Abstract

Next-to-leading order expressions related to Stewart-Lyth inverse problem are used to determine the inflationary models with tensorial power spectrum described by a scale-invariant spectral index. Beyond power-law inflation, solutions are characterized by scale-dependent scalar indices. These models can be used as assumption on the generation of primordial perturbations to test for scale dependence of scalar index at large angular scales. If such a dependence is detected, a nonzero contribution of gravitational waves to the CMB spectrum must be expected.
I. INTRODUCTION

With measurements carried out by experiments Boomerang and Maxima-1 [1] observations of cosmic microwave background (CMB) anisotropies entered a phase where different theories for structure formation at our Universe can be falsified. To date, inflation [2,3] is the cosmological model supported by analysis of these observations [4] as favored theory. Almost any model that produces an accelerated expansion of the early Universe solves the well-known set of fundamental problems faced by the Standard Cosmological Model, provided that the inflationary period lasts long enough (see [5] for detailed explanations). The simplest scenario describes the classical and quantum dynamics of the early Universe dominated by a single scalar field (inflaton) evolving in a nearly flat potential.

Analysis like those in Refs. [4] and [6–9] consist of maximizing a likelihood function over the space of model parameters. A given set of parameters yields a theoretical CMB spectrum to be compared with observations. The precise number of parameters depends on the version of the analysis and commonly is as small as 6 and as large as 11. They are classified as inflationary or cosmological parameters depending on whether they determine the initial power spectrum of fluctuations or its posterior evolution. To set the initial conditions what it is commonly assumed is a particular kind of single scalar field model namely, power-law inflation [10], characterized by scale-invariant scalar and tensorial spectral indices differing in unity each from the other. Amplitudes of tensorial perturbations are neglected. The conclusion to be drawn is that, in the corresponding to observed Universe scales, the actual potential has a strong similarity with that of power-law inflation. No conclusions can be made about potential functional form in other scales, particularly at Planck scales.

Present level of measurements accuracy allows a still large number of inflationary models to remain as candidates for inflaton potential. Any potential that behaves like a slowly decaying exponential in the interval of inflaton values corresponding to the probed range of scales is able to generate the original perturbations growing into the features observed in the CMB spectrum. Nevertheless, resolution to be given by satellites Map [11] and Planck [12] and the scale range to be probed by these and other experiments such as the galaxies surveys, may allow a finest selection.

Next generation of observations could also give information on CMB polarization which, in turn, is linked to $r$, the relative contribution of primordial gravitational waves to CMB anisotropies (see Ref. [13] for a review on this subject). When the relative contribution of kinetic energy to total scalar field energy (known as $\epsilon$) may be regarded as constant then, to leading order (LO) in an expansion in terms of $\epsilon$, the scalar and tensorial indices and $r$ are directly related and could be taken as constants too. In this situation, the model that better accomplish the job of fitting the available data is also power-law inflation, with some distortion taking place for the relation between spectral indices [14]. Assumption $\epsilon = \text{const.}$ is not realistic since the equation of the state must change near the end of inflation in order to return to the Standard Model expansion rate. Hence, several authors had pointed out that some scale dependence for the scalar index may be expected [8,15,16]. If that it is the case, then a power-law index would affect the best-fit values of the entire set of cosmological parameters.

In this work we would like to address the question of which is the theoretical inflationary potential generating the primordial fluctuations characterized by a scale-dependent scalar
index that could give the best fit to current as well as to next generation of observations.

Even if it is already possible to include time dependence of $\epsilon$ at lowest order, we shall show here that some interesting cases able to answer the above stated question are only considered if next-to-leading order (NLO) expressions are used. NLO expressions for the spectra have been tested and found to provide a high accuracy for theoretical perturbations calculations [17]. Even more, some authors have stressed that NLO expressions in terms of $\epsilon$ will be compulsory in order to match analytic results with data to be obtained in the near future [18,19].

Among other alternatives for finding the inflaton potential (see Ref. [18], and references therein), the Stewart-Lyth inverse problem (SLIP) was introduced [20] as a procedure consisting of solving a pair of non-linear differential equations derived in Refs. [21,22] from NLO algebraic Stewart-Lyth equations for spectral indices [23], and determining the corresponding inflaton potential. Actually, this alternative seems to not be useful in the near future for determining the inflaton potential from observations. The reason is the lack of information on the spectrum of primordial gravitational waves. Nevertheless, SLIP solutions can be used to test assumptions on the functional form of the tensorial index by calculating the corresponding scalar index as a function of scales and comparing with experiments.

In this paper we propose two models that could serve to fit the inflationary perturbations, detecting, if there exist, scale dependence for the scalar spectral index at large angular scales while increasing the scale range and resolution of CMB observations. These models are obtained in a straightforward manner as SLIP solutions assuming a constant and almost negligible tensorial index.

In next Sec. we briefly describe the theoretical frame for Stewart-Lyth calculations, the main features of power-law model upon which these calculations are based and current observations are fitted, and present Stewart-Lyth algebraic NLO equations for the spectral indices. In Sec. III SLIP is rewritten using $\epsilon$ as the basic parameter. Further, we introduce a criterion that allows to determine when a given SLIP solution is consistent with assumptions underlying calculations. Sec. IV is devoted to the main aspect of this manuscript, i.e., how to use NLO expressions related to SLIP to determine a theoretical potential yielding a spectrum of perturbations able to fit observations likely to be done in the near future. We summarize main results obtained in Sec.V.

II. PERTURBATIONS PRODUCED BY INFLATIONARY MODELS

Many propositions for inflaton potential can be made fulfilling the conditions for successful inflation (see Ref. [5] for a description of some inflationary models). Hence, the last word while choosing the potential must be the agreement between theoretical predictions and measurements. Ultimately, testing this agreement involves the calculation of primordial perturbations spectra. These spectra are used as initial conditions for the evolution of perturbations which can be computed through the transfer functions (using for example the CMBFAST package [9]), solutions being compared with current measurements of CMB anisotropies. The simplest scenario where that comparison can be carried out is that of a single and real scalar field rolling down a potential.
A. The single scalar field scenario

We assume a flat Friedmann-Robertson-Walker universe containing a single scalar field equivalent to a perfect fluid with equations of motions given by

\[ H^2 = \frac{\kappa}{3} \left( \frac{\dot{\phi}^2}{2} + V(\phi) \right), \]  

\[ \ddot{\phi} + 3H\dot{\phi} = -V'(\phi), \]

where \( \phi \) is the inflaton, \( V(\phi) \) the inflationary potential, \( H = \dot{a}/a \) the Hubble parameter, \( a \) the scale factor, dot and prime stand for derivatives with respect to cosmic time and \( \phi \) respectively, \( \kappa = 8\pi/m_{\text{Pl}}^2 \) is the Einstein constant and \( m_{\text{Pl}} \) the Planck mass.

