LENSING CONSTRAINTS ON THE CORES OF MASSIVE DARK MATTER HALOS

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ABSTRACT

The statistics of wide-separation (6" < θ < 15") gravitational lenses constrain the amount of mass in the cores of dark matter halos on group and cluster mass scales. For a family of halo models with a central cusp $\rho \propto r^{-\alpha}$ (1.0 ≤ $\alpha$ ≤ 1.9), the lack of wide-separation lenses in the large Cosmic Lens All-Sky Survey yields an upper limit on the fraction of the halo mass that is contained within ~4% of the virial radius, $f_{\text{core}} < 0.023$ (95% confidence level, LCDM). This limit offers an important test of the cold dark matter paradigm. While the halo profiles derived from numerical simulations appear to be consistent with this upper limit, larger surveys currently underway such as the 2dF and SDSS should detect wide-separation lenses and thus provide a measurement of the core mass fraction in massive dark matter halos.

Subject headings: cosmology: dark matter – gravitational lensing

1. INTRODUCTION

Cuspy dark matter halos are a robust prediction of the cold dark matter (CDM) paradigm. Numerical simulations have shown that there is a “universal” density profile for dark matter halos with the form $\rho \propto r^{-\alpha}$ ($r_s$ + $r$)$^{-2}$, where $r_s$ is a scale length (Navarro, Frenk & White 1997, hereafter NFW). This result does not depend on particular cosmogonies or on specific initial conditions or formation histories (Huss, Jain & Steinmetz 1999a,b). There is some disagreement about the very inner regions of halos; recent simulations have suggested that the central cusp may be steeper than $r^{-1}$ (e.g., Moore et al. 1998, 1999) and may depend on the halo mass (Jing & Suto 2000).

It is important to compare the predicted halo profiles with observational data to test the CDM paradigm. Galaxy rotation curves can be used to probe dark matter halos (e.g., Flores & Primack 1994; McGaugh & de Blok 1998), but the implications are not clear because beam smearing can make HI rotation curves appear shallower than they really are (van den Bosch & Swaters 2000). Gravitational lensing offers an additional test of halo profiles. More than 60 multiply-imaged quasars and radio sources are known (see Falco et al. 1999), and with image separations of a few arcseconds they primarily probe halos on galaxy mass scales. These lenses can be used to constrain CDM halos (e.g., Rusin & Ma 2000), but the test is complicated by the necessity of including baryons and their effects on dark matter halos. At least 24 giant lensed arcs are also known (see Williams, Navarro & Bartelmann 1999), and with image separations of several tens of arcseconds or larger they probe rich clusters.

Lenses with intermediate image separations, $\theta \sim 10''$, probe halos at intermediate masses. There is just one confirmed lens (Q 0957+561, Walsh, Carswell & Weymann 1979) and one good candidate (RX J0921+4529, Muñoz et al. 2000) with $\theta > 6''$, and each is produced by a galaxy in a cluster. No systematic lens survey has found any lenses in this regime, and this result has been used to place limits on cosmological parameters (e.g., Cen et al. 1994; Kochanek 1995; Wambsganss et al. 1995; Maoz et al. 1997; Mortlock & Webster 2000). Instead, in this Letter we propose to use the statistics of wide-separation lenses as a new test of the structure of dark matter halos (see Flores & Primack 1996). We consider a range of halo profiles suggested by recent CDM simulations, and compare our model predictions with the latest and largest lens survey, the Cosmic Lens All-Sky Survey.

