Simulation of the response of a silicon pixel detector

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Abstract

A model to simulate the response of a silicon pixel detector is described. The effects of geometrical charge sharing, electronic noise threshold dispersion, capacitive coupling between pixel channels and δ -rays production have been taken into account. The model has been tested on the Omega3 chip, which is the direct predecessor of the ALICE pixel detector, to be used in the inner tracking system of the ALICE experiment. The model is able to reproduce the experimental data and the resulting parameters are consistent with the measurements made on the Omega3 chip.

Key words: silicon pixel, simulation

PACS: 07.05.T, 29.40.W

1 Introduction

The Inner Tracking System (ITS) [1] of the ALICE [2] experiment at LHC has the challenging task to identify and reconstruct tracks in the region with the largest track density. In order to achieve this goal, high precision and high granularity detectors are required, sensitive to the hitting coordinates of the charged particles in two dimensions. The two innermost layers of the ITS, in particular, will be formed by silicon pixel detectors with true bidimensional readout.

In the following, we present a model to simulate the pixel detector response, introducing effects which cannot be accounted for by transport packages as GEANT [3] and FLUKA [4], used in AliRoot [5], the standard ALICE simulation and analysis package, based on ROOT [6] framework. In addition, this

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model allows to describe the detailed pixel geometry without introducing millions of small volumes, as it would be required by a full GEANT description of the detector.

The model allows to single out the pixels fired by the passage of a charged particle through the detector, taking into account its geometrical path, the presence of electronic noise, the discriminator threshold dispersion and the δ -rays contribution. The coupling between pixel channels is also considered. The model provides a clustering description of the pixels. This information is crucial in determining the physics performances of the ALICE pixel detector, namely its vertex finding capability, its improvement in the two tracks resolution. It will be shown that the effects introduced in the model produce an increase in the number of fired pixels, which could affect the two-track resolution, and hence the overall track reconstruction efficiency.

To test the model and to set the free parameters which regulate the intensity of the introduced effects, it is necessary to compare the simulated response of the detector with experimental data. Since no data are yet available for the ALICE pixel detector we focused on its direct predecessor, i.e. the Omega3 pixel detector [7]. This chip was developed and studied by the RD19 Collaboration and used in the WA97 [8] and NA57 [9] heavy-ion experiments. The use of an extensively studied chip allows us to check if the tuned values of the parameters of the model agree with the corresponding measured physical quantities.

2 Description of the pixel detectors

The basic element of the ALICE pixel detector is the ladder, consisting of a pixel detector matrix of 256×192 pixel cells bonded to six front-end chips with fully independent readout. The standard cell dimensions are $50 \times 425 \mu\text{m}^2$, with a thickness of $150 \mu\text{m}$. At the boundary between two front-end chips, there are two columns of cells having dimensions of $50 \times 625 \mu\text{m}^2$. The ALICE1 chip, as reported in [10], is a matrix of 256×32 cells, each occupying an area of $50 \times 425 \mu\text{m}^2$. Both the readout logic and the local control functions are integrated on the chip.

Many of the requirements for the ALICE pixel detector are also satisfied by the already existing Omega3 front-end chip. In fact, the improvements in the design of the new ALICE prototype were mainly dictated by the necessity to limit the total detector thickness and to improve its radiation tolerance.

The Omega3 ladder consists of a pixel detector matrix of 128×96 cells; the standard cell dimensions are $50 \times 500 \mu\text{m}^2$, with a thickness of $300 \mu\text{m}$. At

the boundary between two front-end chips, the cells have dimensions of $50 \times 1000 \mu\text{m}^2$. The ladder is bonded to six front-end Omega3 chips, each one serving a sub-matrix of 128×16 active readout cells of $50 \times 500 \mu\text{m}^2$.

In summary, for our purposes, the ALICE detector modules and front-end chips are very similar to those of the existing Omega3 detector, and in the following we will refer to this latter detector in addressing the features of our model.