In this framework, the first three slow-roll parameters were respectively defined in Ref. [24] and can be written as [18],

\[ \epsilon(\phi) \equiv 3 \frac{\dot{\phi}^2}{2} \left[ \frac{\dot{\phi}^2}{2} + V(\phi) \right]^{-1} = \frac{2}{\kappa} \left[ \frac{H'}{H} \right]^2, \]  

\[ \eta(\phi) \equiv -\frac{\ddot{\phi}}{H\dot{\phi}} = \frac{2}{\kappa} \frac{H''}{H} = \epsilon - \frac{\epsilon'}{\sqrt{2\kappa\epsilon}}, \]  

\[ \xi(\phi) \equiv \frac{2}{\kappa} \left( \frac{H'H''}{H^2} \right)^{1/2} = \left[ \epsilon \eta - \left( \frac{2}{\kappa} \right)^{1/2} \eta' \right]^{1/2}. \]

Up to a constant, the first slow-roll parameter (3) is a measure of the relative contribution of kinetic energy to total field energy. By definition \( \epsilon \geq 0 \) and, as it is well known and we shall see in details further in this manuscript, it has to be less than unity for inflation to proceed. This last assertion implies the potential being positive defined. We also note that while defining second and third slow-roll parameters it was assumed the potential to be a monotonically decreasing function. We shall return later to the point of assumptions behind the definitions and calculations to be used in this manuscript.

B. Power-Law Model

Few models of inflation allow exact calculation of scalar and tensorial perturbations. One of them is power-law [10], a scenario of inflation where:

\[ a(t) \propto t^p, \]  

\[ H(\phi) \propto \exp \left( -\sqrt{\frac{\kappa}{2p}} \phi \right), \]  

\[ V(\phi) \propto \exp \left( -\sqrt{\frac{2\kappa}{p}} \phi \right), \]

with \( p \) being a positive constant. It follows from (3), (4), and (5) that in this case the slow-roll parameters are constant and equal each other, \( \epsilon = \eta = \xi = 1/p \). Note that condition \( \epsilon < 1 \) implies \( p > 1 \).
For this model, the power spectrum of scalar perturbations is given by [25]

\[ P_R^{1/2}(k) = \frac{2^{\nu - \frac{3}{2}} \Gamma(\nu)\left(1 - \frac{1}{p}\right)^{\nu - \frac{1}{2}} H^2 \left| \frac{\dd H}{\dd t} \right|_{k=aH}}{m_{Pl}^2 \Gamma(\frac{3}{2})}, \]  

(9)

where \( R \) refers to the curvature perturbation,

\[ \nu \equiv \frac{3}{2} + \frac{1}{p - 1}, \]

and \( k \) is the wavenumber corresponding to the scale matching the Hubble radius. The conversion between the inflaton values and wavenumbers while crossing the Hubble radius can be done using expression [18]

\[ \frac{d \ln k}{d \phi} = \frac{\kappa H}{2 H^2} (\epsilon - 1). \]

(10)

Now, through definition of the scalar index

\[ n_S(k) - 1 \equiv \frac{d \ln A_S^2(k)}{d \ln k}, \]

(11)

where \( A_S(k) \equiv 2P_R^{1/2}(k)/5 \), one can see that for power-law inflation

\[ \frac{n_S(k) - 1}{2} = \frac{1}{1 - p} = -\frac{1}{p} \left(1 + \frac{1}{p} + \frac{1}{p^2} + \cdots \right). \]

(12)

For tensorial perturbations produced during power-law inflation [26], it can be shown that

\[ P_g^{1/2}(k) = \frac{4}{\sqrt{p}} P_R^{1/2}(k), \]

(13)

where \( g \) stands for gravitational waves.

Using definition

\[ n_T(k) \equiv \frac{d \ln A_T^2(k)}{d \ln k}, \]

(14)

with \( A_T(k) \equiv P_g^{1/2}(k) \), it is obtained,

\[ n_T = n_S - 1 \leq 0, \]

(15)

where for inequality was used above mentioned condition \( \epsilon < 1 \). Obviously, relation (15) must be valid to any order in expansion of \( 1/(1 - p) \). Since spectral indices are constant, for power-law inflation, definitions (11) and (14) can be rewritten as

\[ A_S^2(k) = A_S^2(k_0) \left(\frac{k}{k_0}\right)^{n_S - 1}, \]

(16)

\[ A_T^2(k) = A_T^2(k_0) \left(\frac{k}{k_0}\right)^{n_T}, \]

(17)
where \( k_0 \) is a pivot scale usually taken to be the scale probed by COBE. From these expressions it is evident that \( n_S - 1 \) and \( n_T \) measure the deviation of the spectra from scale-invariance.

Summarizing, power-law inflation has a number of characteristic features: slow-roll parameters are constant and equal each other, spectral indices are constant, and spectra given by these indices are red tilted in the same magnitude from Harrison-Zeldovich spectra.

Notice that any inflationary potential that behaves like a decaying exponential for a range of \( \phi \) corresponding to scales currently probed can be, in this range, approximated by a power-law. While this will offer good enough information on the inflationary period corresponding to currently observed Universe, it perhaps will hide clues about physics on higher energy scales.

C. Spectral indices for general models

Up to the present, there are not general analytic expressions to calculate power spectra of inflationary models. Stewart and Lyth [23] derived approximated expressions for both spectra regarding as small the deviation of higher slow-roll parameters from \( \epsilon \) (power-law approximation) and also deviation of \( \epsilon \) with respect to zero (slow-roll approximation). These approximations imply the slow-roll parameters to be slowly varying in time functions. In terms of spectral indices, NLO expressions are

\[
1 - n_S(k) \simeq 4\epsilon - 2\eta + 8(C + 1)\epsilon^2 - (10C + 6)\epsilon\eta + 2C\xi^2, \\
n_T(k) \simeq -2\epsilon [1 + (2C + 3)\epsilon - 2(C + 1)\eta],
\]

where notation is that of Ref. [18] and \( C \approx -0.73 \) is a constant. Symbol \( \simeq \) is used to indicate that these equations were obtained using power-law and slow-roll approximations. Hereafter we shall use equal sign in our calculations, but meaning of approximation should be added whenever it applies.

