Introduction

The reaction for any local quantum field theory is

\[ \mathcal{L} = \frac{1}{2} \left( \partial \phi \right)^2 - V(\phi) \]

where \( \phi \) is the field and \( V(\phi) \) is the potential. This is the basic action for a scalar field theory. In the case of quantum field theory, the potential is given by the interaction terms of the theory.

The basic idea of quantum field theory is to quantize the classical field theories. This is done by replacing the classical fields with quantum fields, which are operators acting on a Hilbert space. The quantum fields are then used to construct the corresponding quantum states and observables.

In quantum field theory, the fundamental objects are the quantum fields, which are represented as operators on the Hilbert space of states. The quantum fields are the primary objects of study in quantum field theory, and the properties of these fields are determined by the Hamiltonian of the system.

The Hamiltonian of a quantum field theory is a Hermitian operator that gives the total energy of the system. The Hamiltonian is constructed by taking the sum of the kinetic and potential energy terms for the quantum fields. The Hamiltonian is a crucial object in quantum field theory, as it determines the dynamics of the system.

The Hamiltonian is given by

\[ \mathcal{H} = \int d^4x \left( \sum_{\phi} \left( \frac{1}{2} \left( \partial \phi \right)^2 + V(\phi) \right) \right) \]

where the sum is over all quantum fields \( \phi \). The potential \( V(\phi) \) is a function of the quantum fields and is determined by the interactions of the system.

The Hamiltonian is a key object in quantum field theory, as it determines the dynamics of the system. The Hamiltonian is used to construct the wave function of the system, which gives the probability amplitude for the system to be in a particular state.

The wave function is given by

\[ \Psi(\mathbf{r}, t) = \int d^4x e^{iS(x)/\hbar} \]

where \( S(x) \) is the action of the system and \( \hbar \) is the reduced Planck constant. The wave function is a fundamental object in quantum field theory, as it gives the probability distribution for the system to be in a particular state.

In summary, quantum field theory provides a framework for understanding the behavior of quantum systems, including the fundamental forces of nature. The quantum fields are the primary objects of study in quantum field theory, and the properties of these fields are determined by the Hamiltonian of the system.

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contain a term:
\[
\frac{\theta}{32\pi^2} \varepsilon_{\mu
u\rho} F_{\mu
u}^a F_{\rho a},
\]
where \(F_{\mu
u}^a\) is the non-Abelian field strength tensor and \(\theta\) is an undetermined constant, the "\(\theta\) angle." However, this term violates CP invariance and hence generates a nonzero neutron EDM. Its strength; i.e., the value of \(\theta\), is therefore constrained by Eq. (1), which yields a very low upper bound \([8, 9, 10]\)
\[
|\theta| < (1 \sim 10) \times 10^{-10}.
\]
Currently there is no satisfactory explanation of why this term is so strongly suppressed and that is the basis of the so-called "strong CP problem:" the goal is to find a reason why it should naturally be identically zero. (In the absence of topologically nontrivial gauge field configurations, Eq. (2) cannot contribute to the action: it is a surface term.)

Herein shall assume \(\theta \equiv 0\) so that the phase in the CKM matrix is the only source of CP and T violation in the standard model. In this case a nonzero neutron EDM is proportional to the CKM phase. (Considerations relevant for \(\theta \neq 0\) are explored, e.g., in Refs. [9, 10].)

Over many years the experimental upper bound on \(d_n\) has steadily decreased and its small value has also proven very effective in ruling out candidates for theories that enlarge the standard model. To illustrate how, we suppose for the moment that the neutron is a collection of three valence-\(q\) as described by a symmetric \(SU(6)\) spin-flavour wave function. Then, by analogy with the magnetic moment,
\[
d_n = \frac{1}{3}(4 d_u - d_d),
\]
where \(d_u,d\) are valence-\(q\) EDMs. In the standard model the first nonzero contribution to a free \(q\) EDM appears at third order and involves a gluon radiative correction (i.e., \(O(a_s^3)\), for the same reason that flavour-changing neutral currents are suppressed: the GIM mechanism) so that \([8, 11]\)
\[
d_u^\text{free} < d_d \lesssim 10^{-34} \text{e cm}.
\]
Using this in Eq. (4) gives a result seven orders-of-magnitude less than the current experimental bound. This result is characteristic: other plausible mechanisms within the standard model, such as hadronic loop corrections, also give a very small value. However, the standard model is peculiar in this regard and candidates for its extension typically contain many more possibilities for CP and T violation, which \(a\ priori\) are not constrained to be small. Thus Eq. (1) is an important and direct constraint on these extensions because Eqs. (4), (5) indicate that the standard model contribution to \(d_n\) cannot possibly interfere at a level that could currently cause confusion. For example, as our calculation will show, the viability of the minimal model of spontaneous CP violation \([12]\), which involves three Higgs doublets, is endangered by Eq. (1).

Our interest in the neutron’s EDM stems from a desire to explore the validity of Eq. (4) in the case that, irrespective of the origin of the valence-\(q\) EDMs, how are they related to the EDM of the bound state? This question has recently been explored \([9, 10, 12]\), using QCD sum rules \([14]\). Our analysis, however, with the purpose of an effort recently developed, well-constrained Poincaré covariant, bound-state picture of the neutron \([15, 16, 17, 18, 19, 20]\) and therefore affords a necessary complement.

The nonzero neutron charge radius is a clear indication that a symmetric \(SU(6)\) wave function is inadequate for the neutron and there are significant additional weaknesses. For example, in making the connection between Eqs. (4) and (5) no consideration is given to the necessary momentum-dependence of the dressed-\(q\) mass function, which is a longstanding prediction of Dyson-Schwinger equation (DSE) studies; e.g., Ref. [21, 22], which has recently been confirmed in lattice-QCD simulations \([23]\) or, for confinement and the concomitant feature that dressed-\(q\) in the neutron are not on shell. (Reference [21] provides an heuristic guide to DSEs and References [24, 25] give an overview of their contemporary application, with particular emphasis on continuum strong QCD.)

