Neutralino–Nucleus Elastic Cross Section in the Minimal Supersymmetric Standard Model with Explicit CP Violation

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Abstract

We study the elastic scattering of the lightest neutralino with a nucleus in the framework of the minimal supersymmetric standard model (MSSM) with explicit flavor preserving CP violation, including the one–loop CP–violating neutral Higgs–boson mixing effects induced dominantly by the CP phases in the top and bottom (s)quark sectors. We construct the most general form of the effective Lagrangian for the neutralino–nucleus scattering in the limit of vanishing momentum transfers and then we perform a comprehensive analysis of the effects of the complex CP phases on the mass spectra of the lightest neutralino, neutral Higgs bosons and top squarks, and on the the spin–dependent and spin–independent neutralino–nucleus scattering cross section for three nucleus targets F, Si and Ge. The CP phases can reduce or enhance the neutralino–nucleus cross sections significantly, depending on the values of the real parameters in the MSSM.

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1 Introduction

A lot of cosmological and astrophysical observations have revealed that the dominant fraction of the matter in the universe and in the galactic halo is not luminous or dark [1]. Furthermore, big–bang nucleosynthesis and galactic structure formation require some non–baryonic and non–relativistic matter for which various particle physics theories, in particular, supersymmetric theories provide good candidates [2, 3]. The lightest supersymmetric particle (LSP), which is guaranteed by R–parity conservation, is one of the most well–established and studied weakly interacting massive particle (WIMP) candidates of dark matter(DM). The null experimental results in searching for the anomalously heavy isotopes and the considerations in the most models suggest that the LSP in the minimal supersymmetric standard model (MSSM) might be the lightest neutralino, a linear combination of the four neutral superpartners of the U(1)$_Y$ and SU(2)$_L$ gauge bosons and of the two neutral Higgs bosons;

\[
\tilde{\chi}^0_1 = N_{11} \tilde{B} + N_{12} \tilde{W}^3 + N_{13} \tilde{H}^0_1 + N_{14} \tilde{H}^0_2,
\]

where \(N\) is the unitary matrix diagonalizing the \(4 \times 4\) neutralino mass matrix [4].

The lightest neutralino as a dominant cold dark matter (CDM) has very weak, but finite coupling strengths to quarks so that there might be a chance to detect the CDM neutralino even in laboratories directly or indirectly [5]. If the couplings are too small, the neutralinos would not have annihilated in the early universe and it would be too much abundant today. Certainly, the capabilities of both direct and indirect search experiments for the lightest neutralino rely crucially on the size of the elastic neutralino–matter scattering cross section, in particular the neutralino–nucleus cross section because the detection rates for either direct or indirect searches are proportional to the cross sections. Since the first rough estimates were made by Goodman and Witten [6], the estimates of the neutralino–nucleus scattering cross section have been continuously and significantly improved [7, 8]. Furthermore, there are many on–going and planned experiments with the goal of direct or indirect detections of the CDM neutralinos among which some experiments have already reported interesting but controversial results [9]. Therefore, it is very important and timely to evaluate the cross section for neutralino–nucleus elastic scattering by including all the possibly dominant factors in determining the elastic scattering cross section as well as the neutralino–neutralino annihilation cross sections in the supersymmetric theories.

In the present work we re–investigate the elastic scattering of the neutralino–nucleus scattering in the framework of the minimal supersymmetric standard model (MSSM) with R–parity focusing on all the dominant effects of flavor–preserving CP–violating complex phases on the cross section [8]. The effects of the phase \(\Phi_\mu\) of the higgsino mass parameter \(\mu\) on the neutralino–nucleus scattering as well as the neutralino relic density have been
already studied by several works [10]. However, there can exist additional effects through two different types of CP–violating phases\(^*\); one is the phase $\Phi_1$ of the $U(1)_Y$ gaugino mass $M_1$ and the other are the phases of the trilinear terms $A_f$ in the sfermion mass matrices. The latter phases can cause the CP–violating mixing among three neutral Higgs bosons at one–loop level so that the couplings of the Higgs bosons to the (s)particles are significantly modified [11, 12, 13]. We note that the phases of the trilinear parameters for the first and second generation sfermions are strongly constrained by the stringent experimental bounds on the electron and neutron electric dipole moments (EDMs) unless the sfermions are too heavy. On the other hand, since the third generation sfermion sectors give only two–loop suppressed effects on the electron and neutron EDMs when any generational mixing among sfermions is neglected, the CP violating phases involving the third generation sfermions can be very large [11]. In addition, the most dominant contributions to the CP–violating neutral Higgs boson mixing are from the third generation sfermion sectors because of their large Yukawa couplings [11, 12]. So, including the CP–violating neutral Higgs boson mixing due to the phases of the third generation trilinear parameters and the phase $\Phi_1$ as well as the phase $\Phi_\mu$ we will provide a comprehensive analysis of the dependence of the neutralino–nucleus elastic scattering cross section on those phases in the present work.

The typical size of the momentum transfers exchanged in the neutralino–nucleus elastic scattering is of the order of 10 KeV, much smaller than the neutralino mass of the order of 100 GeV. For such tiny momentum transfers, the interactions of the lightest neutralino with spin–1/2 quarks can be described by two effective four–Fermi current $\times$ current terms; the spin–dependent axial–vector $\times$ axial–vector four–Fermi Lagrangian and the spin–independent scalar $\times$ scalar four–Fermi Lagrangian. We note that the spin–independent term becomes more significant for the scattering of the neutralino off nuclei with a large mass number because each spin–independent neutralino–nucleon scattering contributes coherently to the overall spin–independent cross section. As a result, the spin–independent part dominates the spin–dependent part for large target nuclei.

In the CP–noninvariant case, the scalar–pseudoscalar mixing among neutral Higgs bosons in the MSSM modifies the scalar couplings of each neutral Higgs bosons to fermions and neutralinos, leading to the significant changes of the spin–independent neutralino–nucleus cross section. It is, therefore, worthwhile to make a systematic investigation of the effects of all the relevant CP–violating phases on the neutralino–nucleus scattering cross section in the framework of the MSSM. For a systematic quantitative investigation, our analysis in the present work is based on a specific framework with the following assumptions:

- The first and second generation sfermions are assumed to be very heavy so that they are decoupled from the theory. In this case, there are no constraints on the CP–

\(^*\)Without any loss of generality, the SU(2)$_L$ gaugino mass $M_2$ can be taken to be real
violating phases from the neutron and EDMs. On the other hand, the annihilation of the neutralinos into tau pairs through the exchange of relatively light scalar tau leptons guarantees that the cosmological constraints on the DM densities be satisfied.

- The explicit CP violation in the Higgs sector induced through the CP–violating radiative corrections from the top and bottom squark sectors is included.
- Simultaneously, the effects of the induced CP–violating phase between Higgs doublets on the chargino and neutralino systems are explicitly included.
- It is necessary to avoid the possible constraints from the so–called Barr–Zee–type diagrams to the electron and neutron EDMs as well as from the Higgs search experiments at LEP. We take two values $3$ and $10$ for $\tan \beta$, the ratio of the vacuum expectation values of two neutral Higgs fields.