For a giving expression of scale factor, the Hubble parameter and potential are determined, and then substituting definitions (3), (4), and (5) in Stewart-Lyth equations (18) and (19), scale-dependent spectral indices are obtained.

Imposing exact power-law relation \( \epsilon = \eta = \xi = 1/p \), expression (15) is recovered. Further, one can check that for Eq. (19) constrain \( n_T \leq 0 \) remains valid, even for scale-dependent tensorial index. Assume \( n_T > 0 \) then, from Eq. (19), we obtain inequality \( 1 \leq 1 + \epsilon < 2(C + 1)(\eta - \epsilon) \). Evaluating \( C = -0.73 \) yields \( \eta - \epsilon \geq 1.852 \), in contradiction with power-law approximation \( |\eta - \epsilon| < 1 \) used to derive Eq. (19). Thus, in general,

\[
n_T(k) \leq 0.
\]

For the kind of models we are considering here, even if tensorial perturbations are large enough to have a detectable imprint in CMB anisotropies, according with definition (14), their amplitudes will vanish in wave longitudes to be probed by most gravity waves interferometers [5].

A similar analysis for Eq. (18) yields that, although models with \( n_S < 1 \) are favored, not particular constrain exist upon values of \( n_S \), thus allowing models with so called, blue spectra, i.e., \( n_S > 1 \).
III. STEWART-LYTH INVERSE PROBLEM

In Ref. [20] it was shown that denoting $T \equiv \dot{\phi}^2/2$ and using definitions (3), (4), (5), together with Eqs. (1) and (2), in a straightforward manner, is obtained that:

\[ \epsilon = 3 \frac{T}{T + V} = \frac{\kappa T}{H^2}, \quad (21) \]
\[ \eta = \frac{\kappa dT}{dH^2}, \quad (22) \]
\[ \xi^2 = \kappa \epsilon \frac{dT}{dH^2} + 2 \kappa \epsilon H^2 \frac{d^2 T}{d(H^2)^2}. \quad (23) \]

Defining $\tau \equiv \ln H^2$, $\delta(k) \equiv n_T(k)/2$ and $\Delta(k) \equiv [n_s(k) - 1]/2$, and substituting expressions (21), (22), and (23) in Eqs. (18) and (19), the indices equations in terms of the first slow-roll parameter $\epsilon$ and its derivatives with respect to $\tau$ ($\dot{\epsilon} \equiv d\epsilon/d\tau$ and $\ddot{\epsilon} \equiv d^2\epsilon/d\tau^2$) become [21,22]:

\[ 2C \epsilon \dot{\epsilon} - (2C + 3) \epsilon \ddot{\epsilon} - \dot{\epsilon} + \epsilon^2 + \epsilon + \Delta = 0, \quad (24) \]
\[ 2(C + 1) \epsilon \ddot{\epsilon} - \epsilon^2 - \epsilon - \delta = 0. \quad (25) \]

Given expressions for scale-dependent spectral indices, corresponding inflaton potential can be found by solving Eqs. (24) and (25) for $\epsilon$, and using definitions of first slow-roll parameter (21). This procedure is what we called Stewart-Lyth inverse problem [20].

When SLIP was introduced in Ref. [20], the inflaton potential corresponding to a given solution of differential equations (24) and (25), was determined as a parametric function of $\tau$. Solutions of these equations are expressed with $\tau$ as an explicit function of $\epsilon$ and are difficult to convert to expressions for $\epsilon$ as explicit functions of $\tau$. Hence, it seems reasonable to look for a similar procedure, in terms of the first slow-roll parameter. Moreover, as we shall see later, using $\epsilon$ as a parameter allows to analyze the solution restricted to the interval of $\phi$ where inflation is feasible, i.e., $0 \leq \epsilon < 1$.

Expression for the potential as a function of $\epsilon$ remains the same that in Ref. [20] and is obtained from definition (21),

\[ V(\epsilon) = \frac{1}{\kappa} (3 - \epsilon) \exp [\tau(\epsilon)], \quad (26) \]

but here, instead substituting first slow-roll parameter as function of $\tau$, we substitute $\exp[\tau(\epsilon)]$. On the other hand, using Eq. (21) [20],

\[ \phi(\tau) = -\frac{1}{\sqrt{2\kappa}} \int \frac{d\tau}{\sqrt{\epsilon(\tau)}} + \phi_0, \quad (27) \]

where $\phi_0$ is an integration constant. Changing variables,

\[ \phi(\epsilon) = -\frac{1}{\sqrt{2\kappa}} \int \frac{d\epsilon}{\sqrt{\epsilon}} + \phi_0, \quad (28) \]

and substituting $\dot{\epsilon}$ from the first order equation (25),
\[\phi(\epsilon) = -\frac{2(C + 1)}{\sqrt{2\kappa}} \int \frac{\sqrt{\epsilon}d\epsilon}{\epsilon^2 + \epsilon + \delta} + \phi_0.\] (29)

This way, the inflaton potential is given by the parametric function,

\[V(\phi) = \begin{cases} 
\phi(\epsilon), \\
V(\epsilon). \end{cases}\] (30)

Here is very important to recall that for SLIP solution (30) to be unique, \(\epsilon(\tau)\) must be solution of both equations (24) and (25). Need of information on the tensorial modes is also a conclusion stressed by Lidsey et al. [18] in their report about perturbative reconstruction of inflaton potential. With this regards, one can see that solution \(\epsilon = 1/p\) to Eqs. (24) and (25) automatically implies \(\Delta = \delta = 1/(1 - p)\) in full correspondence with relation (15).

Let us prove now that using \(\Delta = C_1\) and \(\delta = C_2\) as SLIP input (with \(C_1\) and \(C_2\) being some constants) just yields power-law solution, i.e., \(C_1 = C_2\). To proceed with, we note that in Eqs. (24) and (25) \(\epsilon\) and its derivatives depend on \(\tau\) while \(\Delta\) and \(\delta\) explicitly depend on \(k\). So, we shall rewrite these equations in terms of \(k\) in the same fashion it was done in Ref. [27]. Using (10) it is obtained,

\[\frac{d \ln k}{d \tau} = \frac{1}{2} \frac{\epsilon - 1}{\epsilon}.\] (31)

After conversion to derivatives in term of \(\ln k\), Eqs. (24) and (25) become [27],

\[\begin{align*}
\frac{C(\epsilon - 1)^2}{2\epsilon} \tilde{\epsilon} + \frac{C(\epsilon - 1)}{2\epsilon^2} \tilde{\epsilon}^2 - \left[(2C + 3)\epsilon + 1\right] \frac{\epsilon - 1}{2\epsilon} \tilde{\epsilon} + \epsilon^2 + \epsilon + \Delta &= 0, \\
(C + 1)(\epsilon - 1)\tilde{\epsilon} - \epsilon^2 - \epsilon - \delta &= 0,
\end{align*}\] (32) (33)

where \(\tilde{\epsilon} \equiv d\epsilon/d\ln k\) and \(\tilde{\epsilon}^2 \equiv d^2\epsilon/d(\ln k)^2\).