Hitherto, lacking a Poincaré covariant bound state picture of the neutron, these aspects of dressed-\(q\) behaviour and hadron compositeness have only been explored in studies of the dipole moments of the \(\rho\)-meson \([26, 27]\), a bound state for which sound DSEs models exist. Assuming an explicit EDM contribution to the photon-current-\(q\) vertex:
\[
f \gamma_{\mu} \gamma_{\nu} q_{\nu} \equiv \frac{\epsilon}{2 \sqrt{m_f}} f_i \gamma_{\mu} \gamma_{\nu} q_{\nu},
\]
where: \(\epsilon\) is the positron charge; \(m_{q,u,d}\) are the current-\(q\) masses; \(\epsilon_{q,u,d}\), their electric charge fractions; \(h_{u,d}\), their gyroelectric ratios; and \(q_{\mu}\), the momentum transfer, an analogue of Eq. (4) for a pointlike \(\rho\)-meson is \([26]\)
\[
d_f = d_q + d_q.
\]
In this case the direct calculations showed that Eq. (7) underestimates the EDM of a bound state \(\rho\)-meson, which is composed of a dressed-\(q\) and a dressed-antiquark, and described by a Bethe-Salpeter amplitude, by as much as two orders-of-magnitude. That can materially affect the inferred constraints on extensions of the standard model.

A quantum field theoretical description of the nucleon is a composite of a \(q\) and nonpointlike colour-antitriplet \(q\)-diquark was proposed and explored in Refs. [15, 16], and recent studies \([17]\) have shown that this approach, based on a Poincaré covariant Faddeev equation, is capable of providing a good description of the spectrum of octet and decuplet baryons. With this foundation, a product \(\text{Ansatz}\) for the nucleon’s Faddeev
amplitude has been employed [18, 19, 20] to describe a wide range of elastic nucleon form factors. This is the model we use to explore the implications of Refs. [26, 27] for the neutron’s EDM.

In Sec. II we describe our model in detail, including a discussion of the form of a CP and T violating coupling of a photon to a dressed-quark. An analogous dressed-quark-gluon coupling is also admissible [8] and may yield an equally important contribution to $d_n$ [13]. We neglect it and hence ours is not a complete calculation of $d_n$. Nevertheless, this and like terms are additive, and their omission does not qualitatively affect our discussion nor the points we wish to emphasise. Our results are presented in Sec. III, and we report more than just the EDM so as to establish a context for that result and demonstrate the level of accuracy to be anticipated using our model. Section IV is an epilogue.

II. MODEL ELEMENTS

A. Dressed-quarks

The general form of the dressed-quark propagator is [28]

$$ S(p) = -i \gamma \cdot p \sigma(p^2) + \sigma(p^2), \quad (8) $$

$$ = \frac{[i \gamma \cdot p A(p^2) + B(p^2)]^{-1}}{A(p^2)}, \quad (9) $$

and it is a model-independent result of quark-DSE studies that the wave function renormalisation and dressed-quark mass:

$$ Z(p^2) = 1/A(p^2), \quad M(p^2) = B(p^2)/A(p^2), \quad (10) $$

respectively, exhibit significant momentum dependence for $p^2 \leq 1$ GeV$^2$ which is nonperturbative in origin. (See, e.g., Ref. [29].) This behaviour is a longtime prediction of DSE studies [21] and has recently been observed in lattice-QCD simulations [23]. The infrared enhancement of $M(p^2)$ is an essential consequence of dynamical chiral symmetry breaking (DCSB) and is the origin of the constituent-quark mass. With increasing $p^2$ the mass function evolves to reproduce the asymptotic behaviour familiar from perturbative analyses, and that behaviour is unambiguously evident for $p^2 \geq 10$ GeV$^2$ [22].

While numerical solutions of the quark DSE are readily obtained, the utility of an algebraic form for $S(p)$ when calculations require the evaluation of numerous multidimensional integrals is self-evident. With this in mind an efficacious parametrisation of the dressed-quark propagator which exhibits the essential features described above was introduced in Ref. [30] and has been used extensively in studies of meson properties; e.g., Refs. [30, 31, 32, 33, 34, 35, 36]. We use it herein.

The parametrisation is expressed via

$$ \tilde{\sigma}_S(x) = 2 \tilde{m} \mathcal{F}(2(x + \tilde{m}^2)), \quad (11) $$

$$ \tilde{\sigma}_V(x) = \frac{1}{x + \tilde{m}^2} \left[ 1 - \mathcal{F}(2(x + \tilde{m}^2)) \right], \quad (12) $$

with $x = p^2/\lambda^2$, $\tilde{m} = m/\lambda$, $\mathcal{F}(x) = \int [1 - \exp(-x)]/x$, $\tilde{\sigma}_S(x) = \lambda \sigma_S(p^2)$ and $\tilde{\sigma}_V(x) = \lambda^2 \sigma_V(p^2)$. The mass scale, $\lambda = 0.566$ GeV, and parameter values

$$ \begin{align*}
\tilde{m} & = 0.00897, \\
b_0 & = 1.31, \\
b_1 & = 2.90, \\
b_2 & = 0.603, \\
b_3 & = 0.185,
\end{align*} \quad (13) $$

were fixed in a least-squares fit to light-meson observables [31]. The dimensionless $u = \tilde{m}$ current-quark mass in Eq. (13) corresponds to $m = 5.1$ MeV.