The rest of the paper is organized as follows. In Sec. 2, we present a brief review of the CP–violating mixing among scalar and pseudo–scalar Higgs bosons in the MSSM Higgs sector on the basis of the recent work by Choi, Drees and Lee [12]. In addition, we discuss the neutralino mixing including the CP–violating phase induced from the MSSM Higgs sector. In Sec. 3, we derive the complete analytic expressions for the spin–dependent and spin–independent effective four–Fermi Lagrangian relevant to our calculations explicitly, taking into account all the Higgs–boson, scalar–quark and $Z$–boson exchange diagrams in the limit of vanishing momentum transfers. Section 4 is devoted to a detailed numerical analysis for investigating the dependence of the neutralino–nucleus elastic scattering cross section for the three target nuclei F, Si, and Ge on the CP violating phases as well as other relevant real SUSY parameters such as $\tan \beta$ by taking a few typical parameter sets which effectively cover the whole parameter space. Finally, we summarize our findings and conclude in Sec. 5.

2 CP–violating mixing in the MSSM

2.1 Neutral Higgs–boson mixing

In this section, we give a brief review of the calculation [12] of the Higgs-boson mass matrix based on the full one-loop effective potential, valid for all values of the relevant third–generation soft–breaking parameters.

The MSSM contains two Higgs doublets $H_1$, $H_2$, with hypercharges $Y(H_1) = -Y(H_2) = \ldots$
Here we are only interested in the neutral components, which we write as

\[ H_1^0 = \frac{1}{\sqrt{2}} (\phi_1 + i a_1), \quad H_2^0 = \frac{e^{i\xi}}{\sqrt{2}} (\phi_2 + i a_2), \]

(2)

where \( \phi_{1,2} \) and \( a_{1,2} \) are real scalar fields. The constant phase \( \xi \) can be set to zero at tree level, but will in general become non-zero once loop corrections are included. The mass matrix of the neutral Higgs bosons can be computed from the effective potential [14]

\[
V_{\text{Higgs}} = \frac{1}{2} m_1^2 (\phi_1^2 + a_1^2) + \frac{1}{2} m_2^2 (\phi_2^2 + a_2^2) - |m_{12}^2| (\phi_1 \phi_2 - a_1 a_2) \cos(\xi + \theta_{12})
\]

\[
+ |m_{12}^2| (\phi_1 a_2 + \phi_2 a_1) \sin(\xi + \theta_{12}) + \frac{\hat{g}^2}{8} D^2 + \frac{1}{64 \pi^2} \text{Str} \left[ \mathcal{M}^4 \left( \log \frac{\mathcal{M}^2}{Q^2} - \frac{3}{2} \right) \right], \tag{3}
\]

where we have allowed the soft breaking parameter \( m_{12}^2 = |m_{12}^2| e^{i\theta_{12}} \) to be complex, and we have introduced the quantities

\[
D = \phi_2^2 + a_2^2 - \phi_1^2 - a_1^2, \quad \hat{g}^2 = \frac{g^2 + g'^2}{4},
\]

(4)

where the symbols \( g \) and \( g' \) stand for the SU(2)\(_L\) and U(1)\(_Y\) gauge couplings, respectively. \( Q \) in Eq. (3) is the renormalization scale; the parameters of the tree-level potential, in particular the parameters \( m_1^2, m_2^2 \) and \( m_{12}^2 \), are running masses, taken at scale \( Q \). The potential (3) is then independent of \( Q \), up to two-loop corrections.

The matrix \( \mathcal{M} \) in Eq. (3) is the field-dependent mass matrix of all modes that couple to the Higgs bosons. The by far dominant contributions come from the third generation quarks and squarks. The (real) masses of the former are given by

\[
m_{b_b}^2 = \frac{1}{2} |h_b|^2 (\phi_1^2 + a_1^2), \quad m_t^2 = \frac{1}{2} |h_t|^2 (\phi_2^2 + a_2^2),
\]

(5)

where \( h_b \) and \( h_t \) are the bottom and top Yukawa couplings. The corresponding squark mass matrices can be written as

\[
\mathcal{M}_t^2 = \begin{pmatrix}
\frac{m_Q^2}{2} + m_t^2 - \frac{3}{8} (g^2 - \frac{g'^2}{3}) D & -h_t^* \left[ A_t (H_2^0)^* + \mu H_1^0 \right] \\
-h_t \left[ A_t H_2^0 + \mu^* (H_1^0)^* \right] & m_U^2 + m_t^2 - \frac{g'^2}{6} D
\end{pmatrix},
\]

\[
\mathcal{M}_b^2 = \begin{pmatrix}
\frac{m_Q^2}{2} + m_b^2 + \frac{1}{8} (g^2 + \frac{g'^2}{9}) D & -h_b^* \left[ A_b (H_1^0)^* + \mu H_2^0 \right] \\
-h_b \left[ A_b H_1^0 + \mu^* (H_2^0)^* \right] & m_D^2 + m_b^2 + \frac{g'^2}{12} D
\end{pmatrix}.
\]

(6)

Here, \( H_1^0 \) and \( H_2^0 \) are given by Eq. (2) while \( m_t^2 \) and \( m_b^2 \) are as in Eq. (5) and \( D \) has been defined in Eq. (4). In Eq. (6) \( m_Q^2, m_U^2 \) and \( m_D^2 \) are real soft breaking parameters, \( A_b \) and
\( A_t \) are complex soft breaking parameters, and \( \mu \) is the complex supersymmetric Higgs(ino) mass parameter.

The mass matrix of the neutral Higgs bosons can now be computed from the matrix of second derivatives of the potential (3), where (after taking the derivatives) \( m_{1}^2, m_{2}^2 \) and \( m_{12}^2 \sin(\xi + \theta_{12}) \) are determined by the stationarity conditions. The massless state \( G^0 = a_1 \cos \beta - a_2 \sin \beta \) is the would–be Goldstone mode “eaten” by the longitudinal \( Z \) boson. We are thus left with a squared mass matrix \( \mathcal{M}_H^2 \) for the three states \( a = a_1 \sin \beta + a_2 \cos \beta, \phi_1 \) and \( \phi_2 \). This matrix is real and symmetric, i.e. it has 6 independent entries. The diagonal entry for \( a \) reads:

\[
\mathcal{M}_H^2 \bigg|_{aa} = m_A^2 + \frac{3}{8\pi^2} \left\{ \frac{|h_t|^2 m_i^2}{\sin^2 \beta} g(m_{i_1}^2, m_{i_2}^2) \Delta^2_t + \frac{|h_b|^2 m_b^2}{\cos^2 \beta} g(m_{b_1}^2, m_{b_2}^2) \Delta^2_b \right\},
\]

and the CP–violating entries of the mass matrix, which mix \( a \) with \( \phi_1 \) and \( \phi_2 \) read:

\[
\mathcal{M}_H^2 \bigg|_{a\phi_1} = \frac{3}{16\pi^2} \left\{ \frac{m_i^2 \Delta_t}{\sin \beta} \left[ g(m_{i_1}^2, m_{i_2}^2) \left( X_t \cot \beta - 2 |h_t|^2 R_t \right) - \hat{g}^2 \cot \beta \log \frac{m_{i_2}^2}{m_{i_1}^2} \right] + \frac{m_b^2 \Delta_b}{\cos \beta} \left[ -g(m_{b_1}^2, m_{b_2}^2) \left( X_b + 2 |h_b|^2 R_b \right) + \left( g^2 - 2 |h_b|^2 \right) \log \frac{m_{b_2}^2}{m_{b_1}^2} \right] \right\},
\]