Differentiating Eq. (33) with respect to \(\ln k\) we can replace expressions for \(\tilde{\epsilon}\) and \(\tilde{\epsilon}^2\) obtained from this equation into Eq. (32) and the following algebraic expression for \(\epsilon\) is obtained:

\[\epsilon(k)^4 + P\epsilon(k)^3 + Q(\tilde{\delta}, \delta, \Delta)\epsilon(k)^2 + R(\tilde{\delta}, \delta)\epsilon(k) + S(\delta) = 0,\] (34)

where

\[P = C + 2,\]
\[Q(\tilde{\delta}, \delta, \Delta) = -(C + 1) \left[ C\tilde{\delta} - (2C + 3)\delta + 2(C + 1)\Delta - 1 \right],\]
\[R(\tilde{\delta}, \delta) = (C + 1)C\tilde{\delta} + (2C + 1)\delta,\]
\[S(\delta) = C\delta^2.\]

Roots of Eq. (34) can be calculated but they are very complicated expressions not necessary for further analysis. They can be simply written as

\[\epsilon_i = \epsilon_i(\Delta, \delta, \tilde{\delta}),\] (35)

where \(i = 1, \ldots, 4\). Thus, for \(\Delta = C_1\) and \(\delta = C_2\) the solution is \(\epsilon = \text{const.}\) Substituting back this solution into Eqs. (32) and (33) we finally obtain \(\Delta = \delta\). Note again that this result follows from the use of both equations. In principle, to take just one of the spectral indices as constant does not implies the other one to be constant too.
A. Consistency criterion for SLIP solutions

Solving SLIP is not enough to state that solutions have any physical meaning. We recall that our calculations are based on several assumptions regarding the form of potential (behavior of inflaton as function of cosmic time) and range of slow-roll parameters. Hence, for SLIP to yield consistent results, conditions arising from this assumptions should be fulfilled by obtained inflaton and its potential. Let us analyze these conditions in detail.

To derive Eqs. (18) and (19) and SLIP solution (30), no particular assumption was made about initial conditions for $\dot{\phi}$ nor for its expected value thus, solutions of SLIP are not constrained to chaotic or new inflation nor to a particular energy scale. Furthermore, no assumption was neither made about the potential convexity so, in principle, it could be in any of categories related to classification given in Ref. [7] and moreover, the same potential could have features characteristic of different categories. This is a consequence of dealing with NLO expressions, i.e., allowing a larger variation of slow-roll parameters during inflation. Particularly, SLIP solutions can be associated with a hybrid scenario of inflation [28]. Next, we shall analyze possible constrains upon solutions of SLIP. Deriving Eq. (1) with respect to cosmic time and inserting Eq. (2) it is obtained,

$$T = -\frac{1}{k} \dot{H}.$$  

(36)

Now, taking into account that $H' = \dot{H}/\dot{\phi}$, to determine a sign for $H'$ it is necessary to fix the inflaton behavior as a function of cosmic time. In this paper, it was assumed that $\dot{\phi} > 0$, and, correspondingly, $H' < 0$. Further, comparing definitions (3) and (21) for the first slow-roll parameter and eliminating $\epsilon$ yields,

$$T = \frac{2}{k^2} H'^2,$$  

(37)

and after substituting in equation of motion (1) and deriving with respect to $\phi$ we obtain,

$$V' = \frac{2}{k} \left( 3 - \frac{2}{k} \frac{H''}{H} \right) H H',$$  

(38)

or, regarding definition of second slow-roll parameter (4),

$$V' = \frac{2}{k} (3 - \eta) H H'.$$  

(39)

Eqs. (18) and (19) were obtained using power-law and slow-roll approximations, i.e, the absolute value of $\eta$ should be close to the value of $\epsilon$ which, in turn, should be near zero. Therefore, $(3 - \eta) > 0$ and the sign of $V'$ is determined by that of $H'$. Hence, according to the previous assumptions, the potential must be a monotonically decreasing function of the inflaton. Summarizing, any feasible solution of SLIP should fulfill the following conditions:

$$\begin{align*}
\dot{\phi} &> 0, \\
V'(\phi) &< 0.
\end{align*}$$  

(40)

These correspond to conditions for the inflaton to roll down the potential from lower to higher values of the scalar field. At least in the case we shall analyze here, it seems to be
useful to write conditions (40) in an equivalent manner. Note that \(d\tau/dt = 2\dot{H}/H < 0\), then conditions (40) implies

\[
\begin{align*}
\hat{\phi} < 0, \\
\hat{V} > 0,
\end{align*}
\]  
\(\iff\)
\[
\begin{align*}
\hat{\epsilon} \frac{d\hat{\phi}}{d\epsilon} < 0, \\
\hat{\epsilon} \frac{d\hat{V}}{d\epsilon} > 0.
\end{align*}
\]  
(41)

Similar conditions could be derived for the case of an inflaton rolling down from higher to lower values by properly choosing the behavior of the inflaton as a function of cosmic time. For that case conditions (41) will read,

\[
\begin{align*}
\hat{\epsilon} \frac{d\hat{\phi}}{d\epsilon} > 0, \\
\hat{\epsilon} \frac{d\hat{V}}{d\epsilon} > 0
\end{align*}
\]  
(42)

In fact, of all the expressions used here, the sign of \(\dot{\phi}\) only affects Eq. (29), the modification being \(\phi(\epsilon) \to -\phi(\epsilon)\). That changes the SLIP solutions by the mirror equivalent solutions.