$\epsilon = 10^{-4}$ in Eq. (11) acts only to decouple the large and intermediate-$p^2$ domains.)

Dynamical chiral symmetry breaking is expressed in the parametrisation. It gives a Euclidean constituent-quark mass

$$ M_{u,d} = 0.33 \text{ GeV}, \quad (15) $$

defined [22] by the solution of $p^2 = M^2(p^2)$, whose magnitude is typical of that employed in constituent-quark models [37] and for which the value of the ratio $M_{u,d}/m = 65$, is characteristic and definitive of light-quarks [34]. In addition, DCSB is manifest through a nonzero vacuum quark condensate

$$ \langle \bar{q}q \rangle_0 = \frac{3 \lambda^3}{4\pi^2} \frac{b_0}{b_1 b_3} \frac{1}{\Lambda_{\text{QCD}}} \ln \left( \frac{1 \text{ GeV}^2}{221 \text{ GeV}^3} \right) = (0.221 \text{ GeV})^3, \quad (16) $$

where we have used $\Lambda_{\text{QCD}} = 0.2$ GeV. The condensate is calculated directly from its gauge invariant definition [38] after making allowance for the fact that Eqs. (11), (12) yield a chiral-limit quark mass function with anomalous dimension $\gamma_m = 1$. This omission of the additional $\ln(p^2/\Lambda_{\text{QCD}})$-suppression that is characteristic of QCD is a practical but not necessary simplification.

Motivated by model DSE studies [39], Eqs. (11), (12) express the dressed-quark propagator as an entire function. Hence $S(p)$ does not have a Lehmann representation, which is a sufficient condition for confinement [40]. Employing an entire function for $S(p)$, whose form is only constrained by the calculation of spacelike observables, can lead to model artefacts when it is employed directly to calculate observables involving large timelike momenta [41]. An improved parametrisation is therefore being sought. Nevertheless, on the subdomain of the complex plane explored in the present calculation the integral support provided by an equally efficacious alternative cannot differ significantly from that of this parametrisation, which explains why we use it herein.
B. Nucleon

The idea that diquark correlations play a significant role in baryon structure and interactions is almost as old as that of quarks themselves [42]. It was a motivation for the meson-diquark bosonisation enunciated in Ref. [43], which provides a picture of baryons as dressed-quark–diquark composites, and the subsequent derivation [15] of an homogeneous, Poincaré covariant Faddeev equation for baryons that exploits the role of diquark correlations.

Our picture of the nucleon is based on the latter approach. We represent the nucleon as a relativistic three-quark bound state, involving a nonpointlike, Lorentz-scalar diquark correlation, via a product. Ansatz: for the Faddeev amplitude:

\[
\Psi_{\alpha}(p; \alpha_i; \tau_l) = \epsilon_{\alpha_1 \alpha_2 \alpha_3} \Delta^{0+}(K) \\
\times \left[ \Gamma_{\alpha}(K) \right] \psi(\ell; P) u(P) \right]_{\alpha_3},
\]

where \((\epsilon \cdot P + M) u(P) = 0, \) \(P = p_1 + p_2 + p_3 = \) \(p_{(123)} \) the nucleon's total momentum and \(M \) its mass; \(\epsilon_{\alpha_1 \alpha_2 \alpha_3} \) is the Levi-Civita symbol that provides the colour-singlet factor; \(K = p_1 + p_2 = p_{(12)} \); \(p_{(123)} \) \(\equiv p_1 - p_2, \) \(\ell = (p_{(123)} + p_{(23)})/3; \) and \((\alpha_i; \tau_l)\) are the quark spinor and isospin labels.

Equation (17) describes the general form of the amplitude in the scalar diquark subspace. In this equation \(\Delta^{0+}(K)\) is the collective propagator for a scalar diquark formed from quarks 1 and 2, and \(\Gamma_{\alpha} + \) is a Bether-Salpeter-like amplitude describing their relative momentum correlation. As explained, e.g., in Ref. [44], these functions can be obtained from an analysis of the quark-diquark scattering matrix. However, following Refs. [18, 19, 20], for simplicity herein we employ parametrisations:

\[
\Delta^{0+}(K^2) = \frac{1}{m_{0+}^2} \mathcal{F}(K^2/\omega_{0+}^2),
\]

\[
\Gamma_{\alpha}(K; K) = \frac{1}{N_{\alpha+}} C_{\alpha \beta} \Omega_{\alpha+} \mathcal{F}(k^2/\omega_{0+}^2),
\]

where \(C = g_2 \gamma_5\) is the charge conjugation matrix and \(N_{\alpha+}\) is a calculated, canonical normalisation constant that ensures, e.g., that a \((ud)\)-diquark has electric charge fraction \((1/3)\) for \(K^2 = -m_{0+}^2.\) The parameters are a width, \(\omega_{0+},\) and a pseudoscalar mass, \(m_{0+},\) which have ready physical interpretations: the length \(r_{0+} = 1/\omega_{0+}\) is a measure of the mean separation between the quarks in the scalar diquark; and the distance \(l_{0+} = 1/m_{0+}\) represents the range over which a true diquark correlation in this channel can persist inside a baryon. (NB: The absence of a particle-like singularity in the pseudoscalar propagator presented in Eq. (18) is sufficient to ensure that the diquark is confined inside the baryon [40].)