3 Description of the model

Pb–Pb events at 160 A GeV/c, generated by VENUS 4.12 [11], are tracked in a uniform magnetic field of 1.4 T using the GEANT 3.21 package [3]. A silicon detector plane, consisting of 8 Omega3 ladders covering a sensitive area of about $5 \times 5 \text{ cm}^2$, is positioned 1 m downstream the target to accept particles generated at central rapidity. The impact angles on the plane are close to 90° with respect to the detector plane for most of the particles, although lower angles are also present.

The basic element introduced in the GEANT geometry is the Omega3 ladder, whereas its pixel structure is described *via* a software algorithm.

The model determines the fired pixels and groups them in clusters, taking into account the geometrical charge sharing, the electronic noise and the threshold dispersion.

It can eventually simulate also the capacitive coupling between pixels and the coupling between electronic readout channels.

To study how does each effect influence the cluster size, they have been introduced one at a time.

The δ -rays production is ruled by GEANT and its effect has been studied in addition to the others.

3.1 Geometrical charge sharing

The geometrical charge sharing occurs when the incident particle crosses two or more pixel cells. When this happens, the energy lost by the incident particle is shared among all the crossed cells and more than one pixel could be fired. The effect depends on the impact point of the incident particle on the

cell and on its original direction. Moreover, it depends on the setting of the discrimination threshold of the readout chip.

In our model the energy lost in the pixel cells is assumed to be deposited proportionally to the length of the path of the track in each cell. As the pixels are not individually described as GEANT volumes, the path of each track is evaluated in the simulation by a dedicated algorithm.

In order to determine if a pixel is fired or not, we introduce a software threshold E_t to the energy lost in a single cell, E_l . The quantity E_t is a free parameter of the model. The fired pixels are then grouped by a clustering algorithm according to their position in the detector matrix. Diagonal pixels are also included into the clusters.

In fig. 1 the results of the simulation are shown for different values of the threshold. There, the column cluster length distribution is reported. The column cluster length is equal to the number of fired pixels in the cluster counted along a column. The clustering is performed separately for each ladder of the detector plane. It can be seen that the column cluster length distribution is affected by the charge sharing since the increase in the threshold value produces a decrease in the number of fired pixels.

The same effect can be observed in the row direction, where the average cluster length is smaller and the decrease of the distribution is much steeper.

We have also noticed that the cluster multiplicity is not sensitive to the threshold range chosen, since charge sharing only changes the size of the clusters, not their number.

The geometrical effect depends on the thickness of the detector. In fig. 2 the row and column cluster lengths obtained by a simulation with 300 μm of thickness (the Omega3 value) and with 150 μm (the expected Alice1 value) are compared. The threshold is $E_{th} = 30 \text{ keV}$ in both the simulations.

In summary, the introduction of the geometrical charge sharing in the simulation produces a cluster length distribution which is sensitive to the threshold value. This effect is more evident for contiguous pixels along a column (where they share the large edge), especially for length-2 clusters.

In order to check the geometrical approximation, we have also described the pixel cells as independent GEANT volumes. In this way we can look for the energy lost in the cells as evaluated directly by GEANT, where several independent Landau fluctuation for each cell are considered. In fig. 3 the results from the simulation which use the GEANT segmentation are compared with those from the model. The two descriptions are fully equivalent.

A more sophisticated effect occurs in real detectors, due to the presence of a shared region of a few microns between adjacent cells. When a particle crosses this region, the generated charge carriers are shared between the two cells. The effective size of this region can depend on the drift of the charge carriers, where the drift length is expected to be a few μm . The effect of the drift has also been tested in our simulation. For this purpose we have replaced the linear path of the particles with a tube, whose diameter λ gives a measure of the drift length. In this way the energy lost in a cell is now assumed to be proportional to the volume of the tube inside the pixel cell. The results are shown in fig. 4 for different values of the λ parameter. No significative effects are visible up to $\lambda = 50\mu\text{m}$.