On the other hand, inflation is defined as a period where scale factor grows accelerately, i.e., \(\ddot{a} > 0\). This is equivalent to say that \(d(H^{-1}/a)/dt < 0\), definition remarking that, during inflation, the comoving Hubble radius decrease with time. Deriving, using expression (36) and definition (21) we obtain already mentioned upper value for first slow-roll parameter, i.e., \(\epsilon < 1\). By definition \(\epsilon \geq 0\), then criteria (41) and (42) must be tested in the interval \(\epsilon \in [0, 1)\). Looking at graphs of inflaton and its potential as functions of \(\epsilon\), and plots of \(\epsilon(\tau)\) in the corresponding range, one is able to find out whether SLIP solutions will be consistent with underlying assumptions. Note that these conditions are sufficient for given potential to be inflationary but they do not ensure the inflationary epoch to be long enough.

**IV. MODELS WITH CONSTANT TENSORIAL SPECTRAL INDEX**

Standard procedure for determining cosmological parameters [4,6–9] consists of fitting observations of CMB anisotropies using a given cosmological model. The most popular version assumes a Λ-cold dark matter model. The transfer functions, which depend on the cosmological parameters, are used to calculate the evolution of the density perturbations, assuming an inflationary origin for the primordial perturbations. The set containing the parameters that determine the realization of the model together with the parameters that determine the initial conditions is tuned in order to maximize a likelihood function. Commonly, the initial power spectra are set to the form corresponding to power-law inflation with negligible amplitudes for primordial gravitational waves. The assumption behind is that during the inflationary lapse where quantum fluctuations were imprinted in scales currently reentering our causal Universe, slow-roll parameters behave roughly as constants.

An important parameter is the tensor-scalar ratio of contribution to the CMB spectrum which can be defined as (see Ref. [5] for alternative definitions),

\[
r \equiv 12.4 \frac{A_T^2}{A_S^2}.
\]  
(43)

In principle, the value of \(r\) can be estimated from observations of CMB polarization [13]. To NLO, \(r\) is related with the tensorial index by [18]
\[ n_T \simeq -2 \frac{A_T^2}{A_S^2} (1 + 3\epsilon - 2\eta) . \] (44)

From Eqs. (18) and (19), LO expressions are recovered by neglecting second order terms of slow-roll parameters, and from Eq. (44) by setting the expression within parenthesis equal to unity. This way, if in the desired scale range \( \epsilon \) and \( \eta \) can be approximated by constant values, to LO, \( n_S, n_T \) and \( r \) must be regarded as constants too. Hence, if some information on \( r \) is available, power-law inflation can still be the model providing the best fit to data given some distortion of the relation between indices. We remark that a rather large number of parameters are fitted in the standard procedure, then this method is highly sensitive to the priors assumed. Particularly for inflationary parameters, if some degree of scale dependence is hidden in the CMB, power-law is not the best guess to fit the scalar index. The error of assuming power-law will, in turn, be reflected in the best-fit values of remaining parameters. Hence, to consider \( \epsilon \simeq \text{const.} \) is a big restriction regarded as fair if the fit of data to a constant scalar index give a good result. Nevertheless, such a good overall fit can also be achieved for some potentials yielding scale dependent scalar index, for example, running mass models [8]. Moreover, several authors have pointed out that observations with higher resolution and wider scale-range to be provided by satellites Planck and Map, and galaxies surveys should be able to discern a time dependent \( \epsilon \) [17], making of power-law a poor assumption for the inflationary period. Therefore, question arises of which model could provide the best fit to upcoming data. To answer this question, an option is to find out if there exist models with slowly-varying tensorial and scalar spectral indices which, using current data, can be accurately fitted by a power-law and smoothly departs from power-law while making broader the range of scales or increasing the resolution of measurements. Notice that, in the same way that the fact of power-law being the model that better fits current data does not imply it to be the inflaton potential, the theoretical potential yielding the best fit to future observations must be regarded just as a good approximation in probed scales to the actual potential.

In general, the tensorial spectral index should be a scale-dependent function slowly varying close to zero. As it was already mentioned it is a magnitude faraway from direct measurement capability of present-day and, maybe, near-future instruments. This is a major handicap for both, comparison with observations of spectra calculated through Eqs. (18) and (19) and also for resolution of SLIP. Nevertheless, one can make assumptions on the functional form for \( n_T \) and look for solutions of SLIP and corresponding functional forms of the scalar spectral index which, in turn, can be compared with observations of CMB anisotropies. Thus, a well-based assumption for \( \delta \) is

\[ \delta(\ln k) = \delta(\ln k_0) + a_1 \ln \frac{k}{k_0} + a_2 \ln^2 \frac{k}{k_0} + \cdots . \] (45)

The first reliable observation of \( r \) to be obtained is hardly expected to detect any dependence on the scale [13]. Hence, from this information only a constant value for \( n_T \) would be estimated. With this regard, in this paper, the analysis will be restricted to zero order in expansion (45). We remark that to LO it is a non sense to consider a constant tensorial index while regarding a time dependent \( \epsilon \). Solutions presented here can be obtained only by using NLO expressions related to SLIP.
A. The scale-dependent scalar index for constant tensorial index

In order to compare with experiments, one must have an expression for the scalar spectral index as function of scale. Searching for that relation, let us first to express $\Delta$ as a function of $\epsilon$. It can be derived from algebraic Eq. (34) considering constant $\delta$,

$$
\Delta(\epsilon) = \frac{1}{2(C+1)^2} \left\{ \epsilon^2 + (C+2)\epsilon + (C+1)\left[ (2C+3)\delta + 1 \right] + (2C+1)\frac{\delta}{\epsilon} + C\frac{\delta^2}{\epsilon^2} \right\} . \quad (46)
$$

The comoving scale as a function of $\epsilon$ is obtained by integrating Eq. (33):

$$
\ln k(\epsilon) = \frac{(C+1)}{2} \left( \ln |\epsilon^2 + \epsilon + \delta| - 3 \int \frac{d\epsilon}{\epsilon^2 + \epsilon + \delta} \right) . \quad (47)
$$

This way, the scalar spectral index as a parametric function is given by

$$
n_S(k) = \begin{cases} 
    k(\epsilon), \\
    2\Delta(\epsilon) + 1.
\end{cases} \quad (48)
$$

Finally, having Eq. (48), a parametric expression for $A_S(k)$ can be readily derived using definition (11) and Eq.(33):

$$
A_S(k) = \begin{cases} 
    k(\epsilon), \\
    A_S(\epsilon),
\end{cases} \quad (49)
$$

with $A_S(\epsilon)$ given by

$$
A_S(\epsilon) = A_0 \exp \left[ (C+1) \int \Delta(\epsilon) \frac{\epsilon - 1}{\epsilon^2 + \epsilon + \delta} d\epsilon \right] , \quad (50)
$$

where $A_0$ is an integration constant and $\Delta(\epsilon)$ is given by expression (46). In Secs. IV B and IV C expressions (47) and (50) will be integrated for $\delta = 0$ and $\delta < 0$.