\[
\mathcal{M}_H^2 \bigg|_{a\phi_2} = \frac{3}{16\pi^2} \left\{ \frac{m_i^2 \Delta_t}{\sin \beta} \left[ -g(m_{i_1}^2, m_{i_2}^2) \left( X_t + 2 |h_t|^2 R_t' \right) + \left( g^2 - 2 |h_t|^2 \right) \log \frac{m_{i_2}^2}{m_{i_1}^2} \right] + \frac{m_b^2 \Delta_b}{\cos \beta} \left[ g(m_{b_1}^2, m_{b_2}^2) \left( X_b \tan \beta - 2 |h_b|^2 R_b \right) - g^2 \tan \beta \log \frac{m_{b_2}^2}{m_{b_1}^2} \right] \right\},
\]

where \( \Delta_t \) and \( \Delta_b \), which describe the amount of CP violation in the squark mass, read

\[
\Delta_t = \frac{\Im(m \mu e^{i\xi})}{m_{i_2}^2 - m_{i_1}^2}; \quad \Delta_b = \frac{\Im(m \mu e^{i\xi})}{m_{b_2}^2 - m_{b_1}^2},
\]

and \( g(x, y) = 2 - [(x + y)/(x - y)] \log(x/y) \). The definition of the mass squared \( m_A^2 \) and the dimensionless quantities \( X_{t,b}, R_{t,b} \) as well as the other CP–preserving entries of the mass matrix squared \( \mathcal{M}_H^2 \) can be found in Ref. [12]. As noted earlier, the size of these CP–violating entries is controlled by \( \Delta_t \) and \( \Delta_b \).

The real and symmetric \( 3 \times 3 \) matrix \( \mathcal{M}_H^2 \) can be diagonalized by an \( 3 \times 3 \) rotation \( O \);

\[
\begin{pmatrix}
a \\
\phi_1 \\
\phi_2
\end{pmatrix} = O \begin{pmatrix}
H_1 \\
H_2 \\
H_3
\end{pmatrix}
\]
with the increasing order of the three mass eigenvalues, $m_{H_1}^2 \leq m_{H_2}^2 \leq m_{H_3}^2$, taken as a convention. Note that the loop–corrected neutral Higgs–boson sector is determined by fixing the values of various parameters; $m_A$, $\mu$, $A_t$, $A_b$, a renormalization scale $Q$, $\tan \beta$, and the soft–breaking third generation sfermion masses, $m_{\tilde{Q}}$, $m_{\tilde{U}}$, and $m_{\tilde{D}}$. The radiatively induced phase $\xi$ is no more an independent parameter and it can be absorbed into the definition of the $\mu$ parameter so that the physically meaningful CP phases in the Higgs sector are the phases of the re–phasing invariant combinations $A_t \mu e^{i\xi}$ and $A_b \mu e^{i\xi}$. This neutral Higgs–boson mixing changes the couplings of the Higgs fields to fermions, gauge bosons, and Higgs fields themselves so that the effects of CP violation in the Higgs sector can be probed through various processes [15].

2.2 Top and bottom squark mixing

On the other hand, the $2 \times 2$ Hermitian top and bottom squark mass matrices squared $M_{\tilde{t}}^2$ and $M_{\tilde{b}}^2$ can be obtained after plugging the vacuum expectation values for the Higgs fields:

$$M_{\tilde{t}}^2 = \begin{pmatrix}
  m_{\tilde{Q}}^2 + m_t^2 + m_Z^2 \cos 2\beta \left( \frac{1}{2} - \frac{2}{3} s_W^2 \right) & -m_t \left( A_t^* + \mu e^{i\xi} \cot \beta \right) \\
  -m_t \left( A_t + \mu^* e^{-i\xi} \cot \beta \right) & m_{\tilde{U}}^2 + m_t^2 + \frac{2}{3} m_Z^2 \cos 2\beta s_W^2
\end{pmatrix},$$

$$M_{\tilde{b}}^2 = \begin{pmatrix}
  m_{\tilde{Q}}^2 + m_b^2 - m_Z^2 \cos 2\beta \left( \frac{1}{2} + \frac{1}{3} s_W^2 \right) & -m_b \left( A_b^* + \mu e^{i\xi} \tan \beta \right) \\
  -m_b \left( A_b + \mu^* e^{-i\xi} \tan \beta \right) & m_{\tilde{D}}^2 + m_b^2 - \frac{1}{3} m_Z^2 \cos 2\beta s_W^2
\end{pmatrix} \ (11)$$

These Hermitian mass matrices can be diagonalized by the unitary matrices, $U_{\tilde{t}}$ and $U_{\tilde{b}}$,

$$\begin{pmatrix}
  \tilde{t}_L \\
  \tilde{t}_R
\end{pmatrix} = U_{\tilde{t}} \begin{pmatrix}
  \tilde{t}_1 \\
  \tilde{t}_2
\end{pmatrix}, \quad \begin{pmatrix}
  \tilde{b}_L \\
  \tilde{b}_R
\end{pmatrix} = U_{\tilde{b}} \begin{pmatrix}
  \tilde{b}_1 \\
  \tilde{b}_2
\end{pmatrix}, \quad (12)$$

respectively, where $\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$ are the mass eigenstates with their masses, $m_{\tilde{t}_{1,2}}$ and $m_{\tilde{b}_{1,2}}$, respectively. We note that the off–diagonal terms of each mass matrix squared are proportional to the corresponding fermion mass so that their contributions can be significant only for the third generation sfermions – top and bottom squarks.

2.3 Neutralino mixing

In general, the induced phase $\xi$ in Eq. (2) remains as a non–trivial physical phase and leads to a modification in the neutralino mass matrix; analytically, the induced phase plays a role of rotating the vacuum expectation value $v_2$ into $v_2 e^{i\xi}$ so that it modifies the neutralino mass
matrix describing the gauginos and higgsinos through electroweak symmetry breaking. In the weak–interaction basis \((\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)\) the \(4 \times 4\) symmetric, but complex neutralino mass matrix reads

\[
M_N = \begin{pmatrix}
|M_1| e^{i\Phi_1} & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W e^{-i\xi} \\
0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W e^{-i\xi} \\
-m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -|\mu| e^{i\Phi_\mu} \\
m_Z s_\beta s_W e^{-i\xi} & -m_Z s_\beta c_W e^{-i\xi} & -|\mu| e^{i\Phi_\mu} & 0
\end{pmatrix},
\]

(13)

where the phases of the U(1) gaugino mass \(M_1\) and the higgsino mass parameter \(\mu\), \(\{\Phi_1, \Phi_\mu\}\) are explicitly given. The neutralino mass matrix is diagonalized in a symmetric way through a unitary matrix \(N\);

\[
\begin{pmatrix}
\tilde{\chi}_1^0 \\
\tilde{\chi}_2^0 \\
\tilde{\chi}_3^0 \\
\tilde{\chi}_4^0
\end{pmatrix} = N \begin{pmatrix}
\tilde{B} \\
\tilde{W}_3^3 \\
\tilde{H}_1^0 \\
\tilde{H}_2^0
\end{pmatrix},
\]

(14)

with the increasing order of the mass eigenvalues, \(m_{\tilde{\chi}_1^0} \leq m_{\tilde{\chi}_2^0} \leq m_{\tilde{\chi}_3^0} \leq m_{\tilde{\chi}_4^0}\), as a convention. One can find that after an appropriate field redefinition there are two rephrasing–invariant phases \(\Phi_1\) and \(\Phi_\mu + \xi\) in the neutralino sector, where \(\Phi_1\) is the phase of the U(1) gaugino mass \(M_1\) and \(\Phi_\mu\) the phase of the the higgsino mass parameter \(\mu\).

