((k = 0)\) causes frequency differences that are independent of \(m\) and thus only contribute to the mean frequency of the \((n, \ell)\) multiplet. Higher order terms give frequencies that are functions of \(m\) and thus contribute to the splitting coefficients.

The angular integrals in Eq. (2) can be evaluated to give

\[
\int_0^{2\pi} \int_0^\pi \sin \theta \ d\theta \ d\phi \ Y^m_l(m) P_k(\cos \theta) = \frac{1}{\ell} Q_{l \ell} P^{(l)}_{2\ell}(m),
\]

where \(Q_{l \ell}\) depends only on \(\ell, k\) and \(P^{(l)}_{2\ell}(m)\) are the orthogonal polynomials defined by Eq. (1). The extra factor of \(1/\ell\) ensures that \(Q_{l \ell}\) approach a constant value at large \(\ell\). Thus with this choice of expansion [Eq. (3)] the inversion problem is decomposed into independent inversions for each even splitting coefficient and \(c_k(r), \rho_k(r)\) can be computed by inverting the splitting coefficient \(a_{2k}\). This would be similar to the 1.5d inversions used to determine rotation rates (Ritzwoller & Lavely 1991), called so because a two dimensional solution is obtained as a series of one dimensional inversions:

\[
\frac{\ell a_{2k}(n, \ell)}{\nu_{\ell \ell} Q_{l \ell}} = \int_0^R K_{c^2, \rho} \ c_k(r) \ dr + \int_0^R K_{\rho, \rho} \ \rho_k(r) \ dr.
\]

To determine the sound speed or density at a particular colatitude \(\theta\) we can combine these solutions using Eq. (3). Thus to invert for \(\delta c^2/c^2\) at colatitude \(\theta\), we can use the usual equation for spherically symmetric case with the frequency difference given by:

\[
\delta \nu = \delta \nu_{n \ell} + \sum_k \frac{\ell a_{2k}(n, \ell)}{Q_{l \ell}} P_{2k}(\cos \theta),
\]

where \(\delta \nu_{n \ell}\) is the difference in mean frequency for the \(n, \ell\) multiplet, between the Sun and a solar model. Thus with this choice of \(\delta \nu\) we can apply the technique used by Basu & Antia (1997) to determine convection zone depth at a given colatitude.

### 3.2 Overshoot below the convection zone

In addition to the depth of the convection zone, we attempt to determine changes in the extent of overshoot below the convection zone. The sudden change in the temperature gradient at the base of the convection zone introduces an oscillatory signal in the frequencies as a function of radial order \(n\) (Gough 1990). The presence of an adiabatically stratified overshoot layer causes the temperature gradient to have a discontinuity; the magnitude of discontinuity increasing with the extent of overshoot. Thus the amplitude of the oscillatory signal also increases with the extent of overshoot. To a first approximation, this signal has the form \(A \cos(2\nu t + \phi)\), where \(\nu\) is the frequency of the mode, \(\tau\), the ‘frequency’ of the signal is the acoustic depth of the transition layer, \(A\) is the amplitude and \(\phi\) a phase. This signal has been used earlier to estimate the extent of overshoot below the solar convection zone (Monteiro, Christensen-Dalsgaard & Thompson 1994; Basu, Antia & Narasimha 1994). The oscillatory signal can be amplified by taking the fourth differences of the frequencies as a function of the radial order \(n\) enabling a more precise measurement of the amplitude, \(A\), of oscillations (Basu et al. 1994). Since the amplitude of the signal increases with increase in extent of overshoot, it can be calibrated against amplitudes for models with known extents of overshoot. We use the method described by Basu (1997) to isolate the oscillatory signal and measure its characteristics.

Data sets used so far indicate that the amplitude of the oscillatory signal is consistent with no overshoot, however, this does not preclude any change of the amplitude with time. The precision of this measurement is much less than that of the depth of the convection zone, but since there is some suggestion that this amplitude may be varying with time (Monteiro et al. 1998) it needs to be checked independently.

We can also calculate the latitude dependence of extent of overshoot by adding the contribution from even splitting coefficients as given in Eq. (6) to the frequencies. The resulting frequencies can be used to calculate the fourth difference and the oscillatory part can be isolated as explained above to calculate the amplitude. Any variation in amplitude of this oscillatory signal can be attributed to change in extent of overshoot with latitude. Since the errors in estimating the extent of overshoot are much larger than those in estimating the depth of the convection zone, probability of detecting either temporal or latitudinal variation in extent of overshoot is much smaller than those in convection zone depth. But for completeness we have attempted to study this variation too.

### 3.3 The tachocline

To determine the properties of tachocline we use the three techniques described by Antia et al. (1998), which are (1) a calibration method in which the properties at each latitude are determined by direct comparison with models; (2) a one dimensional (henceforth, 1d) annealing technique in which the parameters defining the tachocline at each latitude are determined by a nonlinear least squares minimization using simulated annealing method and (3) a two-dimensional (henceforth, 2d) annealing technique, where the entire latitude dependence of tachocline is fitted simultaneously, again using simulated annealing. The properties we are interested in are the position and the thickness of the tachocline and the change in rotation rate across the tachocline. We study these properties as a function of latitude using all the techniques listed above. The use of three different techniques allows us a check on the results. In all techniques the tachocline is represented by a model of the

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