B. Null tensorial index, $\delta = 0$

It has been stressed the relevance of Eq. (25) for constraining solutions of Eq. (24). As was already proved, for constant $\delta$, a trivial SLIP solution is power-law inflation, corresponding to $\delta = \Delta$. We shall analyze here remaining solutions involving scale-dependent $\Delta$. As it was discussed in Sec. II C while analyzing constrain given by Eq. (20), $\delta \leq 0$.

Case $\delta = 0$ is a good example of using SLIP to test the reliability of given functional form for tensorial index and of corresponding analysis of solutions. We shall see that it fails to match current observations. Explicit integration of Eq. (25) with $\delta = 0$ yields,

$$
\exp(\tau - \tau_0) = (\epsilon + 1)^{2(C+1)} , \quad (51)
$$

where $\tau_0$ is the integration constant. This solution, first reported in Ref. [20], is plotted in Fig. 1.
FIG. 1. Solution of the first order equation for $\delta = 0$.

Corresponding to $\delta = 0$ expression for $\phi(\epsilon)$ is obtained by straightforward integration of Eq. (29),

$$\phi(\epsilon) = -\frac{4(C + 1)}{\sqrt{2\kappa}} \arctan \left( \sqrt{\epsilon} \right) + \phi_0. \quad (52)$$

Expression for $V(\epsilon)$ is obtained by substituting solution (51) in Eq. (26),

$$V(\epsilon) = V_0(3 - \epsilon)(\epsilon + 1)^{2(C + 1)}, \quad (53)$$

where $V_0 = \exp(\tau_0)/\kappa$. Separating $\epsilon$ in the expression for $\phi$ and substituting in Eq. (53) finally yields for the inflaton potential,

$$V(\phi) = V_0 \frac{3 - \tan^2 \left[ \frac{2\kappa}{4(C+1)} (\phi - \phi_0) \right]}{\cos^{4(C+1)} \left[ \frac{2\kappa}{4(C+1)} (\phi - \phi_0) \right]}.$$

$$\quad (54)$$

FIG. 2. SLIP solution for $\delta = 0$. 

13
Plot of this solution for range corresponding to $0 \leq \epsilon < 1$ is presented in Fig. 2.

Looking at expression (54) and Fig. 2, one can conclude that exist an interval of $\phi$ where the potential is not consistent with the assumption $\dot{\phi} > 0$, i.e., where the potential increases with the inflaton value. One can wonder whether there is another sector of this potential where inflation can take place. Indeed, such a sector can exist but the primordial fluctuations generated during the corresponding inflaton rolling down could be different to those assumed as SLIP input. Note that the range of $\phi$ used to plot solution (54) was possible to determine using Eq. (52) which describe the inflaton as a function of the first slow-roll parameter. The valid range of values for $\epsilon$ determine the corresponding inflaton values.

Let us look at plots of $\phi$ and $V$ as functions of $\epsilon$, Fig. 3. Regarding criterion (41)

\begin{align*}
&\text{FIG. 3. Inflaton and its potential as functions of the first slow-roll parameter for } \delta = 0. \\
&\text{FIG. 4. Consistent SLIP solution for } \delta = 0.
\end{align*}

and Figs. 1, and 3, it can be concluded that solution is consistent only for $\epsilon \in [0, \epsilon_0^*]$ with
\[ \epsilon_0^* \equiv \frac{(6C + 5)}{(2C + 3)} \simeq 0.4. \] This way, the correct functional form for the potential with \( \delta = 0 \) is that in Fig. 4.

Observe that for this case the potential curvature undergoes changes. Such a behavior for the potential is obtained as a SLIP solution due to the use of NLO expressions.

After integrating Eq. (33), the comoving number \( k \) as a function of \( \epsilon \) is given by

\[ k = k_0 \left( \frac{(\epsilon + 1)^2}{\epsilon} \right)^{C+1}, \]  

with \( k_0 \) being the integration constant. From here, \( \epsilon(k) \) is obtained,

\[ \epsilon(k) = \frac{1}{2} \left\{ \left( \frac{k}{k_0} \right)^{1/(C+1)} - 2 - \sqrt{\left( \frac{k}{k_0} \right)^{1/(C+1)} \left[ \left( \frac{k}{k_0} \right)^{1/(C+1)} - 4 \right]} \right\}, \]  

where the root was chosen corresponding to \( 0 \leq \epsilon < 1 \).

Substituting expression (56) in Eq. (46) with \( \delta = 0 \), the corresponding expression for \( \Delta(k) \) is,

\[ \Delta(k) = \frac{1}{8 (C + 1)^2} \left\{ \left( \frac{k}{k_0} \right)^{1/(C+1)} - \sqrt{\left( \frac{k}{k_0} \right)^{1/(C+1)} \left[ \left( \frac{k}{k_0} \right)^{1/(C+1)} - 4 \right]} \right\} \times \left\{ \left( \frac{k}{k_0} \right)^{1/(C+1)} + 2C - \sqrt{\left( \frac{k}{k_0} \right)^{1/(C+1)} \left[ \left( \frac{k}{k_0} \right)^{1/(C+1)} - 4 \right]} \right\}. \]  

Plot for \( \Delta(k) \) and \( \delta = 0 \) is presented in the left part of Fig. 5. The range of scales plotted was chosen to allow observation of features at lowest scales. The logarithmic plot of a proper range of scales is presented in the right part of this figure, where in the vertical axis is plotted the logarithm of squared scalar amplitudes given by,
\[
\ln \frac{A_S(k)^2}{A_0^2} = -\ln \left( \frac{1}{2} \left( \frac{k}{k_0} \right)^{1/(C+1)} - 2 - \sqrt{\left( \frac{k}{k_0} \right)^{1/(C+1)} \left[ \left( \frac{k}{k_0} \right)^{1/(C+1)} - 4 \right]} \right)
\]
\[
+ \frac{1}{8(C+1)} \left( \frac{k}{k_0} \right)^{1/(C+1)} - 2 - \sqrt{\left( \frac{k}{k_0} \right)^{1/(C+1)} \left[ \left( \frac{k}{k_0} \right)^{1/(C+1)} - 4 \right]} \right)
\]
\[
\times \left\{ \left( \frac{k}{k_0} \right)^{1/(C+1)} - 2 + 4C - \sqrt{\left( \frac{k}{k_0} \right)^{1/(C+1)} \left[ \left( \frac{k}{k_0} \right)^{1/(C+1)} - 4 \right]} \right\} .
\] (58)

The value for \( A_0 \) must be chosen taking into account the observational constrain \( A_S^2 \sim 10^{-5} \) given by COBE measurements.