**Figure 1:** The six mechanisms contributing to the neutralino–quark elastic scattering process \(\tilde{\chi}_1^0 q \rightarrow \tilde{\chi}_1^0 q\); the spin–1 \(Z\) exchange, the three spin–0 neutral–Higgs–boson exchanges and the two squark \(\tilde{q}_{1,2}\) exchanges. Here, the index \(k\) denotes 1, 2 or 3.
3 Neutralino–nucleus elastic cross section

3.1 Feynman rules

In this section, we present all the Feynman rules in terms of mass eigenstates that are necessary for the neutralino–quark elastic scattering process (see Fig. 1).

The interactions of the neutral gauge bosons $Z$ to quarks are described by the same Lagrangian as given in the SM:

$$\mathcal{L}_{Zqq} = \frac{e}{s_W c_W} \bar{q} \gamma^\mu \left[ (Q_q s_W^2 - T_3^q) P_L + Q_q s_W^2 P_R \right] q Z^\mu,$$

where $Q_q$ and $T_3^q$ are the electric charge and the isospin of the quark, respectively. The interactions of the neutral Higgs bosons with quarks are described by the Lagrangian

$$\mathcal{L}_{H_k qq} = -\frac{h_d}{\sqrt{2}} \bar{d} \left[ O_{2k} + i O_{1k} s_\beta \gamma_5 \right] d H_k - \frac{h_u}{\sqrt{2}} \bar{u} \left[ O_{3k} + i O_{1k} c_\beta \gamma_5 \right] u H_k,$$

where the Yukawa couplings of down- and up-type quarks are given by

$$h_d = \frac{e m_d}{\sqrt{2} s_W m_W c_\beta}, \quad h_u = \frac{e m_u}{\sqrt{2} s_W m_W s_\beta}.$$

(17)

On the other hand, the SU(2) and U(1) gauginos, $\tilde{W}^3$ and $\tilde{B}$, have zero isospin and electric charges so that they do not couple to the $Z$ boson. However, the Higgsinos $\tilde{H}^0_1$ and $\tilde{H}^0_2$ have non–trivial isospins so that the couplings of the $Z$ boson to the lightest neutralinos, which are Majorana fermions, are described by the Lagrangian

$$\mathcal{L}_{Z\chi\chi} = \frac{e}{4 s_W c_W} \left[ |N_{13}|^2 - |N_{14}|^2 \right] \left( \tilde{\chi}^0_1 \gamma^\mu \gamma_5 \tilde{\chi}^0_1 \right) Z^\mu.$$

(18)

where the Majorana nature of the neutralino is reflected in the fact that only the axial vector coupling but not the vector coupling is allowed.

On the other hand, the interactions of the lightest neutralino with a pair of quark and squark come from both the gauge and Yukawa interactions. We consider the general flavor–diagonal squark mixing as well as the neutralino mixing and then we obtain the following Lagrangian

$$\mathcal{L}_{\chi q q} = \frac{e}{\sqrt{2} s_W} \left\{ \bar{q}^1 \tilde{\chi}^0 \left[ B^1_{qL} P_L + B^1_{qR} P_R \right] q \right\} + \text{H.c.},$$

(19)

where the coefficients $B^i_{qL,R}$ ($i = 1, 2$) in the mass eigenstate basis expressed in terms of the squark mixing angle $\theta_q$ and phase $\phi_q$ read

$$B^1_{qL} = \cos \theta_q A^{LL}_q + e^{-i\phi_q} \sin \theta_q A^{RL}_q, \quad B^1_{qR} = \cos \theta_q A^{LR}_q + e^{-i\phi_q} \sin \theta_q A^{RR}_q,$n

$$B^2_{qL} = \cos \theta_q A^{RL}_q - e^{i\phi_q} \sin \theta_q A^{LL}_q, \quad B^2_{qR} = \cos \theta_q A^{RR}_q - e^{i\phi_q} \sin \theta_q A^{LR}_q.$$n

(20)
with the coefficients

\[ A_{uLL}^u = N_{12}^* + \frac{1}{3} t_W N_{11}^*, \quad A_{uLR}^u = \frac{\sqrt{2} s_W h_u N_{14}}{e}, \]
\[ A_{uRL}^u = \frac{\sqrt{2} s_W h_u N_{14}^*}{e}, \quad A_{uRR}^u = \frac{4}{3} t_W N_{11}, \tag{21} \]

for the up–type quarks \( u, c \) and \( t \) and

\[ A_{dLL}^d = -N_{12}^* + \frac{1}{3} t_W N_{11}^*, \quad A_{dLR}^d = \frac{\sqrt{2} s_W h_d N_{13}}{e}, \]
\[ A_{dRL}^d = \frac{\sqrt{2} s_W h_d N_{13}^*}{e}, \quad A_{dRR}^d = \frac{2}{3} t_W N_{11}, \tag{22} \]

for the down–type quarks \( d, s \) and \( b \). The chirality–preserving coefficients \( A_{q}^{LL,RR} \) originate from the gauge interactions, but the chirality–flipping coefficients \( A_{q}^{LR,RL} \) from the Yukawa interactions, respectively.

Finally, the interactions of the neutral Higgs bosons to the lightest neutralinos involve both the neutralino mixing and the Higgs boson mixing. For the sake of notation we introduce the symbol \( G_k \) defined in terms of the induced phase \( \xi \), the neutralino diagonalization matrix \( N \), and the Higgs diagonalization matrix \( O \):

\[ G_k \equiv (N_{12} - t_W N_{11}) \left[ i (N_{13} s_\beta + N_{14} c_\beta e^{i\xi}) O_{1k} + N_{13} O_{2k} + N_{14} O_{3k} e^{i\xi} \right]. \tag{23} \]

With this abbreviation, the interaction Lagrangian for the coupling of the Higgs boson to the lightest neutralino pair is cast into a simple form:

\[ \mathcal{L}_{H\chi\chi} = \frac{e}{2 s_W} \sum_{k=1,2,3} \bar{\chi}_1^0 \left[ \Re(G_k) + i \Im(G_k) \gamma_5 \right] \chi_1^0 H_k, \tag{24} \]

where \( \Re(G_k) \) and \( \Im(G_k) \) denote the real and imaginary parts of the coefficient \( G_k \) in Eq. (23), respectively.