2. CUSPY HALOS

To obtain a family of halos with a range of central cusps, we adopt a generalization of the NFW profile (see Jing & Suto 2000; Wyithe, Turner & Spergel 2000),

$$\rho(r) = \frac{\rho_s}{(r/r_s)^\alpha(1+r/r_s)^{3-\alpha}},$$

(1)

where $r_s$ is a scale length and $\rho_s$ is a characteristic density. The density scales as $\rho \propto r^{-\alpha}$ for small $r$ and $\rho \propto r^{-3}$ for large $r$, so it reduces to the NFW profile when $\alpha = 1$. The characteristic density $\rho_s$ is given by

$$\rho_s = \rho_{\text{crit}}(z) \frac{200(3-\alpha)}{3 \cdot \text{F}_1[3-\alpha,3-\alpha,4-\alpha,-r_{200}/r_s]},$$

(2)

where $\text{F}_1(a, b, c, z)$ is a hypergeometric function, and $r_{200}$ is the radius within which the mean density is 200 times the critical density of the universe $\rho_{\text{crit}}$. Moore et al. (1998, 1999) advocate a profile of the form $\rho \propto x^{-1.5}(1+x^{1.5})^{-1}$ where $x = r/r_s$. Although this differs slightly from a cuspy profile with $\alpha = 1.5$, the two profiles are similar at small radii and differ by ≤15% in the enclosed inner mass, which we will argue below is the most important quantity in determining the number of lenses. We do not consider the Moore profile further because there is no simple family of models connecting it to the NFW model. It is often convenient to replace the scale radius of a cuspy halo with a “concentration” parameter. For NFW ($\alpha = 1$) halos, Navarro et al. (1997) define $C_{\text{NFW}} \equiv r_{200}/r_s$. For the generalization to $\alpha \neq 1$, we define $C_{\alpha} \equiv r_{200}/r_s^\alpha$ in terms of

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the radius $r_{-2}$ at which the logarithmic slope of the density profile is $-2$, which is equivalent to $C_{\text{NFW}}$ for $\alpha = 1.3$.

A cuspy halo is fully specified by its mass, redshift, cusp slope, and scale radius or concentration, but these parameters are not all independent. For NFW halos, the concentration is correlated with the mass and redshift in a way that appears to contain information about the formation epoch of the halo (e.g., Navarro et al. 1997). However, there is a scatter in concentrations at fixed mass and redshift that may reflect differences in formation histories (e.g., Bullock et al. 1999; Klypin et al. 2000). The scatter is important for our calculations because lensing is very sensitive to concentration; more concentrated halos are much more efficient lenses. We use results from the simulations by Bullock et al. (1999), namely a scatter that is consistent with a log-normal distribution with standard deviation $\Delta (\log C_{-2}) \approx 0.18$, and a median concentration that scales with mass and redshift as $\text{med}(C_{-2}) \propto M^{-1/3}(1+z)^{-1}$ (Bullock et al. 1999; Navarro & Steinmetz 2000). These trends were derived from fits of NFW ($\alpha = 1$) profiles to simulated halos, but because their origin is physical we assume that they apply to other values of $\alpha$ as well.

### 3. Lens Theory

To compute the total number of lenses expected to be produced by a population of halos it is sufficient to use spherical halos. Departures from spherical symmetry mainly affect the relative numbers of two-image and four-image lenses, which we do not differentiate. Wyithe et al. (2000) and Li & Ostriker (2000) discuss the lensing properties of cuspy halos of the form given in eq. (1), but their formalism can be simplified so the deflection is written as

$$\phi_R (r) = \frac{R}{\pi \Sigma_c} \int_R^\infty \frac{M(r)}{r^2 \sqrt{r^2 - R^2}} \, dr, \quad (3)$$

where $M(r)$ is the mass inside radius $r$ and $\Sigma_c = (c^2 D_c)/(4 \pi GD_D D_s)$ is the critical surface density for lensing (e.g., Schneider, Ehlers & Falco 1992). Here $D_c$ and $D_D$ are angular diameter distances to the lens and source, respectively, and $D_s$ is the angular diameter distance from the lens to the source. The positions of lensed images are found by solving the lens equation $u = R - \phi, R(\hat{r})$, where $u$ is the impact parameter of the source relative to the lens. The magnification of an image at $R$ is then

$$\mu = [1 - \phi, R(\hat{r})]^{-1} \left[ 1 - \phi, R(R) \right]^{-1},$$

where $\phi, R = d(\phi, R)/dR$.