From these figures it is observable that the scalar index could be regarded as scale independent for relevant to measurements scales. The corresponding constant value is \( n_S = (C + 2)/(C + 1) \approx 4.7 \), which is too far from values allowed by theory and experiments. It seems to be not possible an inflationary model to exist such that the tensorial spectrum generated in its framework will be of Harrison-Zeldovich type and the corresponding scalar index will be scale-dependent. Therefore, to this order and considering Eqs. (44) and (19) with \( \delta = 0 \), it can be concluded that any model with a scale-dependent scalar index matching observations will give a nonzero tensorial contribution to the CMB spectrum.

Note that, while applying criterion (42) to the mirror image with respect to the ordinate axis of solution (54), the same result is obtained.

C. SLIP solution for negative tensorial index, \( \delta < 0 \)

We shall find the inflationary potentials producing perturbations that, to next-to-leading order, are characterized by constant and negative tensorial index and scale-dependent scalar index. Since for \( \delta < 0 \), SLIP equations are not explicitly solvable in terms of \( \phi \), we must look for a parametric expression for the inflaton potential.

The solution of Eq. (25) for \( \delta < 0 \) is [20]

\[ \exp(\tau - \tau_0) = \left| \epsilon^2 + \epsilon + \delta \right|^{\frac{C+1}{2\epsilon + 1}} \frac{2\epsilon + 1 + \sqrt{1 - 4\delta}}{2\epsilon + 1 - \sqrt{1 - 4\delta}} . \] (59)

The three branches corresponding to this expression are plotted in Fig. 6.

It is observed that, along with stationary solution

\[ \epsilon_+ = -\frac{1}{2} + \frac{1}{2}\sqrt{1 - 4\delta} , \] (60)

solutions can increase unbounded or decrease bounded by the value \( \epsilon = 0 \). Regarding that \( \tau \) and cosmic time have opposite signs, there is some time interval where \( \epsilon \approx \epsilon_+ \), i.e., the solution asymptotically behaves like that of power-law. After integrating Eq. (29) for \( \delta < 0 \) and inserting solution (59) in expression (26), the parametric potential is given by

16
FIG. 6. Solution of the first order equation for $\delta < 0$. Each curve branch that goes from left to right is a solution with different initial values.

\[
V(\phi) = \begin{cases} 
\phi(\epsilon) = \frac{2(C+1)}{\sqrt{\kappa}} \frac{1}{\sqrt{1-4\delta}} \left[ -\sqrt{1+\sqrt{1-4\delta}} \arctan \left( \frac{\sqrt[4]{2} \epsilon}{\sqrt{1+\sqrt{1-4\delta}}} \right) \right] + \frac{1}{2} \sqrt{-1 + \sqrt{1-4\delta}} \ln \left| \frac{\sqrt[4]{2} \epsilon + \sqrt{-1 + \sqrt{1-4\delta}}}{\sqrt[4]{2} \epsilon - \sqrt{-1 + \sqrt{1-4\delta}}} \right| + \phi_0, \\
V(\epsilon) = V_0(3-\epsilon) |\epsilon^2 + \epsilon + \delta|^{C+1} \frac{2C+1+\sqrt{-1+\sqrt{1-4\delta}}}{2C+1-\sqrt{-1+\sqrt{1-4\delta}}} \sqrt{1-4\delta}.
\end{cases}
\]

The inflaton and corresponding potential as functions of $\epsilon$ are respectively plotted in Fig. 7. Note that the graph of the potential has also a maximum around $\epsilon^* \equiv \frac{(6C+5) + \sqrt{(6C+5)^2 - 4\delta(2C+3)}}{(4C+6)} \simeq 0.4$.