It is now straightforward to derive the coefficients \( A_q \) and \( B_q \) involving the effective four–fermion Lagrangian in the non–relativistic limit for the neutralino–quark elastic scattering:

\[ \mathcal{L}_{\text{eff}} = A_q (\bar{\chi}_1 \gamma^\mu \gamma_5 \chi_1) (\bar{q} \gamma_\mu \gamma_5 q) + B_q (\bar{\chi}_1 \chi_1) (\bar{q} q). \tag{25} \]

The \( t \)–channel \( Z \)–exchange diagram contributes to the spin–dependent part, and the \( t \)–channel Higgs–exchange diagrams to the spin–independent part, while the \( s \)–channel squark–exchange diagrams contribute to both the spin–dependent and spin–independent parts.
After appropriate Fierz transformations, the coefficients $A_q$ and $B_q$ in the non–relativistic limit read:

$$A_q = \frac{g^2}{16} \sum_{i=1,2} \frac{\left| B_{iL}^q \right|^2 + \left| B_{iR}^q \right|^2}{m_{q_i}^2 - (m_{X_0} - m_q)^2} - \frac{G_F}{\sqrt{2}} \left| N_{13} \right|^2 - \left| N_{14} \right|^2 T_3,$$

$$B_q = -\frac{g^2}{8} \sum_{i=1,2} \frac{\Re(B_{iL}^q B_{iR}^q^*)}{m_{q_i}^2 - (m_{X_0} - m_q)^2} - \frac{g h_q}{2\sqrt{2}} \sum_{k=1}^{3} \frac{\Re(G_k)}{m_{H_k}^2} \left\{ \begin{array}{ll} O_{2k} & \text{for } q = d \\ O_{3k} & \text{for } q = u \end{array} \right.,$$

(26)

where $G_F$ is the Fermi constant, and $h_q$ and $T_3$ are the Yukawa coupling and the third isospin component of the quark $q$, respectively. We have confirmed that in the CP–invariant theories, the expressions (26) are consistent with those in [3]. Note that the coefficient $A_q$ contains the $Z$–exchange contribution as well as the squark–exchange contributions, while the coefficient $B_q$ has the Higgs–exchange contributions as well as the squark–exchange contributions. The spin–dependent part is not suppressed by quark masses so that it can be large for the lightest neutralino of higgsino type, i.e, for large values $\left| N_{13} \right|$ and $\left| N_{14} \right|$. On the other hand, the spin–independent part from the $B_q$ terms is always proportional to quark masses and in particular the Higgs–exchange contributions are sizable only when the lightest neutralino is a well–balanced mixture of gaugino and higgsino states as seen from Eq. (23) and the size of the scalar–pseudoscalar mixing. Therefore, it might be naively expected that the small first and second generation quark masses will give rise to very small spin–independent cross sections. However, the spin–independent cross section can be significantly enhanced by coherent neutralino–quark scattering effects so that they dominate over the spin–dependent cross section for heavy nuclei.

### 3.2 Neutralino–nucleus elastic scattering cross sections

In order to obtain the scattering cross section of the lightest neutralino off heavy nuclei we need to know the detailed information on the configuration of the protons and neutrons inside each heavy nucleus and on that of quarks and gluons inside each proton and neutron. In the present work, we will not touch on this issue in detail, but simply take $^{19}$F, $^{29}$Si, and $^{73}$Ge as three heavy nuclei and use the parametrizations of the form factors for each neutralino–nucleus scattering cross section as presented in Ref. [3].

The total cross section for each elastic scattering process $\tilde{\chi}_1^0 N \to \tilde{\chi}_1^0 N$ with $N$=F, Si, and Ge at zero momentum transfers can be divided into the axial–vector and scalar parts:

$$\sigma = \sigma_A + \sigma_S.$$  

(27)
First of all, let us consider the spin–dependent part that is parametrized as follows:

$$\sigma_A = \frac{32}{\pi} G_F^2 J(J+1) m_r^2 \Lambda^2,$$

(28)

where $m_r$ is the neutralino–nucleus reduced mass $m_r = m_{\tilde{\chi}^0} m_N / (m_{\tilde{\chi}^0} + m_N)$ and $J$ the total angular momentum of the nucleus, the value of which is 1/2 for $^{19}$F and $^{29}$Si, 9/2 for $^{73}$Ge, respectively, and the spin– and target–dependent quantity $\Lambda$ is given by

$$\Lambda = \frac{1}{J} \left[ a_p \langle S_p \rangle + a_n \langle S_n \rangle \right],$$

(29)

where $\langle S_p \rangle = \langle N|S_p|N \rangle$ and $\langle S_n \rangle = \langle N|S_n|N \rangle$ and are the expectation values of the spin content of the proton and neutron group in the nucleus, respectively.

These expectation values $\langle S_{p,n} \rangle$ are different for each target nucleus and their explicit values for the target nuclei $^{19}$F, $^{29}$Si and $^{73}$Ge are given in the shell model by

$$\begin{align*}
\langle S_p \rangle_F &= +0.415, & \langle S_n \rangle_F &= -0.047, \\
\langle S_p \rangle_{Si} &= -0.002, & \langle S_n \rangle_{Si} &= +0.130, \\
\langle S_p \rangle_{Ge} &= +0.011, & \langle S_n \rangle_{Ge} &= +0.491.
\end{align*}$$

(30)

On the other hand, the coefficients $a_p$ and $a_n$ are parametrized in terms of the quark spin contents of the proton and neutron, $\Delta_{q}^p$ and $\Delta_{q}^n$, respectively, and the effective axial–vector couplings $A_q$ as

$$a_p = \sum_{q=u,d,s} \frac{A_q}{\sqrt{2} G_F} \Delta_{q}^p, \quad a_n = \sum_{q=u,d,s} \frac{A_q}{\sqrt{2} G_F} \Delta_{q}^n.$$  

(31)

In the present work, we take the factors $\Delta_{q}^{p,n} (q = u, d, s)$ [16] to be:

$$\begin{align*}
\Delta_u^p &= +0.77, & \Delta_d^p &= -0.38, & \Delta_s^p &= -0.09, \\
\Delta_u^n &= -0.38, & \Delta_d^n &= +0.77, & \Delta_s^n &= -0.09.
\end{align*}$$

(32)

More detailed information on the expectation $\langle S_{p,n} \rangle$ in Eq. (30) and the factors $\Delta_q^p$ and $\Delta_q^n$ in Eq. (32) can be found in Ref. [3].