The number of lenses expected to be found in a survey depends on the optical depth for lensing as well as on the “magnification bias,” which accounts for lenses that are intrinsically fainter than the flux limit of the survey but are brought into the sample by the magnification (e.g., Turner, Ostriker & Gott 1984). The number of lenses with a total flux greater than $S$ expected to be found in a survey with selection functions described by $F$ is then

$$N_{\text{lens}}(S; F) = \frac{1}{4\pi} \int \frac{d\zeta}{z} \int dV \int \frac{dM}{dM} \frac{dn}{dM} \int_m^\infty du \int_\theta \frac{dN_{\text{src}}(S/\mu) dz_s}{dz_s},$$

where $\zeta$ is the source redshift, $dV$ is the comoving volume element, and $dn/dM$ is the mass function of halos. Also, $u$ is the angular position of the source relative to the lens center, and $\mu$ is the total magnification of that source; the $\mu$ integral extends over the range of impact parameters that produce multiple images. Finally, $[dN_{\text{src}}(S)/dz_s] dz_s$ is the number of sources brighter than flux $S$ that lie in the redshift range $z_s$ to $z_s + dz_s$. The factor $F(u)$ indicates whether a lens associated with a source at $u$ would be detected given the selection functions.

We follow Narayan & White (1988), Kochanek (1995), and Porciani & Madau (2000) and compute lens statistics using a mass function of dark matter halos given by Press-Schechter theory combined with the spherical collapse model. Numerical simulations suggest that this mass function overestimates the number of halos below $\sim 10^{14} h^{-1} M_{\odot}$ and underestimates the number of halos above this mass (e.g., Jenkins et al. 2000). Because we are interested in the high-mass end of the mass function, our results should slightly underestimate the number of lenses, and lead to conclusions that are conservative. We compute the CDM power spectrum using the fitting formula given by Eisenstein & Hu (1999). We present results for two different flat cosmologies: SCDM, with matter density $\Omega_M = 1$ and Hubble constant $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$; and LCDM, with $\Omega_M = 0.3$, $\Omega_{\Lambda} = 0.7$, and $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$. From the abundance of clusters we set the variance of mass fluctuations on $8 h^{-1} \text{ Mpc}$ scales to be $\sigma_8 = 0.52$ for SCDM and $\sigma_8 = 0.93$ for LCDM (Eke, Cole & Frenk 1996).

Li & Ostriker (2000) have recently performed a similar calculation. However, they use a different power spectrum, omit magnification bias, and do not consider a scatter in concentrations, so their quantitative results are somewhat different from ours.

### 4. The CLASS Lens Survey

To test the models we must compare predicted lens statistics with a well-defined observational sample. The Cosmic Lens All-Sky Survey (CLASS; e.g., Brown & Myers 2000) is the largest statistically homogeneous search for gravitational lenses. The sample comprises 10,499 flat-spectrum radio sources whose flux distribution can be described as a power law $dN_{\text{src}}/dS \propto S^{-2.1}$ (see Rusin & Tegmark 2000). The survey includes 18 gravitational lenses with image separations $\theta < 3''$. An explicit search for lenses with image separations $6'' < \theta < 15''$ has found no lenses (Phillips et al. 2000).

The redshift distribution of the full CLASS sample is not known. Marlow et al. (2000) report redshifts for a small subsample of 27 sources. They find a mean redshift of $\langle z \rangle = 1.27$, which is comparable to that found in other radio surveys at comparable fluxes. We present results assuming that the full sample has the same source redshift distribution as the spectroscopic subsample of Marlow et al. (2000), but our results are not substantially different if we simply place all sources at the mean redshift of the spectroscopic subsample, so the source redshift distribution is not a significant source of uncertainty.