FIG. 7. Inflaton and its potential as functions of the first slow-roll parameter for $\delta = -0.01$. 

```plaintext
\[
\epsilon^* \equiv \frac{(6C+5) + \sqrt{(6C+5)^2 - 4\delta(2C+3)}}{(4C+6)} \simeq 0.4
\]```
for $\delta = -0.01$, not shown in the figure in order to observe the details for small values of $\epsilon$. Hence, the analysis for $\delta < 0$ should be done for three intervals of $\epsilon$, namely, $I_1 = [0, \epsilon_+]$, $I_2 = (\epsilon_+, \epsilon^*)$, and $I_3 = (\epsilon^*, 1)$.

Making use of criterion (41), consistent SLIP solutions are determined to exist in the intervals $I_1$ and $I_2$. Corresponding plots are presented in Fig. 8. Here, the value of the constant $V_0$ could be chosen to make the potential flat enough for conditions of successful inflation to be satisfied. For $\epsilon \in I_3$, the solution fails to fulfill criterion (41).

The main difference between both solutions is the curvature of the inflaton potential near the origin, but is not a trivial one. For the same parametric expression of the potential we have two rather different realizations of inflation depending not only on initial conditions for $\phi$ but also for $\dot{\phi}$ (i.e., depending on the initial conditions for $\epsilon$). In the case corresponding to the left graph on Fig. 8, the term in Eq. (2) given by the first derivative of the potential will generically dominate during the inflationary epoch meanwhile, in the remaining case, the evolution of $\phi$ will be dominated by the friction term due to Universe expansion, $3H\dot{\phi}$.

For $\delta < 0$, after integration of Eq. (47) we have that

$$k = k_0 \epsilon^2 + \epsilon + \delta \frac{2\epsilon + 1 + \sqrt{1 - 4\delta}}{2\epsilon + 1 - \sqrt{1 - 4\delta}}^{\frac{3(C+1)}{2(C+1)}},$$

where $k_0$ is the integration constant. With $\Delta(\epsilon)$ given by Eq. (46), parametric plots for $\Delta(k)$ corresponding to $\epsilon \in I_1$ and $\epsilon \in I_2$ are presented in Fig. 9. Again, the scale range was chosen to allow details observation at lowest scales. Observe that in both cases the present error for $n_S$ (e.g., $n_S = 0.99 \pm 0.09$ in Refs. [4]) can mask the scale dependence at large $k^{-1}$.

After integrating Eq. (50) for $\delta < 0$, the logarithm of squared scalar amplitudes is given by

$$\ln \frac{A_S(\epsilon)^2}{A_0^2} = \frac{\delta C - 1 - C}{C + 1} \ln \epsilon + \delta (C + 1) \ln \left| \epsilon^2 + \epsilon + \delta \right|$$
FIG. 9. $\Delta$ as a function of the comoving number for $\epsilon \in I_1$ and $\epsilon \in I_2$, respectively, and $\delta = -0.01$.

$$\Delta = \delta + 3\delta (C + 1) \frac{2\epsilon + 1 + \sqrt{1 - 4\delta}}{\sqrt{1 - 4\delta}} \ln \frac{2\epsilon + 1 + \sqrt{1 - 4\delta}}{2\epsilon + 1 - \sqrt{1 - 4\delta}} + \frac{\epsilon^3 + 2C\epsilon^2 + 2C\delta}{2(C + 1)\epsilon}. \quad (63)$$

As observed in Fig 10, where the parametric plots of scalar amplitudes for $\epsilon \in I_1$ and $\epsilon \in I_2$

FIG. 10. $\ln A_{S}^2$ as a function of the $\ln k$ for $\delta = -0.01$ and for $\epsilon \in I_1$ (lower branch) and $\epsilon \in I_2$ (upper branch).

is presented in the same figure, differences are almost impossible to note when the full range of scales is considered. Differences arise at large angular scales which could be out of reach for measurements. It means that, from the observational point of view, there could be not differences between these two different realizations of inflation if the scales where differences
arise are not probed or the resolution is not high enough to detect the scale dependence. These scales are precisely those where higher energies physics could leave an imprint.

Recalling the behavior of solution (51) for small \( \tau \), i.e., large \( t \) (see Fig. 6) and taking the limit \( \epsilon \to \epsilon_+ \) of Eqs. (46) and (62) it is found out that for small \( k^{-1} \), \( \Delta \approx \delta \) (see also Fig. 9). That the scale interval where this behavior is observed corresponds to sufficient large number of e-folds to solve the Standard Model problems is provided by the asymptotic behavior of \( \epsilon \) near the value \( \epsilon_+ \).

These models have the desired feature of an almost negligible \( \delta = \text{const.} \) and the scalar index being nearly constant in a wide range of scales, with the possibility of choosing values that can accurately match current observations. In fact, the value \( \delta = -0.01 \) chosen in the figures of this section corresponds to \( n_S = 0.98 \) compatible with values given in Refs. [4]. Any other value for \( n_S \) arising from analysis similar to that of Refs. [4,6–9], except for blue spectra, can also be fitted with models given by Eq. (61). Furthermore, this value of \( \delta \) approximately corresponds to \( r \approx 0.12 \) which is greater than \( r^* = 0.1 \), the value given in Ref. [5] as the lower limit for \( r \) to be detectable with a 95-percent confidence regarding the error reported in Ref. [6] as the estimate for Planck measurements. Lower values for \( r \) can be appropriately taken into account.

Because models given by Eq. (61) do not have a graceful exit to the Standard Model stage of Universe evolution (\( \epsilon \) converges to \( \epsilon_+ \) not to 1), \( \phi \) must be regarded here as the dominant scalar field in a hybrid scenario with \( \epsilon_+ \) being the value corresponding to the critical value of \( \phi \) near which the false vacuum becomes unstable and the multiple scalar fields roll to the true potential minimum [5].

Finally, all of the statements done in this section with regards to solution (61) are also valid after changing \( \phi(\epsilon) \) by \( -\phi(\epsilon) \) and using criterion (42).

V. CONCLUSIONS

We presented a version of Stewart-Lyth inverse problem using the first slow-roll parameter as basic variable in the procedure of finding the inflaton potential. That allows to analyze the solutions in the range of this parameter where inflation is feasible. A criterion was introduced to check for solutions consistent with the assumptions underlying the derivation of the Stewart-Lyth inverse problem equations.

It was shown that expressions related to Stewart-Lyth inverse problem can be used to determine inflationary models corresponding to given observations. We proved that power-law inflation is a trivial solution of this problem when constant spectral indices are used as input in related equations. Next-to-leading-order in the slow-roll expansion makes possible to consider more general scenarios where slow-roll parameters can slowly vary with time. In a near future, these scenarios could be more realistic than common assumption regarding slow-roll parameters as constants during inflation.

Looking for a potential generating the primordial perturbations able to grow into CMB anisotropies and matching current and future observations, we solved the Stewart-Lyth inverse problem with constant tensorial index as input. Inflationary models were found which, unlike power-law inflation, yield scalar modes characterized by a scale-dependent index. For negative tensorial index, solutions were given as an expression depending on the first slow-roll parameter.
The special case of a Harrison-Zeldovich spectrum of tensorial perturbations, i.e., constant amplitudes of gravitational waves, is ruled out by comparison of our results with current observations. It means that it seems to not exist inflationary models with scale dependent scalar index and null tensorial index. Hence, for any model exhibiting some degree of scale dependence of the index of curvature perturbations it must be expected a nonzero contribution of primordial gravitational waves to the amplitudes of the CMB spectrum.

Potentials obtained for strictly negative tensorial index fulfill the conditions for successful inflation. Evolution of the scalar field given by these potentials can be considered as the dynamical element in a hybrid inflation scenario, the value of the inflaton corresponding to power-law solution acting like the instability value for the false vacuum.

These models can be used as assumption on the origin of primordial perturbations to test for scale dependence of the scalar index. Using them to fit inflationary perturbations is almost as easy as using power-law inflation. Only one parameter, the constant tensorial index, need to be fitted. If CMB polarization fails to give a value of the tensor-scalar ratio greater than the threshold value \( r^* = 0.1 \), then the tensorial index must be fitted in the interval \((-0.02, 0)\). Otherwise, if some value for the tensor-scalar ratio is measured, then an approximated value for the tensorial index can be estimated to serve like pivot value for the fitting procedure.

We would like to stress that if any of the potentials here presented make possible to reach an overall good fit for CMB anisotropies and to detect scale dependence of the scalar index from the next generation of observations, the conclusion to be drawn is that the actual inflaton potential is similar in the probed scale to the used one. In turn, if the quality of the overall fit is not improved compared to the result obtained using power-law inflation, either the scale dependence of the scalar index is practically negligible, or the information on the scale dependence of the tensorial index is fundamental in order to account for the features in the CMB anisotropies.

The spectra of scalar perturbations produced by these models differ from those of power-law inflation at scales corresponding to earlier times in the Universe evolution. Thus, if while increasing the quality of observations a good fit using any of these potentials is achieved, it could give some hints about physics taking place at very high energies.

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