Secondly, the spin–independent cross section to which each neutralino–proton or neutralino–neutron scattering contributes in a coherent manner is parameterized as

$$\sigma_s = \frac{4 m_r^2}{\pi} \left[ Z f_p + (A - Z) f_n \right]^2,$$

(33)
where \( Z \) and \( A \) denote the atomic number and mass number of the nucleus, respectively. In the limit of \( m_\chi \ll m_{\tilde{q}} \) and \( m_q \ll m_{\tilde{q}} \), the effective couplings of the lightest neutralino to protons and neutrons, \( f_p \) and \( f_n \), are approximated to be

\[
\frac{f_{p,n}}{m_{p,n}} = \sum_{q=u,d,s} f_{p,n}^q \frac{B_q}{m_q} + \frac{2}{27} f_{p,n}^G \sum_{q=c,b,t} \frac{B_q}{m_q},
\]

up to the lowest order in \( 1/m_{\tilde{q}} \), where the parameters \( f_{p,n}^q \) are defined as

\[
f_{p,n}^q = \langle n, p | m_q \bar{q} q | n, p \rangle / m_{n,p}, \quad f_{p,n}^G = 1 - \sum_{q=u,d,s} f_{p,n}^G.
\]

In our present numerical analysis these quantities are taken to be

\[
f_{p}^u = 0.019, \quad f_{p}^d = 0.041, \quad f_{p}^s = 0.140, \\
f_{n}^u = 0.023, \quad f_{n}^d = 0.034, \quad f_{n}^s = 0.140.
\]

as in Ref. [17].

We note in passing that some contributions from the other interactions such as so–called twist–2 operators are numerically small.

4 Numerical results

We are now ready to present some numerical results. It is known that loop–induced CP violation in the Higgs sector can only be large if both \( |\mu| \) and \( |A_t| \) ( or \( |A_b| \) if \( \tan \beta \gg 1 \) ) are sizable. In addition, for moderate values of \( \tan \beta \) the contributions from the (s)bottom sector are still quite small so that our numerical results are not sensitive to \( m_{\tilde{D}} \) and \( A_b \). In light of these points, we therefore choose

\[
|A_t| = |A_b| = 2 m_{\tilde{Q}} = 2 m_{\tilde{U}} = 2 m_{\tilde{D}} = 1.0 \text{ TeV},
\]

and take the phases of \( A_t \) and \( A_b \) to be equal. Numerically we find that for the large squark masses in Eq. (37) the squark–exchange contributions to the neutralino–nucleus scattering are very small compared to the other \( Z \)–boson and Higgs–exchange contributions.

Since the main focus will be on the effects of the three CP–violating phases \( \{\Phi_1, \Phi_\mu, \Phi_A\} \) on the neutralino–nucleus scattering cross section, we do not perform any parameter scan on all the real SUSY parameters in the present work. Nevertheless, it is crucial to cover the
overall parameter space effectively. So, except for the gaugino mass unification condition for the moduli of the SU(2) and U(1) gaugino masses

\[ |M_1| = \frac{5}{3} \tan^2 \theta_W M_2 \approx 0.5 M_2, \]  

(38)

we consider three different sets for the gaugino and higgsino mass parameters:

- **Gaugino**: \( M_2 = 100 \text{ GeV}, \ |\mu| = 1.0 \text{ TeV}, \)
- **Higgsino**: \( M_2 = 500 \text{ GeV}, \ |\mu| = 100 \text{ GeV}, \)
- **Mixed**: \( M_2 = |\mu| = 150 \text{ GeV}. \)

(39)

As will be shown later, the lightest neutralino is Bino–like in the gaugino case, while it is higgsino–like in the Higgsino case.

Considering the Higgs search experiments and the constraints on the CP–phases of the top and bottom squark sectors from the electron and neutron EDMs at the two–loop level, we take two typical values of \( \tan \beta; \ \tan \beta = 3 \) and 10. In addition, we take two values, 150 and 500 GeV for the pseudoscalar mass parameter \( m_A \); In the former case, all the neutral Higgs bosons have masses less than 200 GeV, while in the latter case two neutral Higgs bosons are much heavier than the lightest Higgs boson. As a result, a significant CP–violating Higgs–boson mixing is expected only for \( m_A = 150 \) GeV. Finally, we take the running masses \( \tilde{m}_t(m_t) = 165 \text{ GeV} \) and \( \tilde{m}_b(m_b) = 4.2 \text{ GeV} \) as the top and bottom quark masses. We note in passing that the spin–independent cross section is rather strongly dependent on these quark masses [18].

The loop–induced phase \( \xi \) between the vacuum expectation values of two neutral Higgs bosons can be adjusted to be zero by taking an appropriate renormalization scheme. With this adjustment and the universality assumption for the trilinear parameters, we can find that the Higgs–boson sector as well as the top and bottom squark sectors involves only the combination \( \Phi_{A\mu} \equiv \Phi_A + \Phi_\mu \) of the two rephasing–invariant phases \( \Phi_A \) and \( \Phi_\mu \), while the light neutralino sector involves both \( \Phi_1 \) and \( \Phi_\mu \), but not \( \Phi_A \).

### 4.1 Mass spectra

Based on the above parameter sets as in Eq. (39), let us first investigate the effects of the rephasing–invariant CP phases on the mass spectra for the the lightest neutralino, top squarks and Higgs bosons. Figure 2 shows the dependence of the lightest neutralino mass on the phase \( \Phi_\mu \) for five different values \( \{0^0, 45^0, 90^0, 135^0, 180^0\} \) for the phase \( \Phi_1 \); the

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\( ^{\dagger} \)The phase \( \Phi_1 \) of \( M_1 \) is zero up to the one–loop level for a positive \( M_1 \) in the minimal supergravity model.
upper left (right) frame is for $\tan \beta = 3$ (10) in the gaugino case, the middle left (right) frame is for $\tan \beta = 3$ (10) in the higgsino case, and the lower left (right) frame for $\tan \beta = 3$ (10) in the mixed case. The approximate form of the lightest neutralino mass is given by

$$m_{\tilde{\chi}_0^1} \simeq \begin{cases} 
\text{Gaugino} & : |M_1| - \frac{m_\chi^2}{|\mu|} \sin 2\beta \cos(\Phi_1 + \Phi_\mu), \\
\text{Higgsino} & : |\mu| - \frac{m_\chi^2}{2} (1 + \sin 2\beta) \left[ \frac{\mu^2}{|M_1|} \cos(\Phi_1 + \Phi_\mu) + \frac{\mu}{M_2} \cos \Phi_\mu \right],
\end{cases} \quad (40)$$

in the gaugino and higgsino cases, respectively.

Combined with the approximate expressions (40), Figure 2 shows several interesting features about the sensitivity of the neutralino mass to the phases $\Phi_A$ and $\Phi_\mu$.
• The mass is mainly determined by the modulus of $M_1$ and $\mu$ in the gaugino and higgsino cases, respectively, while in the mixed case the mass is affected by both $M_1$ and $\mu$.

• As the phase–dependent parts are accompanied by a factor $\sin 2\beta$ as shown in the expressions, the sensitivity of the neutralino mass to the phases is smaller for larger $\tan \beta$ in every case.

• In the mixed case, the sensitivity of the neutralino mass to the phase $\Phi_1$ is larger when $\Phi_\mu = 0$ or $2\pi$.

Consequently, it is expected that only in the mixed and higgsino cases with relatively small $|M_1|$ and $|\mu|$ the neutralino–nucleus scattering cross sections can be affected by the phases $\Phi_\mu$ and $\Phi_1$ through the lightest neutralino mass $m_{\tilde{\chi}_1^0}$.