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3 For $\alpha \neq 1$, $C_{-2}$ differs from the parameter $C_2 \equiv r_{200}/r_s$ used by Wyithe et al. (2000). We believe that $C_{-2}$ is the better generalization to $\alpha \neq 1$ because the radius $r_{-2}$ has physical significance, while $r_s$ does not; in any case, the alternate parameters have a simple relation in cuspy halos, $C_s = (2 - \alpha)C_{-2}$.
We focus on wide-separation lenses, $6'' < \theta < 15''$, which are not present in the CLASS sample and correspond to halos on the mass scale of groups and clusters. The exact bounds on the image separation have no intrinsic significance but are chosen to correspond to the analysis of the CLASS survey data (see Phillips et al. 2000). We compute the number of lenses for particular values of the cusp slope and median concentration; we consider a wide range of models with $1.0 \leq \alpha \leq 1.9$ and $2 \leq \text{med}(C_{-2}) \leq 11$. Models with $\text{med}(C_{-2}) > 11$ are ruled out at 99% confidence or better for all values of $\alpha$ that we consider.

![Figure 1](image1.png)

**Figure 1.** The number of wide-separation lenses expected in the CLASS survey as a function of the cusp slope and concentration parameter, for the LCDM cosmology. The dotted curves show contours drawn at 1, 2, 3, and 4 lenses, and the solid curves show the 1σ, 90%, and 95% confidence upper limits.

Figure 1 shows the expected number of wide-separation lenses as a function of $\alpha$ and $C_{-2}$. The results are not easy to interpret for two reasons. First, there is some ambiguity in the meaning of the concentration parameter when halos are allowed to have arbitrary cusps (see §5 of Wyithe et al. 2000 for a detailed discussion). Second, lensing is not very sensitive to the cusp slope and the concentration separately, but depends mainly on the amount of mass in the central regions of halos. It is more instructive then to plot the number of lenses versus the “core mass fraction” $f_{\text{core}}$, or the fraction of the total mass of the halo that is contained within some small fiducial radius, as in Figure 2. The best fiducial radius is the one that minimizes the scatter in the relation between the core mass fraction and the number of lenses. It is 0.045 $r_{200}$ for SCDM and 0.032 $r_{200}$ for LCDM, which is approximately equal to the mean Einstein radius of lenses with $6'' < \theta < 15''$. Because of systematic trends and the scatter in concentrations, the core mass fraction is measured for a median halo of mass $10^{14} h^{-1} M_\odot$ at redshift $z = 0$, which is the mass scale relevant for wide-separation lenses.

![Figure 2](image2.png)

**Figure 2.** The number of wide-separation lenses plotted versus the fraction of halo mass that is contained within a small fiducial radius. The dotted curves show contours drawn at 1, 2, 3, and 4 lenses, and the solid curves show the 1σ, 90%, and 95% confidence upper limits.

The predicted number of lenses admits the following quantitative interpretation. The probability of finding $k$ lenses when $N$ are expected is well approximated by the Poisson distribution as $P(k|N) = N^k e^{-N} / k!$. Given that CLASS contains no wide-separation lenses ($k = 0$), the upper limits on the predicted number of lenses are $N < (1.15, 2.30, 3.00)$ at the (1σ, 90%, 95%) confidence levels. Figures 1 and 2 then yield upper limits on the core mass fraction, cusp slope, and concentration, which are summarized in Table 1. For NFW ($\alpha = 1$) halos, the median concentration of a $10^{14} h^{-1} M_\odot$ halo must be less than about 7 at 1σ and less than 9 at 90%. The simulations...
lations by Navarro et al. (1997) and Bullock et al. (1999) yield concentrations of slightly less than 6, so they are consistent with the lens data (see Figure 2). Halos that are significantly more concentrated would disagree with the lens data. We could compute the limits on the concentration parameter for other cusp slopes (and Table 1 includes the limits for Moore-like \( \alpha = 1.5 \) halos), but we believe it is more instructive to focus on the core mass fraction.

Finally, we note that modeling all halos as singular isothermal spheres yields a prediction of 9 (11) wide-separation lenses for SCDM (LCDM). Thus, the statistics of wide-separation lenses strongly exclude the hypothesis that the halos of groups and clusters are singular isothermal spheres.

