Pseudoscalar production in electromagnetic fields
by a Schwinger-like mechanism

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Abstract

In this talk I report on some recent calculations on the production of pseudoscalars from intense electromagnetic fields.

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1 Decay of classical background fields into pseudoscalars

We would like to calculate the effective action for the background electromagnetic $E$ and $B$ fields,

$$e^{iS_{\text{eff}}[E,B]} = \int \mathcal{D}\phi \ e^{iS[\phi,E,B]}$$

when integrating a pseudoscalar $\phi$ of mass $m$, that has an action $S$ with a coupling to the $E$ and $B$ fields of the form

$$S[\phi, E, B] = \int d^4x \ \frac{1}{2} \phi(x) \left[-\partial^2 - m^2 + f(x, E, B)\right] \phi(x)$$

Using the identities

$$i \frac{\partial S_{\text{eff}}[E, B]}{\partial m^2} = -\int \mathcal{D}\phi \ \phi^2 \ e^{iS[\phi,E,B]}$$

$$= -\frac{1}{2} \int d^4x \ G(x, x; E, B)$$

$$= -\frac{1}{2} \int d^4x \ \int \frac{d^4p}{(2\pi)^4} \ G(p; E, B)$$

we can express the effective Lagrangian of the background fields in terms of the Green's function $G(p)$ of $\phi$ propagating in these fields.

$$\mathcal{L}_{\text{eff}}[E, B] = \frac{i}{2} \int dm^2 \int \frac{d^4p}{(2\pi)^4} \ G(p; E, B)$$

Our objective now is to determine 1) $G(p)$ and 2) $\mathcal{L}_{\text{eff}}$.

1) The action (1) contains the interaction Lagrangian

$$\mathcal{L}_I(x) = \frac{1}{2} f(x) \phi^2(x)$$

and leads to the following equation for the Green function

$$\left[\partial^2 + m^2 - f(x)\right] G(x, 0) = \delta^4(x)$$

We will approximate $f(x)$ by its Taylor series near the reference point $x = 0$ up to second order,

$$f(x) = \alpha + \beta_\mu x^\mu + \gamma_{\mu\nu} x^\mu x^\nu$$

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The equation (6) is then approximated by
\[ \partial^2 + m^2 - \alpha - \beta_{\mu} x^{\mu} - \gamma_{\mu\nu} x^{\mu} x^{\nu} \] \( G(x,0) = \delta^4(x) \) (8)

or, in momentum space,
\[ -p^2 + m^2 - \alpha + i\beta_{\mu} \frac{\partial}{\partial p_{\mu}} + \gamma_{\mu\nu} \frac{\partial}{\partial p_{\mu}} \frac{\partial}{\partial p_{\nu}} \] \( G(p) = 1 \) (9)

As is shown in detail in [1], the solution for \( G(p) \) that satisfies the boundary conditions is
\[ G(p) = i \int_0^{\infty} ds e^{-i\gamma s} \left[ e^{ip\cdot A} + B\cdot p + C \right] \] (10)

where
\[ A = \frac{1}{2} \gamma^{-1} \cdot \tan(2\gamma s) \] (11)
\[ B = -\frac{i}{2} \gamma^{-2} \cdot [1 - \sec(2\gamma s)] \cdot \beta \] (12)
\[ C = i\alpha s - \frac{1}{2} \text{tr} \left[ \ln \cos(2\gamma s) \right] + \frac{i}{8} \beta \cdot \gamma^{-3} \cdot [\tan(2\gamma s) - 2\gamma s] \cdot \beta \] (13)

2) The effective Lagrangian is obtained by substituting \( G(p) \) in (4) and carrying out the integration over \( m^2 \),
\[ \mathcal{L}_{\text{eff}} = -\frac{i}{2} \int_0^{\infty} \frac{ds}{s} \int \frac{d^4p}{(2\pi)^4} \exp \left\{ -ism^2 + ip \cdot A + B \cdot p + C \right\} \] (14)

After evaluation of the Gaussian integral we finally get
\[ \mathcal{L}_{\text{eff}} = -\frac{1}{32\pi^2} \int_0^{\infty} \frac{ds}{s^3} e^{-is(m^2-\alpha)} \left[ \det \left( \frac{2\gamma s}{\sin 2\gamma s} \right) \right] \frac{1}{2} e^{il(s)} \] (15)

where
\[ l(s) = \frac{1}{4} \beta \cdot \gamma^{-3} \cdot [\tan(\gamma s) - \gamma s] \cdot \beta \] (16)

When the effective Lagrangian has an imaginary part, there is particle production with a probability density given by
\[ w = 2 \text{Im} \mathcal{L}_{\text{eff}}[E,B] \] (17)

In turn, a non-zero value for \( \text{Im} \mathcal{L}_{\text{eff}} \) may arise depending on the sign of the \( \gamma \)-matrix eigenvalues.
2 Effective $F^2φ^2$ interactions

We assume the standard pseudoscalar-two photon coupling

$$L_{φγγ} = \frac{1}{8}gφ\epsilon^{μνρσ}F_{μν}F_{ρσ}$$

(18)

We shall now show that, for the purposes of calculating $L_{eff}$ of external $E$ and $B$, we can use an interaction Lagrangian of the type displayed in (5).