**Figure 3:** The dependence of the light and heavy top squark masses $m_{\tilde{t}_1,2}$ on the phase $\Phi_{A\mu}$ for (a) $|\mu| = 1$ TeV and $\tan \beta = 3$, (b) $|\mu| = 100$ GeV and $\tan \beta = 3$, (c) $|\mu| = 1$ TeV and $\tan \beta = 10$, (d) $|\mu| = 100$ GeV and $\tan \beta = 10$ with the parameter set (37) for the squark mass parameters and trilinear parameters.
For moderate values of $\tan \beta$ the loop–induced effects to the neutral Higgs boson sector is dominated by the top (s)squark contributions due to the largest top Yukawa coupling. In this light, we present in Fig. 3 the dependence of the top squark masses $m_{\tilde{t}_1,2}$ on the phase $\Phi_{A_{\mu}}$ for (a) $|\mu| = 1$ TeV and $\tan \beta = 3$, (b) $|\mu| = 100$ GeV and $\tan \beta = 3$, (c) $|\mu| = 1$ TeV and $\tan \beta = 10$, (d) $|\mu| = 100$ GeV and $\tan \beta = 10$ with the parameter set (37) for the squark mass parameters and trilinear parameters. Analytically, we find that the phase dependence of the top squark masses is determined by the quantity:

$$\Delta_{12} = m_t \Im(A_\mu \mu e^{i\xi}) \cot \beta.$$  

(41)

Figure 3 and Eq. (41) clearly show that the top squark masses are strongly dependent on the phase $\Phi_{A_{\mu}}$ only in the gaugino case with large $|\mu|$ and small $\tan \beta$. In the other cases, the top squark masses are almost insensitive to the phase $\Phi_{A_{\mu}}$.

**Figure 4:** The Higgs–boson mass spectrum with respect to the phase $\Phi_{A_{\mu}}$ in (a) the gaugino case and (b) the higgsino case with $\tan \beta = 3$, and in (c) the gaugino case and (d) the higgsino case with $\tan \beta = 10$; the pseudoscalar mass parameter $m_A$ is set to be 150 GeV.
The mass spectrum of the neutral Higgs bosons is sensitive to the combination $\Phi_{A\mu} = \Phi_A + \Phi_\mu$. So, we exhibit in Fig. 3 the Higgs–boson mass spectrum with respect to the phase $\Phi_{A\mu}$ with $m_A = 150$ GeV. The mass difference between the the lightest Higgs boson and the heavier Higgs boson is more significant for smaller $\tan \beta$. The reason is that for the pseudoscalar mass comparable to $m_Z$, the off–diagonal entries of the Higgs–boson mass matrix are essentially proportional to $\sin 2\beta$. On the other hand, the sensitivity of the Higgs–boson masses to the phase $\Phi_{A\mu}$ is strongly suppressed for large $m_A$ due to the suppressed mixing. Because of this feature we have shown the Higgs mass spectrum only for $m_A = 150$ GeV in Fig. 3. For moderate values of $\tan \beta$ the size of the one–loop induced CP–violating neutral Higgs–boson mixing is dictated by the factor

$$\Delta_i = \frac{3m(A_1\mu e^{i\xi})}{m_{t_2}^2 - m_{t_1}^2}. \quad (42)$$

Therefore, only when both $|A|$ and $|\mu|$ are large, the Higgs boson masses become sensitive to the phase $\Phi_{A\mu}$. This feature is clearly reflected in Fig. 3 for the Higgs mass spectrum; in the gaugino case with a large value of $|\mu| = 1$ TeV, the Higgs boson masses depend strongly on the phase, while in the higgsino case with a small value of $|\mu| = 100$ GeV, the Higgs boson masses are almost insensitive to the phase. Consequently, the large effects of the phase $\Phi_{A\mu}$ on the Higgs masses and mixing are expected for large $|\mu|$ as well as small $m_A$ and $\tan \beta$.

### 4.2 Neutralino–nucleus scattering cross sections

With the previous comprehensive investigations of the mass spectra for the lightest neutralino, top squarks and neutral Higgs bosons, let us study in this section the dependence of the neutralino–nucleus cross sections on the CP–violating phases $\{\Phi_1, \Phi_\mu, \Phi_A\}$ as well as the other real SUSY parameters.

Firstly, we evaluate the elastic cross sections in the CP–invariant theories. We take the three phases to be vanishing; $\Phi_A = \Phi_\mu = \Phi_1 = 0$ and consider F, Si and Ge as three typical target nuclei. Table 1 shows the neutralino–nucleus elastic scattering cross sections; spin–independent and spin–dependent parts, $\sigma_S$ and $\sigma_A$, for the three different scenarios and two values of $m_A$; the upper table is for $\tan \beta = 3$ and the lower part for $\tan \beta = 10$. Note that the size of the cross sections depends significantly on the scenarios and on the target nuclei as well as on the pseudoscalar mass parameter $m_A$:

- The cross sections are strongly suppressed in the gaugino case, but they are very much enhanced in the mixed and higgsino cases. This suppression feature is more predominant in the spin–dependent cross section, for the coupling of the $Z$ boson to the neutralinos is strongly suppressed in the gaugino case, i.e. the absolute magnitudes of $N_{13}$ and $N_{14}$ are extremely small.
• The spin–independent cross sections are much more enhanced for heavier nucleus and become dominant over the spin–dependent cross section as can be expected from the coherent scattering effects.

• The spin–independent cross sections are strongly suppressed for large pseudoscalar mass $m_A$ and large $\tan \beta$, especially in the gaugino case.

The elastic scattering process $\tilde{\chi}_1^0 q \rightarrow \tilde{\chi}_1^0 q$ is related to the neutralino–neutralino annihilation process, $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow q\bar{q}$, contributing to the relic density of the SUSY dark matter. We note that in most cases the gaugino scenario is favored by the estimates of the relic density.

Table 1: The neutralino–nucleus elastic scattering cross sections; spin–independent and spin–dependent parts for the three different scenarios and two values of $m_A$ in the CP–invariant case, $\Phi_\mu = \Phi_1 = \Phi_A = 0$. F, Si and Ge are considered as three different target nuclei; the upper table is for $\tan \beta = 3$ and the lower part for $\tan \beta = 10$.

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Figure 5: The ratio $R_{\text{Ge}}[\Phi_\mu; \Phi_A, \Phi_1]$ with respect to the phase $\Phi_\mu$ for the parameter set (37) and $m_A = 150$ GeV in (a) the gaugino case and (b) the higgsino case with $\tan\beta = 3$, and in (c) the gaugino case and (d) the higgsino case with $\tan\beta = 10$. Here, the phase $\Phi_1$ is taken to be $0^\circ$.