The remaining element in Eq. (17), \(\psi,\) is a Bethe-Salpeter-like amplitude that describes the relative momentum correlation between the dormant quark and the diquark's centre-of-momentum. Using Eqs. (18), (19), \(\psi\) can be obtained by solving a Poincaré covariant Faddeev equation for the nucleon [16]:

\[
\psi(k; P) u(P) = -2 \int \frac{d^4 \ell}{(2 \pi)^4} \Delta^{0+}(K_\ell) \Gamma_{\alpha+}(k + \ell/2; K) \times S(\epsilon_{\ell}) \tilde{F}_{\alpha+}(\ell + k/2; -K_\ell) \times S(\epsilon_\ell) \psi(P) u(P),
\]

where \(K_\ell = -\ell + (2/3) P, \epsilon_{\ell} = -\epsilon + k + P/3, \ell_+ = \epsilon + P/3.\) For a positive energy nucleon, the solution has the general form

\[
\psi(\ell; P) = f_1(\ell; P) \left[ 1 - \frac{1}{M} (\gamma \cdot \ell - \ell \cdot P) \right] f_2(\ell; P),
\]

where \(P^0 = -1\) and, in the nucleon rest frame, \(f_{1,2}\) describe, respectively, the upper, lower component of the dressed-nucleon spinor.

To learn about \(\psi\) we solved Eq. (20) and also its extension to include an axial-vector diquark correlation, a sound foundation for the latter step having been laid in Ref. [17]. We used the dressed-quark propagator described in Sec. III A and, to further simplify and so expedite the calculations, we retained only the first Chew-Low moment of the functions \(f_{1,2}\) in Eq. (21); i.e., we assumed \(f_{1,2}(\ell; P) = f(\ell; P^2; P^2).\) In this way we found [20] that a simultaneous description of the nucleon and \(\Delta\) masses is possible when the axial-vector diquark correlations are included. That description requires (in GeV)

\[
\begin{array}{cccc}
\omega_{1+} & m_{1+} & \omega_{1+} & m_{1+} \\
0.42 & 0.64 & 1.09 & 0.86 \\
& 0.94 & 1.23
\end{array}
\]

where \(\omega_{1+}, m_{1+}\) are obvious analogues in the respective axial-vector channel of the scalar diquark parameters in Eqs. (18), (19), and corresponds to (in fm) \(r_{1+} = 0.47, r_{1+} = 0.23, I_{1+} = 0.31, I_{1+} = 0.18.\) (The last two columns in Eq. (22) are the calculated \(\Delta\) masses. A description of the \(\Delta,\) for which \(J = 3/2\) is obviously impossible with only a scalar diquark.) In our calculation the value of \(m_{1+} \) was taken from Refs. [18, 19], and that of the ratio \(m_{1+}/m_{1+} = 0.78,\) as by the Bethe-Salpeter equation studies of Ref. [45], which is consistent with that obtained in lattice-QCD simulations [46]. The parameters \(\omega_{1+}, i_{1+}\) were then varied to fit \(M_{\Delta}, D_{\Delta}.\) The observations: \(m_{1+}/m_{1+} = 0.78 \approx 0.76 = M_{\Delta}/D_{\Delta}; M_{u,d}^{F} + m_{1+} \approx 0.97;\) and \(M_{u,d}^{F} + m_{1+} \approx 1.19.\) Somehow, however. Our own and other [17, 47] Faddeev equation studies show that the nucleon contains a significant axial-vector diquark component (neglecting it, the nucleon mass is \(\sim 40\%\) too large [20]), and hence the origin of the \(N\) and \(\Delta\) masses and mass splitting does not live simply in summing over constituent masses.

For the purpose of developing an intuitive understanding, our eigenvector solution for the nucleon can adequately be approximated as [20]

\[
\psi(\ell; P) = \frac{1}{N_{\psi}} \mathcal{F}(k^2/\omega_{0+}^2) \left[ 1 - \frac{1}{M} (\gamma \cdot \ell - \ell \cdot P) \right] f_2(\ell; P),
\]
with calculated values of \( \omega_0 \approx 0.4 \text{ GeV} \) \( (r_0 = 0.49 \text{ fm}) \) and \( r \approx 0.5 \). This makes clear that \( r_0 \approx r_{0+} \approx r_{1+} \), which is a necessary condition for the internal consistency of the quark–diquark Faddeev equation description since it signifies that the mean separation between the quarks in the constituent diquarks is no more than the size of the nucleon. Furthermore, the value of it indicates that the lower component is a significant piece of a relativistic nucleon’s spinor. (\( \omega_0 \) is a calculated normalisation constant that ensures, e.g., that the proton has unit charge.)

Following this study we can complete the specification of a well-informed product Ansatz. Equation (17), with Eqs. (18), (19), (23), provides a two-parameter model: we fix the values of \( r \approx 0.5 \) and \( r_{0+} \approx 0.64 \text{ GeV} \), motivated by the Faddeev equation studies, and allow \( \omega_0 \) and \( \omega_\psi \) to vary so as to obtain a least-squares fit to the proton’s electric form factor, as described in Refs. [18, 19, 20]. With this expedient we have an algebraic scalar diquark model that provides accurate estimates of known observables, as we show in Sec. III, and hence can easily be used to obtain realistic constraints on the neutron’s EDM that reflect the influence of those aspects of strong QCD that we identified in Sec. I: DCSB, quark confinement, and hadron compositeness.

C. Quark-Photon Coupling

A calculation of the electromagnetic interaction of a composite particle cannot proceed without an understanding of the coupling between the photon and the bound state’s constituents. This is illustrated with particular emphasis in Refs. [30, 33, 42], which consider effects associated with the Abelian anomaly. When quarks are dressed as described in Sec. II A, only a dressed-quark-photon vertex, \( \Gamma_\mu \), can satisfy the vector Ward-Takahashi identity:

\[
q_\mu \Gamma_\mu (\ell_1, \ell_2) = S^{-1}(\ell_1) - S^{-1}(\ell_2),
\]

where \( q = \ell_1 - \ell_2 \) is the photon momentum flowing into the vertex. The constraints that this identity and other features of a renormalisable quantum field theory place on the form of \( \Gamma_\mu \) have been explored extensively in Refs. [49].