### Upper Limits From Wide-Separation Lenses

<table>
<thead>
<tr>
<th>Cosmology</th>
<th>( f_{\text{core}} )</th>
<th>( C_{-2}(1) )</th>
<th>( C_{-2}(1.5) )</th>
<th>( C_{-2}(2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCDM</td>
<td>0.031</td>
<td>0.037</td>
<td>0.040</td>
<td></td>
</tr>
<tr>
<td>LCDM</td>
<td>0.017</td>
<td>0.021</td>
<td>0.023</td>
<td></td>
</tr>
</tbody>
</table>

\*NOTE.—\( f_{\text{core}} \) is the fraction of a median halo’s mass that is contained within 0.045 \( r_{200} \) for SCDM, or 0.032 \( r_{200} \) for LCDM. \( C_{-2}(\alpha) \) is the median concentration of halos with cusp slope \( \alpha \).

### 6. CONCLUSIONS

We have studied the statistics of wide-separation (6\( \arcsec \) < \( \theta < 15\arcsec \)) lenses produced by dark matter halos with general cusps of the form \( \rho \propto r^{-\alpha} \) for 1.0 \( \leq \alpha \leq 1.9 \). In these models the number of expected lenses is determined almost entirely by the fraction of the halo mass that is contained within a fiducial radius that is \( \sim 4\% \) of the virial radius. Combining our results with the lack of observed wide-separation lenses in the CLASS lens survey yields an upper limit on how concentrated halos can be. The concentration parameter for the standard NFW profile must be \( C \lesssim 7 \) (1\( \sigma \)); see Table 1 for more details. Massive halos found in numerical simulations of CDM cosmologies have \( C \sim 6 \) (e.g., Navarro et al. 1997; Bullock et al. 1999) and thus are consistent with the lensing constraints, but halos cannot be much more concentrated than that. Halos with cusps steeper than \( r^{-1} \) are also allowed provided that they have somewhat lower concentrations. Although we quote limits on the concentration, we believe that it is more attractive to state our results in terms of the core mass fraction, e.g., \( f_{\text{core}} < 0.023 \) at 95% confidence for LCDM. This quantity should be a more robust prediction of numerical simulations than the slope of the central density and should allow a simple comparison of our models with simulations.

Two points are important for interpreting our results. First, the scatter in halo properties requires that we quote median properties. Any given halo may be more concentrated than the median and hence violate the mass limits that we have derived for median halos. In other words, the lensing limits must be applied statistically, not to individual halos. Second, we have considered only dark matter models, neglecting any baryons that might occupy the cores of massive dark matter halos as cD galaxies in clusters or central ellipticals in groups. This approach should lead us to conservative conclusions, because any central baryonic component would only increase the number of expected lenses and thus aggravate the discrepancy between the data and the CDM models.

While our results indicate that the lack of wide-separation lenses in the CLASS survey is perhaps not surprising, they also suggest that we should expect to discover them soon. In our models, NFW halos with a median concentration \( C = 6 \) predict 0.6 lenses with 6\( \arcsec \) < \( \theta < 15\arcsec \) out of the 10,499 CLASS targets. Several new surveys are large enough that we should expect a few group and cluster lenses; they include 2dF and SDSS, which should measure the redshifts of more than 25,000 and \( 10^5 \) quasars, respectively. If the new surveys continue to lack wide-separation lenses, the lensing constraints on the cores of dark matter halos will become strong enough to question the CDM paradigm. If, on the other hand, the new surveys do detect wide-separation lenses, the lensing constraints on the cores of massive dark matter halos will strongly question the CDM paradigm. We thank M. Steinmetz and V. Eke for helpful discussions about dark matter halos, and P. Phillips and I. Browne for information about the CLASS survey. Support for this work was provided by NASA through ATP grant NAG5–4236 and grant AR–0633710–94A from the Space Telescope Science Institute (P.M.).

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