3.2 *Electronic noise and threshold dispersion*

The noise can be described as a random fluctuation (in time domain) of the energy lost in each cell of the pixel detector. This fluctuation can be approximately parametrised as a Gaussian, due to the characteristics of the shaper used in the Omega3 and ALICE1 readout chips. In addition, in real detectors a spatial non-uniformity is observed, due to the spatial dispersion of the discriminator thresholds in the different cells of the readout chip. The hardware thresholds distribution has been measured for the Omega3 chip [12] and it is approximately Gaussian around the nominal value. A similar behaviour is also expected for the ALICE1 chip.

To simulate both effects in the model, we introduced a pedestal E_p , which fluctuates from event to event and from pixel to pixel, with a Gaussian distribution centered at zero with a standard deviation σ . The effective energy that can give a signal in the pixel cells is then given by $E_{eff} = E_l + E_p$ and this value should now be compared with the previously introduced software threshold E_t . A pixel can be fired only by noise if its pedestal fluctuation is larger than E_t .

It is worthwhile noting that the sum of the Gaussian distributions of the discriminator thresholds (in space) and of the pedestals (in time) is equivalent for our purposes to a single Gaussian, which we actually introduce in the model, representing both fluctuations. Therefore, the square of our σ parameter has to be interpreted as the sum of the variances of the two physical distributions.

In fig. 5 the column cluster length distributions for different settings of the parameter σ are shown. The distribution corresponding to the absence of any fluctuation in the pixel detector is drawn in full line, while the dashed and dotted lines show the distributions obtained increasing the Gaussian fluctuation of the pixel pedestal. The value of the parameter E_t is set to 30 keV .

It can be noticed that as the σ parameter increases, the number of length-1 clusters is enhanced with respect to those with higher dimension. This is due to the fact that the pixels fired by noise are uncorrelated.

3.3 δ -rays production

The production of δ -rays gives an important contribution to the size of the cluster associated to a single track. A sizeable fraction of the energy lost by the primary particle is transferred to δ -rays, therefore a large number of pixels is additionally fired. The δ -rays production is provided directly by GEANT. In this case, the parameter E_δ , defining the lower limit of the δ -rays energy for their tracking, should be set.

In fig. 7 the effect of the δ -rays production is shown for different values of the E_δ parameter. The simulation without δ -rays is also reported for comparison. In all the simulations the threshold E_t is set to 30 keV. It can be seen that the tails of the cluster length distributions are strongly enhanced when δ -rays production is included. This enhancement is larger in column direction, whereas in row direction it is limited by the δ -rays range. The distributions appear to be nearly insensitive to the values of E_δ in the considered range (10 ÷ 150 keV).

4 Comparison with experimental data

In order to set the free parameters of the model and to test its reliability, we need to compare the simulated performances of the pixel plane with a real experimental situation. As already stated, the only available data on Omega3 pixel detector come from the experiments WA97 and NA57. We have chosen a raw data sample of 10,000 events coming from one of the most efficient planes of the NA57 telescope during its 1998 Pb-Pb run at 160 A GeV/c. Therefore, in the simulation program, we put the Omega3 plane in the same position with respect to the target as it had in the NA57 experiment; the experimental trigger is also introduced and tuned in order to reproduce the measured charged multiplicity distribution [13].

We performed several simulations with the δ -rays always included, using $E_\delta = 30$ keV, and we tuned the intensities of the introduced effect in order to reproduce the experimental data.

4.1 *Tuning of the thresholds*

In fig. 8 the row and column cluster length distributions are shown for real data (full circles) and for simulated data with different values of the software threshold E_t . All the distributions are normalised, the δ -rays are included and only the geometrical charge sharing effect is considered.

The comparison of the cluster length distributions allows a fine tuning of the value of the E_t parameter, since the simulated distribution is very sensitive to its changes, especially concerning the number of length-2 clusters, as shown in fig. 1. The tuning has been performed considering the first three bins in both the row and column cluster length distributions and the best agreement with experimental data is achieved for values of the E_t parameter between 25 keV and 30 keV.