\( \Gamma_\mu \) is the solution of an inhomogeneous BSE, and the pointwise behaviour of the solution has been elucidated in the numerical studies of Refs. [30]. However, for our purposes we again prefer an efficacious algebraic parametrisation and that is provided by [51]

\[
\begin{align*}
\Gamma_\mu (\ell_1, \ell_2) &= i \Sigma_A (\ell_1, \ell_2) \gamma_\mu (\ell_1 + \ell_2) \\
&\times \left[ \frac{1}{2} \gamma_5 (\ell_1 + \ell_2) \Delta_A (\ell_1, \ell_2) + \Delta_B (\ell_1, \ell_2) \right],
\end{align*}
\]

\[
\begin{align*}
\Sigma_F (\ell_1, \ell_2) &= \frac{1}{2} \left[ F (\ell_1^2) + F (\ell_2^2) \right],
\end{align*}
\]

\[
\Delta_F (\ell_1, \ell_2) = \frac{F (\ell_1^2) - F (\ell_2^2)}{\ell_1^2 - \ell_2^2},
\]

where \( F = A, B \), i.e., the scalar functions in Eq. (9). A feature of Eq. (25) is that \( \Gamma_\mu \) is completely determined by the dressed-quark propagator. Furthermore, when using the dressed-quark propagator, renormalisation, improvements on the vertex Ansatz can only modify our results by \( \lesssim 10\% \), as illustrated, e.g., in Refs. [36, 52].

Equations (25)-(27) and Refs. [49, 50, 51] consider only the CP preserving part of the dressed-\( \gamma_5 \)-vertex. When the possibility of CP and T violation is admitted, additional contributions are possible and the form that has most often been considered is that of Eq. (6):

\[
\Gamma_\mu (\ell_1, \ell_2) = \frac{e}{2 m} Q H^- (q^2) \gamma_5 \sigma_{\mu\nu} q_\nu,
\]

where \( e \) is the positron charge, \( m \) is the \( u = d \) current-quark mass, \( Q = \text{diag}[2/3, -1/3] \) is the quark charge fraction isospin matrix; and \( H^- = \text{diag}[h^-_u (q^2), h^-_d (q^2)] \) defines an analogous quark EDM matrix, wherein we acknowledge that the quarks’ EDMs may be momentum-dependent. Plainly, adding this term preserves Eq. (24).

As discussed in Ref. [27], another contribution is possible:

\[
\begin{align*}
\Gamma_\mu^{\pm} (\ell_1, \ell_2) &= \frac{e}{2 m} Q H^\pm (q^2) \gamma_5 \\
&\times \left[ \bar{u}(\ell_1) \gamma_5 (\ell_1 + \ell_2) \gamma_\mu u(\ell_2) \right],
\end{align*}
\]

where \( q^2 = 1 \); \( H^\pm (q^2) \) is an obvious analogue of \( H^- (q^2) \); and adding this term also preserves Eq. (24). Note though that in any calculation where the quarks are assumed to be on-shell, and hence described by spinors for which \( (i \gamma_5 \cdot \ell + m) u(\ell) = 0 \), then

\[
\bar{u}(\ell_1) \gamma_5 (\ell_1 + \ell_2) \gamma_\mu u(\ell_2) = \bar{u}(\ell_1) \gamma_5 \sigma_{\mu\nu} q_\nu u(\ell_2),
\]

and consequently the two structures are equivalent. However, Eq. (30) is not satisfied by the dressed-quarks in the bound state nucleon: they are confined and may not even have a mass shell, so that in the general case the two contributions are distinguishable and must be treated separately. As an example, for the \( \rho \)-meson [44] different operator structures generate individual contributions that differ by \( \lesssim 20\% \) from their average value [27].

In our calculations then we employ the following algebraic Ansatz for the dressed-quark-photon vertex

\[
\begin{align*}
\Gamma_Q (\ell_1, \ell_2) &= Q \Gamma_\mu (\ell_1, \ell_2) + \Gamma_\mu^{\pm} (\ell_1, \ell_2) + \Gamma_\mu (\ell_1, \ell_2).
\end{align*}
\]

III. RESULTS

The electromagnetic nucleon current is

\[
\begin{align*}
J_{\mu} (P', P) &= i e \bar{u}(P') A_{\mu} (q, P) u(P), \\
&= i e \bar{u}(P') \left( \gamma_\mu F_1 (q^2) + \frac{1}{2 M} \sigma_{\mu\nu} q_\nu F_2 (q^2) \right) u(P),
\end{align*}
\]

\[
+ \frac{1}{2 M} \gamma_5 \sigma_{\mu\nu} q_\nu H (q^2) u(P),
\]

where \( A_{\mu} = A_{\mu} (q, P) \) is the current quark’s quark propagator. The parameter \( A_{\mu} \) is obtained by matching to the current quark propagator and to the nucleon electromagnetic form factors. The parameter \( A_{\mu} \) is a function of \( q^2 = (P'^2 - P^2) \) and is determined by the fit to the nucleon electromagnetic form factors.

The parameter \( H (q^2) \) is obtained by matching to the current quark propagator and to the nucleon electromagnetic form factors. The parameter \( H (q^2) \) is a function of \( q^2 = (P'^2 - P^2) \) and is determined by the fit to the nucleon electromagnetic form factors.

The parameter \( F_{1,2} (q^2) \) is obtained by matching to the current quark propagator and to the nucleon electromagnetic form factors. The parameter \( F_{1,2} (q^2) \) is a function of \( q^2 = (P'^2 - P^2) \) and is determined by the fit to the nucleon electromagnetic form factors.
TABLE I: Calculated values of a range of well-known physical observables. The "Obs." column reports experimental values [33] or values employed in a typical meson-exchange model [54]. The remaining columns report our results, obtained using the Faddeev Ansatz parameters in Eq. (36).