Secondly, we study the effects of the CP–violating phases on the neutralino–nucleus scattering cross sections. Note that the spin–dependent cross sections $\sigma_A$ are independent of $\Phi_A$ and almost independent of the phases $\Phi_1$ and $\Phi_\mu$, especially in the gaugino and higgsino cases, because the neutralino mixing elements $N_{13}$ and $N_{14}$ determining the coupling of the $Z$ boson to the lightest neutralinos are suppressed by the factors $m_Z/|\mu|$ and $m_Z/|M_{1,2}|$ in the gaugino and higgsino cases, respectively. Even in the mixed case the lightest neutralino is almost Bino–like because of the gaugino mass unification condition so that this scenario also causes the spin–dependent cross sections are almost independent of the CP–violating phases. Taking into account these points, we consider the ratio of the spin–independent cross sections only for the heaviest target nucleus Ge:

$$R_{\text{Ge}}[\Phi_\mu; \Phi_A, \Phi_1] = \frac{\sigma_s[\Phi_\mu; \Phi_A, \Phi_1]}{\sigma_s[0; 0, 0]}$$

(43)
for several values of the phases \( \{ \Phi_1, \Phi_A \} \) and present the numerical results for the spin–independent cross section ratio (43) by taking \( m_A = 150 \) GeV and the parameter set (37) in three figures; the first figure is for \( \Phi_1 = 0^0 \) (Figure 5), the second one for \( \Phi_1 = 90^0 \) (Figure 6) and the third one for \( \Phi_A = 180^0 \) (Figure 7), respectively. Each figure contains four frames for (a) the gaugino case and \( \tan \beta = 3 \), (b) the higgsino case and \( \tan \beta = 3 \), (c) the gaugino case and \( \tan \beta = 10 \) and (d) the higgsino case and \( \tan \beta = 10 \); in each frame the solid line is for \( \Phi_A = 0^0 \), the dotted line for \( \Phi_A = 90^0 \) and the dashed line for \( \Phi_A = 180^0 \).

Figure 6: The same ratio \( \mathcal{R}_{Ge} [\Phi_\mu, \Phi_A, 90^0] \) as in Fig. 5 but for \( \Phi_1 = 90^0 \).

Comparing the results presented in the three figures, we find several interesting features about the dependence of the spin–independent ratio \( \mathcal{R}_{Ge} \):

- In all the figures and all the scenarios the spin–independent ratio is strongly dependent on the phase \( \Phi_\mu \); it is striking that certain values of \( \Phi_\mu \) render the cross sections almost vanishing.
The ratio is very sensitive to the phase $\Phi_A$ in the gaugino case, where the Higgs–boson couplings to the lightest neutralinos as well as the quarks are significantly affected by the phases $\Phi_A$ and $\Phi_\mu$ as well as the phase $\Phi_1$. This $\Phi_A$ dependence is expected to be more significant for small $m_A$ due to a larger mixing between the scalar and pseudoscalar Higgs bosons. The ratio itself varies more significantly with the phases $\Phi_\mu$ and $\Phi_A$ for $\tan \beta = 10$ than $\tan \beta = 3$.

In the higgsino case, the cross section ratio $R_{Ge}$ is (almost) always suppressed for non–trivial values of $\Phi_\mu$, i.e. the neutralino–nucleus cross section is maximal near $\Phi_\mu = 0$ and 360º.

On the contrary, the cross section ratio is enhanced or suppressed depending on the values of the phases $\{\Phi_1, \Phi_\mu, \Phi_A\}$ in the gaugino case. Interestingly, the cross section ratio is maximal when the sum of the phases $\Phi_\mu$ and $\Phi_1$ is 0º (360º) or 180º, while the ratio is almost vanishing when the sum of the phases $\Phi_\mu$ and $\Phi_1$ is 90º or 270º. This feature can be understood from the fact that in the gaugino case the matrix elements $N_{1k}$ ($k = 1 - 4$) as well as the lightest neutralino mass depend on only the rephasing–invariant combination $\Phi_\mu + \Phi_1$ with a good approximation.

Comparing the lines for three different values of $\Phi_A$ in each frame of Figs. 5, 6, and 7 in the gaugino case, we can confirm that the phase determining the CP–violating Higgs–boson mixing is indeed the rephasing–invariant combination $\Phi_{A\mu}$ of the phases $\Phi_A$ and $\Phi_\mu$.

All these features can be understood by noting that the spin–independent interactions are primarily determined by the coupling strengths of the Higgs bosons to the neutralinos and the quarks given in Eqs. (16) and (24). The scalar couplings of a Higgs boson to quarks can vanish when the Higgs boson becomes (almost) a CP–odd state for certain values of $\Phi_{A\mu}$ through the CP–violating scalar–pseudoscalar mixing. Certainly, for some values of the phases the scalar couplings can be enhanced compared to the CP–invariant case. On the other hand, the scalar couplings of the Higgs bosons to the lightest neutralinos, which is given by the real part of $G_k$ in Eq. (23), involves not only the CP–violating Higgs–boson mixing but also the neutralino mixing. Therefore, these couplings are expected to be strongly dependent on all the phases in the gaugino case.

To summarize, the CP–violating phases could reduce or enhance the neutralino–nucleus cross sections compared to those in the CP–invariant theories. The phase $\Phi_1$ affects the cross section significantly in most cases, while the phase $\Phi_A$ modifies the (spin–independent) cross section in the gaugino case with large $|\mu|$ through the phase combination $\Phi_{A\mu} = \Phi_A + \Phi_\mu$ for relatively small $m_A$. 

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5 Summary and conclusions

In this paper, we have made a comprehensive investigation of the effects of the CP–violating phases on the neutralino–nucleus elastic scattering cross sections in the framework of the MSSM through the neutralino mixing and the one–loop induced Higgs–boson mixing from mainly the top (s)quark sector due to the largest top Yukawa coupling. For the sake of numerical analysis, we have imposed the universal relation for the squark mass parameters and the gaugino mass unification condition only for the moduli of the SU(2) and U(1) gaugino masses. We have taken large sfermion masses and moderate values of tan β, allowing the CP–violating phases to have large non–trivial values without violating the stringent constraints of the electron and neutron EDM measurements, as well as large trilinear parameters for a significant neutral Higgs–boson mixing.
Based on three scenarios – gaugino, higgsino and mixed – for the neutralino sector, we have performed a detailed numerical analysis for the mass spectra of the lightest neutralino, top squarks and neutral Higgs bosons. The lightest neutralino mass is strongly dependent on the phases $\Phi_1$ and $\Phi_\mu$ for small $\tan \beta$ and in the mixed case with comparable gaugino and higgsino mass parameters. On the contrary, the top squark and Higgs boson masses are significantly affected by the combination $\Phi_{A\mu}$ of the phases $\Phi_\mu$ and $\Phi_A$ in the gaugino case with large $|\mu|$ and with small $\tan \beta$.

After estimating the neutralino–nucleus scattering cross sections for the target nuclei, F, Si and Ge for the three scenarios in Eq. (39) in the CP–invariant theories, we have studied the effects of the CP–violating phases on the spin–independent cross section ratio $R_{Ge}$ for the target nucleus Ge, for which the spin–independent cross section is significantly enhanced by large coherent scattering effects due to its large mass number. We have found that the CP–violating phases could reduce or enhance the cross section ratio significantly. The phase $\Phi_\mu$ as well as the phase $\Phi_1$ affects the ratio section in almost all the cases, while the phase $\Phi_A$ modifies the (spin–independent) cross section in the gaugino case with large $|\mu|$ through the phase combination $\Phi_{A\mu} = \Phi_A + \Phi_\mu$, when the pseudoscalar mass $m_A$ is relatively small.

Consequently, the CP–violating phases as well as all the real parameters in the MSSM are very crucial in estimating the neutralino–nucleus cross sections and so in determining the possibility of experimentally detecting the lightest neutralinos if they indeed constitute a major component of cold dark matter in the Universe.

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