4.2 *Tuning of noise and threshold fluctuations*

The effect of the σ parameter is visible in the cluster length distributions especially in the first bin. In fig. 9 the comparison of the row and column cluster length distributions is done by adding in the simulation the effect of noise and threshold non-uniformity to the geometrical charge sharing. The E_t parameter is fixed at 30 keV, while the σ parameter is left variable.

It can be noted that, with a value of the σ parameter not exceeding 7 keV, the distributions show a good agreement with data. This allows to determine an upper limit to the intensity of the fluctuations. For higher values of the σ parameter the hit multiplicity is strongly enhanced, and the agreement with data is lost (not shown). More details can be found in [14].

4.3 *Coupling effects*

We consider the possibility to simulate the coupling between pixel. In this way we account for two different effects: The first is the capacitive coupling, which comes from the geometrical shape of the detector cells and which is effective mainly by column. The second is due to the cross-talk between electronic channels and takes place at different levels in the readout chain. This effect is expected to be lower in the new ALICE1 chip with respect to the Omega3 chip, considered in this analysis.

We use a simple parametrization for an overall description of the above mentioned effects: we assign a fraction $F_r \cdot E_l$ (where $F_r < 1$) of the energy lost in

a cell to the two nearest cells in the row direction. A fraction $F_r \cdot (F_r \cdot E_l)$ is assigned to the two next cells, and so on. A different parameter (F_c) is used for the column direction.

By construction, the coupling is symmetric with respect to the central cell; consequently it mostly involves odd configurations. Therefore the cluster length distributions are enhanced starting from the third bin [14].

In fig. 10 the row and column cluster length distributions are shown for real data and for simulated data with different values of the parameters F_r and F_c . The values of the E_t and σ parameters are set to 30 keV and 5 keV respectively and δ -rays are included.

The dashed line represents the model prediction if the coupling is not included, while in full line are shown the distributions when the coupling by row and by column is added, respectively with parameters $F_r = 0.1$ and $F_c = 0.1$. It can be seen that the simulated column cluster length distribution is not affected by the coupling², and that reproduces satisfactorily the experimental distribution. Therefore, the δ -rays contribution is dominant with respect to the coupling effects acting along a pixel column. On the other hand, the presence of coupling may improve the agreement with experimental data in the row direction, where the δ -rays effect is limited by their range.

4.4 Cluster shape analysis

In order to better characterize the effect of δ -rays production on the cluster size, we performed a cluster shape analysis. We considered clusters with dimension up to $N=3$ and we tagged their different configurations.

In fig. 11 the frequencies of cluster configurations for cluster formed by one or two pixels ($N \leq 2$) are reported, for both real data and simulation. All the distributions are normalised, and this is almost equivalent to assume the same number of single hits for all the configurations. The correspondence between histogram bins and cluster configurations is sketched in the figure. In bin n.4 both the orientations of the diagonal cluster are included. The full line shows the distribution without coupling; the other two lines show the contribution of this effect, with parameters $F_r = 0.1$, $F_c = 0$ (coupling by rows) and $F_c = 0.1$, $F_r = 0$ (coupling by columns).

The agreement between simulation and experimental data is good, even if a slight underestimation of the diagonal configuration (bin n.4) is present. As already stated, the effect of coupling on clusters of dimension 2 is negligible.

² We observed that this happens as far as the F_c value does not exceed 0.2.

In fig. 12 the frequencies of cluster configurations for clusters formed by three pixels ($N = 3$), for both real data and simulation are reported. The data have been normalised with respect to the first four bins of the histograms. Each bin in the histogram corresponds to a given configuration, as explained in the figure. In bins n.7, 8 and 9 all the possible orientations of the corresponding pixel configurations are included. The line types follow the same convention as in the previous figure.