<table>
<thead>
<tr>
<th></th>
<th>Obs.</th>
<th>Set 1</th>
<th>Set 2</th>
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<tbody>
<tr>
<td></td>
<td>$\tau_+^2 (\text{fm}^2)$</td>
<td>$\mu_+ (\mu_N)$</td>
<td>$\mu_+ (\mu_N)$</td>
</tr>
<tr>
<td></td>
<td>$0.87^{+0.04}_{-0.05}$</td>
<td>$-0.34^{+0.05}_{-0.06}$</td>
<td>$1.93^{+0.10}_{-0.11}$</td>
</tr>
<tr>
<td></td>
<td>$(\text{fm}^2)$</td>
<td>$\rho_+ (\mu_0)$</td>
<td>$\rho_+ (\mu_0)$</td>
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<tr>
<td></td>
<td>$2.79^{+0.08}_{-0.09}$</td>
<td>$2.83^{+0.10}_{-0.11}$</td>
<td>$1.61^{+0.12}_{-0.13}$</td>
</tr>
<tr>
<td></td>
<td>$(\text{fm}^2)$</td>
<td>$\langle \xi^2 \rangle (\text{fm}^2)$</td>
<td>$g_A$</td>
</tr>
<tr>
<td></td>
<td>$0.03^{+0.01}_{-0.02}$</td>
<td>$1.36$</td>
<td>$0.98$</td>
</tr>
<tr>
<td></td>
<td>$(\text{fm}^2)$</td>
<td>$f_{\text{sNN}}$</td>
<td>$f_{\text{sNN}}$</td>
</tr>
<tr>
<td></td>
<td>$13.0$</td>
<td>$15.8$</td>
<td>$16.7$</td>
</tr>
<tr>
<td></td>
<td>$(\text{MeV})$</td>
<td>$m_{\text{NN}}$</td>
<td>$m_{\text{NN}}$</td>
</tr>
<tr>
<td></td>
<td>$7.10^{+0.12}_{-0.13}$</td>
<td>$12.2$</td>
<td>$15.5$</td>
</tr>
</tbody>
</table>

where the spinors satisfy $\gamma \cdot P u(P) = iM u(P)$, $\bar{u}(P) \gamma \cdot P = iM \bar{u}(P)$, with $M = 0.94$ GeV; $R = P^\dagger + P$ and $q \cdot P = 0$; and $\Lambda_\gamma$ is the nucleon-photon vertex. $F_1$ and $F_2$ are the usual Dirac and Pauli electromagnetic form factors of the nucleon, in terms of which the electric and magnetic form factors are

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2} F_2(q^2),$$

$$G_M(q^2) = F_1(q^2) + \frac{q^2}{4M^2} F_2(q^2).$$

The remaining term yields $F_3$ and hence the nucleon’s EDM form factor, which vanishes in the absence of a CP and T violating quark-photon coupling. Equation (33) is the general form because Eq. (30) is valid for an on-shell nucleon.

With the specification of the elements in Sec. II, an impulse approximation calculation of the electromagnetic properties of the nucleon is straightforward using the expression for $\Lambda_\gamma$ given in the appendix. Each calculation requires the evaluation of a number of multidimensional integrals, which we accomplish using Monte-Carlo methods, requiring a statistical accuracy of $\lesssim 1 \%$.

Our results for an illustrative range of well-known quantities are presented in Table I, which serves to exemplify the accuracy of our scalar diquark model of the nucleon. To display the sturdiness of the model we used two parameter sets (dimensioned quantities in GeV)

$$m_{\text{NN}} = \begin{cases} 0.562 & 0.562 \\ 1.063 & 0.50 \end{cases}$$

with, in each case, the values of $\omega_{\text{NN}}$ and $\omega_\psi$ determined via a least-squares fit to the proton’s electric form factor. Set 1 is "preferred" because the value of $R$ is fixed to that obtained in the Faddeev equation study described in Sec. IID. (The minor adjustment of $m_{\text{NN}}$ is necessary to preserve neutrality of the neutron subject to the variation of $\omega_{\text{NN}}, \omega_\psi$, see next paragraph.) In comparison with the form factors depicted in Refs. 18, 19, both of which used $R = 0$ and hence overlooked an important qualitative outcome of Faddeev equation studies, our calculated $G_E(q^2)$ is much improved cf. the data, as seen, e.g., in Ref. 20. The other form factors are little affected. (NB. Only the electromagnetic properties reported in Table I can be calculated directly from the information provided herein. The remaining quantities were calculated as described in Ref. 19: $\pi N N' - \pi N N$ using Eq. (22) of that reference; vector meson - using Eq. (38); and axial form factor - using Eq. (60).)

We remark that there is a quantitative discrepancy between the values of $\omega_{\text{NN}}, \omega_\psi$, obtained here and those obtained in the Faddeev equation study, Eqs. (22), (23). That is to be anticipated because here we are requiring a fit to wide range observables in a circumscribed model space; i.e., the discrepancy is an artefact of our scalar-diquark product Ansatz. However, our model’s simplicity and illustrative utility outweigh the defect because in this and its earlier applications the qualitative implications of the defect are readily identifiable.

### A. Electric dipole moment

Having established a context, we now report our results for the EDM. Using Eq. (31) one finds

$$\mathcal{H} = \frac{M}{m} \text{diag} \left( \sum_{f=t,u,d} u^*_f(p) e_f h^*_f, \sum_{f=t,u,d} u^*_f(n) e_f h^*_f \right),$$

an isospin matrix, where the calculated values of $u^*_f(N)$ express the dependence of the nucleon’s EDM on its internal structure. Clearly, because of isospin symmetry,

$$u^*_f(p) = u^*_f(n), \quad u^*_f(p) = u^*_f(n).$$

The $M/m$ multiplicative factor in Eq. (37) makes explicit the importance of the DCSB mechanism in amplifying the contribution from dressed-quarks to the nucleon’s EDM.