We first calculate the two-photon two-pseudoscalar amplitude in momentum space,

$$4 \left(\frac{1}{4}gφ\right)^2 \epsilon^{μνρσ}k_{μ}F_{ρσ} \frac{-i g_{μν'}}{k^2} \epsilon^{μ'ν'ρ'σ'}(-k_{μ'})F_{ρ'σ'}$$

(19)

Due to the presence of the $k^2$ term in the denominator, the effective coupling (19) is non-local. However, when we calculate the effective action for the external electromagnetic field the momentum $k$ is integrated over. One can therefore make use of the identity

$$\int d^4k k_{μ}k_{μ'} g(k^2) = \int d^4k \frac{g_{μν}k^2}{4} g(k^2)$$

(20)

to simplify (19). Thus, we can reduce the effective two-photon two-pseudoscalar to a local interaction vertex. Back in configuration space, it is given by

$$L_I = -\frac{1}{4}g^2φ^2 F_{μν}F^{μν} = \frac{1}{2}g^2φ^2(E^2 - B^2)$$

(21)

so that we can identify $f(x)$ in (5) with

$$f(x) = g^2(E^2 - B^2)$$

(22)

3 Pseudoscalar production in a cylindrical capacitor

In order to have a non trivial $L_{eff}$, one needs non-zero second derivatives of the electromagnetic fields as they appear in expression (22), which imply a non-zero
\(\gamma\)-matrix. We illustrate it in the simple situation of the electric field inside a cylindrical capacitor.

The modulus of the electric field inside a cylindrical capacitor whose axis lies along the \(z\)-axis depends only on \(\rho = (x^2 + y^2)^{\frac{1}{2}}\),

\[
E(\rho) = \frac{\lambda}{2\pi} \frac{1}{\rho}
\]

with \(\lambda\) the linear electric charge density. It follows that

\[
f(\rho) = g_c^2 \left( \frac{1}{\rho^2} \right)
\]

where \(g_c \equiv \lambda g / 2\pi\)

Expanding the fields near some reference point \((x_0, y_0, z_0)\) with \(\rho_0 = (x_0^2 + y_0^2)^{\frac{1}{2}}\), we identify

\[
\alpha = \frac{g_c^2}{\rho_0^2}
\]

\[
(\beta)_i = -\frac{2g_c^2}{\rho_0^2} (x_0, y_0)
\]

and

\[
(\gamma^2)_{ij} = \frac{g_c^2}{\rho_0^2} \begin{pmatrix} -y_0^2 + 3x_0^2 & 4x_0y_0 \\ 4x_0y_0 & -x_0^2 + 3y_0^2 \end{pmatrix}
\]

(We only include the \(x - y\) entries). In diagonal form \(\gamma^2\) reads

\[
(\gamma^2_D)_{ij} = \frac{g_c^2}{\rho_0^2} \begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix} \equiv \begin{pmatrix} a^2 & 0 \\ 0 & -b^2 \end{pmatrix}
\]

Introducing \(\alpha, \beta\) and \(\gamma\) in \(\mathcal{L}_{\text{eff}}\) as given in (15), we get the following expression

\[
\mathcal{L}_{\text{eff}} = -\frac{1}{32\pi^2} \int_0^{\infty} \frac{ds}{s^3} e^{-is(m^2 - \alpha)} \sqrt{\frac{2as}{\sinh 2as}} \sqrt{\frac{2bs}{\sin 2bs}} e^{il(s)}
\]

where

\[
l(s) = \lambda (as - \tanh as) \lambda = g_c^4 \rho_0^{-6} a^{-3} = \frac{g_c}{3 \sqrt{3}}
\]

One can perform the integration in (29) by extending \(s\) to the complex plane. The details of the integration can be found in [1], where it is found that

\[
\text{Im} \mathcal{L}_{\text{eff}} = \frac{a^\frac{1}{2} b^\frac{1}{2} \infty}{8\pi^2} \sum_{n=0}^{\infty} (-1)^n C_n e^{-\chi(2n+1)\pi}
\]
\[
C_n = \int_0^{\pi} du \frac{e^{-\chi u} e^{-\lambda \cot(u/2)}}{[u + (2n + 1)\pi]^2 [\sin u]^{1/2} \left[ \sinh \left( \frac{b}{a} [u + (2n + 1)\pi] \right) \right]^{1/2}}
\]  

(32)

(we can put \( b/a = 1/\sqrt{3} \)). We have defined

\[
\chi = \frac{m^2 - \alpha}{2a} + \frac{\lambda}{2} = \frac{1}{\sqrt{3}} \left( \frac{1}{2} \frac{m^2 \rho_0^2}{g_c} - \frac{1}{3} g_c \right)
\]  

(33)

Finally, the expression for the probability per unit volume and per unit time for pseudoscalar production inside a cylindrical capacitor is

\[
w = \frac{3^4}{4\pi^2} \frac{g_c^2}{\rho_0^4} C_0 e^{-\chi \pi}
\]  

(34)

were we kept only the leading \( n = 0 \) term in (31).

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References