Here again a good agreement with data can be observed. The residual discrepancy in bin n.5 could be reduced by tuning the coupling along rows. The dashed line shows the cluster frequencies when a coupling with $F_r = 0.1$ and $F_c = 0$ is added: the simulated frequency in bin n.5 results to be well above the experimental one. However, if a coupling by column is applied at the same time, as shown in dotted line, the frequency of bin n.5 is lowered. In fact, the contemporaneous presence of the coupling by columns, though does not affect the row cluster length distribution (see fig. 10), reduces the number of $N=3$ clusters, because additional fired pixels are added to already existing clusters. As a consequence, the configuration corresponding to bin n.5 also depends on the setting of the F_c parameter, being fixed the number of pixels forming the cluster ($N=3$).

The cluster shape analysis, in addition, allows to distinguish the δ -ray from the coupling contribution. The most important difference between these two effects is that δ -rays are emitted isotropically, whereas the coupling in our model is effective only by row and by column. As a consequence, δ -rays increase all the cluster configurations and in particular the frequency of bin n.6, corresponding to the cluster configurations where three pixels share their larger side. On the other hand, the coupling is competitive with δ -rays production only in bins n.5 and n.6, where three contiguous pixels are aligned respectively along a row and along a column[14].

We can conclude that the coupling effects are negligible with respect to the δ -rays contribution and that the experimental data can be reproduced using coupling parameters $F_r \leq 0.1$ and $F_c \leq 0.1$, applied simultaneously.

In summary, we obtained a good agreement between experimental and simulated data in the following ranges of the parameters:

- $E_t = 25 \div 30 \text{ keV}$;
- $\sigma \leq 7 \text{ keV}$;
- $F_r \leq 0.1, F_c \leq 0.1$

- $E_\delta \leq 150 \text{ keV}$.

4.5 Consistency of the parameters

In order to verify the consistency of our model with the physical effects which actually take place in the pixel detector, the tuned parameters should be compared with the corresponding quantities directly measured on the detector.

Threshold measurements have been performed on single Omega3 chips and on detector arrays of 4 Omega3 ladders, corresponding to 24 chips [12]. The threshold distribution measured on a single chip has a mean value of $4000e^-$, corresponding to 14.5 keV , and a standard deviation of $400e^-$, corresponding to 1.5 keV . The standard deviation due to electronic noise was also measured for single Omega3 chips, obtaining an estimation of $220e^-$, which corresponds to 0.8 keV [15].

However, an average value of the threshold distribution of $6800e^-$, corresponding to 24.6 keV , has been measured on the 24 chip array [12]. This value is close to the tuned value of our threshold parameter E_t , obtained from a detector plane consisting of 48 chips (8 ladders).

The value of the σ parameter, on the other hand, should be compared to the squared sum of the measured variances of the threshold and noise distributions, as discussed in section 3.2. The squared sum, measured on a single chip, results to be 1.7 keV , which is well below the upper limit of 7 keV obtained with the simulation.

The value $E_\delta = 30 \text{ keV}$ applied to the δ -rays tracking is a consequence of the E_t threshold setting.

No measure of the coupling parameters was found in literature .

5 Conclusions

We have described a model to simulate the response of silicon pixel detectors to minimum ionizing particles. The pixels fired by a charged particle crossing the detector are determined considering the effects of geometrical charge sharing, electronic noise, non-uniformities of the discriminator thresholds, coupling between pixel channels and δ -rays production. The contribution of the included effects to the number of clusters of fired pixels and to their size have been analysed for each of them.

The model has been tested on existing data on the Omega3 pixel detector, whose design is very similar to that of the new ALICE pixel detector. We have shown that the model is able to reproduce the response of the Omega3 pixel detector used in the NA57 experiment, since it describes satisfactorily the experimental cluster length distributions. The experimental frequencies of the cluster configurations are mostly reproduced considering the δ -rays production, whereas the coupling effects give a minor contribution.

The comparison with experimental data has allowed us to tune the parameters that set the intensity of the introduced effects. The results are consistent with the measurements performed on the Omega3 detector.

A new calibration of the parameters should be performed as soon as the experimental data from test beams on the new ALICE chip will be available.

Acknowledgements

We wish to thank the NA57 collaboration for allowing us to use their experimental data for this work, and F. Carminati and G. Paic for encouraging us with fruitful discussions.

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