Existing experiments constrain $\mathcal{H}(q^2 = 0)$ and in Table II we report the relevant values of $w_f^*(n)$. (There are no cancellations between contributions from different $\Lambda_j$, Eqs. (A8) (A7). The values are comparable in magnitude to their analogues obtained using QCD Sum Rules, e.g., Ref. 15.) The table makes clear the extent to which Eq. (30) (the on-shell assumption) is violated by the dressed-quarks in the nucleon bound state: for the doubly represented quark flavour the difference is quantitatively similar to that in the $\rho$-meson [24]. The difference is much larger for the odd flavour quark. However, that is likely an artefact of only retaining a scalar diquark, in which case $\Lambda_j^Q$ in Eq. (A6) provides a sole.
TABLE II: Calculated values of the coefficients in Eq. (37). (All quantities are dimensionless.) In the absence of dressed-quark confinement and off-shell effects, the entries in the two rightmost columns would be zero. The parameter values for Sets 1 and 2 are given in Eq. (36).

\[
\begin{array}{cccccc}
  w_q^+(n) & w_u^+(n) & w_d^-(n) & w_s^-(n) & \frac{w_q^+-w_u^+}{w_d^-+w_s^-} & \frac{w_q^-+w_u^-}{w_d^-+w_s^-} \\
  \text{Set 1} & 0.560 & 0.555 & 0.796 & -0.111 & 0.15 & 2.9 \\
  \text{Set 2} & 0.647 & 0.043 & 0.878 & -0.102 & 0.15 & 2.5 \\
\end{array}
\]

unmatched contribution to \( h^+ \). On the information available we therefore judge that the magnitude of this effect is best estimated from the difference for the doubly represented flavour.

Using Eqs. (13), (37) and the Set 1 results from Table II we obtain

\[
\begin{align*}
  h_n & = 185.2 \frac{1}{3} \left( -0.560 h_d^+ + 0.110 h_u^+ 
  \right. \\
  & \left. \quad - 0.756 h_d^- - 0.222 h_u^- \right) , \\
  h_F & = 185.2 \frac{1}{3} \left( -0.555 h_d^+ + 0.111 h_u^+ 
  \right. \\
  & \left. \quad + 1.12 h_d^- + 1.51 h_u^- \right) . \tag{39}
\end{align*}
\]

The ratio of the coefficients of \( h_d^- \) and \( h_u^- \) in Eq. (39) is 3.4, while the value of this ratio obtained from Eq. (4) is 2.0. The difference between these two values goes to the extent to which \( SU(6) \) spin-flavour symmetry is broken by quark + quark = diquark clustering in the nucleon bound state. This difference may change a little but will not vanish upon the inclusion of axial-vector diquark correlations in the nucleon's Faddeev amplitude. (The limited effect that axial-vector diquarks have on other observables that do not involve cancellations between the \( A^l \) contributions [20, 47] makes us confident of this.) The \( h^+ \) contributions do not have an analogue perturbatively nor in the constituent quark model.

We observe that hadronic loop insertions are unambiguously at leading order to our impulse approximation and, based on analyses of other observables [20, 32, 36, 52, 55], we expect that for realistic hadron masses their contribution will modify Eqs (39), (40) by \( \lesssim 10 \% \). This suppression is due to the compositeness of the hadrons: away from the position of well-known kinematic singularities, the need to extrapolate bound state properties off-shell [56], for the evaluation of integrals over spacelike momenta; and the integration cutoffs applied by the hadrons' finite size, both lead to a quenching of the loop contributions.

If we assume that CP violating current-quark-level interactions yield \( h_d \gg h_u \), as is the case, e.g., in Higgs boson exchange models [27], then

\[
\hat{h}_n = -40.6 \left( 0.85 h_d^+ + 1.15 h_u^+ \right) \tag{41}
\]

and Eq. (1) applies the following bound on the di-current-quark gyroelectric ratios:

\[
\left| 0.85 h_d^+ + 1.15 h_u^+ \right| < 14.7 \times 10^{-14} . \tag{42}
\]

Using Eq. (40) it is clear that, for \( h_d \gg h_u, d_F \approx 0.1 d_n. \)

The DCSB mechanism, which is responsible for turning the current-quark mass into the constituent-quark mass, as reviewed in Sec. II A, is responsible for the suppression of \( h_d \) with respect to \( h_u \) by the factor of \( \lesssim 40 \) in Eq. (41). This value is a lower bound on the magnitude of the effect. It will increase when using any reasonable \( A^l \) that incorporates strong interaction dressing in \( \Gamma^l \) analogous to that described by \( \Lambda^l \). \( \Lambda^l \) in the CP and T preserving part of the vertex. We can estimate the scale of this effect by comparing \( \omega_q^-(p) = \omega_q^-(n) \) and \( \omega_q^+(p) = \omega_q^+(n) \) with lattice estimates [57] of the proton's tensor charges: \( \delta u = 0.839(60), \delta d = -0.180(10) \). It is thus apparent that Eq. (42) overestimates the upper bound by less than a factor of two.

The result in Eq. (5) corresponds to \( |h_d^+ + h_u^+| \leq 10^{-23} \), so Eq. (42) does not challenge the standard model. However, with [2] \( \text{Re}(e'/e) = (2.1 \pm 0.46) \times 10^{-3} \), the calculation in Sec. VI of Ref. [58] corresponds to a Weinberg model [12] prediction of [59]

\[
|h_d^+ + h_u^+| = (0.3 \sim 9.0) \times 10^{-14} . \tag{43}
\]

Hence this model is threatened by our estimate of the bound, Eq. (42), which incorporates a well-constrained modelling of the effects of DCSB, and the binding and confinement of dressed-quarks in the neutron.

We emphasise that the results in Table II are independent of the means used to calculate \( h^+_q \). Hence Eq. (39) can be applied to constrain any extension of the standard model. Here we have only exemplified that using a particular candidate.

IV. EPILOGUE

The results are clear. Quantitatively - Our value for \( d_n \) will be modified by the well-constrained inclusion of axial-vector diquark correlations. However, based on the effect this has on other observables, the correction should not exceed 15 %. Qualitatively - 1) The scale of dynamical chiral symmetry breaking in QCD generates a significant amplification of the contribution from a current-quark's EDM to that of the bound state containing it: the minimal enhancement factor is roughly the ratio of the constituent-quark and current-quark masses, and is understandable and calculable through the necessary momentum-dependence of the dressed-quark mass function. 2) Confinement and compositeness entail that the dressed-quarks comprising the bound state are not on shell and hence the two CP and T violating operator structures that are indistinguishable in the free-quark limit yield materially different contributions to the EDM of the bound state. As a consequence these operator structures must be analysed and their strengths determined independently in any model that provides for CP and T violation. Both these effects should be accounted for in using \( d_n \) as a means of constraining extensions of the standard CKM model. These conclusions bear
equally on the effects of operator structures we have neglected.

APPENDIX A: IMPULSE APPROXIMATION

As made clear in Sec. 2.3 of Ref. [24], when using an antisymmetrized product Ansatz for the nucleon’s Faddeev amplitude the impulse approximation is

\[ \Lambda_\mu(q, P) = \Lambda_\mu^A(q, P) + 2 \sum_{i=1}^{\infty} \Lambda_\mu^i(q, P), \]  

(\text{A1})

where:

\[ \Lambda_\mu^j(q, P) = \frac{1}{2} \int_{0}^{2\pi} \frac{d^2 k}{2\pi} \psi(\ell - \frac{2}{3} P) \Delta_\mu^j(K) \times \psi(\ell P) \Lambda_\mu^j(p_3 + q, p_3), \]  

(\text{A2})

with \( K = \ell + \frac{2}{3} P \), \( p_3 = -i \frac{\sqrt{3}}{3} P - \ell \), \( \Lambda_\mu^j(\ell_1, \ell_2) = S(\ell_1) \Gamma^{\mu}_\ell(\ell_1, \ell_2) S(\ell_2) \) and

\[ \Lambda_\mu^2(q, P) = 6 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4\ell}{(2\pi)^4} \left[ \frac{1}{3} \rho_1(p_1 + p_2 + p_3) \right] \times \psi(\ell P) \Lambda_\mu^2(p_3 + q, p_3), \]  

(\text{A3})

\[ \Lambda_\mu^2 \] describes the photon probing the structure of the scalar diquark correlation, and contributes equally to both the proton and neutron. That contribution is trivially zero for the EDM. This merely reflects the fact that such a moment is forbidden to a scalar particle, so the only nontrivial contribution is that of the diquark’s CP- and T-preserving electromagnetic factor. The remaining terms are

\[ \Lambda_\mu^j(q, P) = \frac{1}{2} \int \frac{d^2 k}{(2\pi)^2} \frac{d^2 \ell}{(2\pi)^2} \left[ \frac{1}{3} \rho_1 p_1 + p_2 + p_3 \right] \times \left[ \frac{1}{3} \rho_1 p_1 + p_2 + p_3 \right] \times S(p_3 + q, p_3), \]

(\text{A5})

\[ \Lambda_\mu^2(q, P) = 6 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4\ell}{(2\pi)^4} \left[ \frac{1}{3} \rho_1 p_1 + p_2 + p_3 \right] \times \Omega(p_1, p_2, p_3) \times \Lambda_\mu^2(p_2 + q, p_2) \times S(p_3 + q, p_3), \]

(\text{A6})

\[ \Lambda_\mu^2(q, P) = 6 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4\ell}{(2\pi)^4} \left[ \frac{1}{3} \rho_1 p_1 + p_2 + p_3 \right] \times \Omega(p_1, p_2, p_3) \times S(p_3 + q, p_3) \times \Lambda_\mu^2(p_2 + q, p_2), \]

(\text{A7})

Of these five terms, the only \( u \)-quark contribution to the neutron EDM comes from \( \Lambda_\mu^2 \) — the other four terms sum the contribution from the \( d \)-quark. (The situation is reversed for the proton.)

Our numerical results are calculated by evaluating these integrals using Monte-Carlo methods and the input specified in Eqs. (11), (12), (18), (19), (23) and (31).

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REFERENCES

[28] In our Euclidean formulation: $p \cdot q = \sum_{\mu=1}^{4} p_{\mu} q_{\mu} = i \int d^{4}x \left\{ \gamma_{\mu} \gamma_{5} \right\} \partial_{\mu} \phi_{\alpha}(x) \rightarrow 2 \delta_{\mu \nu} \gamma_{\mu} \phi_{\alpha}(x) = \gamma_{\nu} \phi_{\alpha}(x)$ and $\frac{\partial}{\partial x} \gamma_{\mu} \gamma_{\nu} = -4 \epsilon_{\mu \nu \alpha \beta} \gamma_{\alpha} \gamma_{\beta}$.
[40] This is a sufficient condition for confinement because of the associated violation of reflection positivity, as discussed, e.g., in Sec. 6.2 of Ref. [21], Sec. 2.2 of Ref. [24] and Sec. 2.4 of Ref. [25].