The physical meaning of phase and its importance for quantum teleportation

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Abstract

We argue that the two parties in a quantum teleportation protocol need to share more resources than just an entangled state and a classical communication channel. As the phase between orthogonal states has no physical meaning by itself, a shared standard defining all relevant phases is necessary. We discuss several physical implementations of qubits and the corresponding physical meaning of phase.

1 Introduction

Quantum teleportation is one of the most important quantum information processing protocols. It was discovered in 1993 [1] and several teleportation experiments have been performed since then [2, 3, 4]. It is well known that the main quantum ingredient is a nonlocal entangled state. It is furthermore assumed that the only additional resource needed is a classical communication channel. Here we will show that that is not quite true. In standard formulations of quantum teleportation the relative phase of the entangled state is considered irrelevant. While it is true that teleportation works for any value of that phase, it must still be known in some way or another. It turns out that this is possible only if the two parties sharing the nonlocal entangled state, Alice and Bob, share a standard against which they both can gauge their local phases. This standard is not always trivial to establish and depends on the precise physical implementation of the quantum states involved in the teleportation protocol.

Suppose Alice and Bob share an entangled state of the form

\[ \frac{\left| 1 \right>\left< 0 \right| - \exp(i\varphi)\left| 0 \right>\left< 1 \right|}{\sqrt{2}}, \quad (1) \]

and wish to use this state to teleport an unknown quantum state

\[ |\psi(\theta, \varphi)\rangle = \sin(\theta/2)|0\rangle + \exp(i\varphi)\cos(\theta/2)|1\rangle, \quad (2) \]
given to Alice by a third party Victor. Although the standard teleportation protocol from [1] works perfectly for any phase $\phi$ and the state (1) is entangled for any phase $\phi$, if that phase is actually unknown to Alice and Bob (more precisely, varies from experiment to experiment in an unknown way), they cannot achieve ideal teleportation and in fact do not share an entangled state. The state (1) averaged over all values of $\phi$ is merely a classically correlated state. One may check that the teleportation fidelity drops to $2/3$. For example, if they wrongly assume $\phi = 0$, then the state obtained by Bob will be

$$|\psi(\theta, \varphi)\rangle = \sin(\theta/2)|0\rangle + \exp(i(\varphi + \phi)) \cos(\theta/2)|1\rangle,$$

which on average has an overlap of $2/3$ with $|\psi\rangle$, since

$$\frac{1}{2\pi} \int_0^{2\pi} d\varphi \int_0^\pi \frac{d\theta}{2} \sin \theta \int_0^{2\pi} \frac{d\varphi}{2\pi} |\langle \psi(\theta, \varphi) | \psi_{\phi}(\theta, \varphi) \rangle|^2 = \frac{2}{3}. \tag{4}$$

This is in fact the maximum fidelity that can be achieved by classical teleportation (that is, without using any entanglement).

Producing a nonlocal entangled state with a well-defined value of the phase $\phi$ is not a trivial task. It amounts to Alice and Bob locking the relative phases $\phi_i$ for $i = A, B$ between their respective implementations of the states $|0\rangle$ and $|1\rangle$,

$$|R(\phi_i)\rangle_i = \left(|0\rangle_i + \exp(i\phi_i)|1\rangle_i\right)/\sqrt{2}, \tag{5}$$

assuming for the moment that they are able to consistently define $|0\rangle$ and $|1\rangle$; see below. Since the states $|0\rangle$ and $|1\rangle$ are orthogonal, their relative phase has no physical meaning and can only be defined relative to a reference. If Alice and Bob use independent phase standards, however, they can never be sure to create the same entangled state from experiment to experiment. Thus, Alice and Bob need to share a phase reference.

## 2 The physical meaning of phase

We will discuss here various proposals for implementing quantum bits (see for example [5]), the meaning of the phase in those cases, and the various phase standards that are needed.

1. **Spin-1/2 particles.** One of the most popular representations of a qubit is a spin-1/2 particle, such as an electron or a carbon nucleus. This is indeed one of the natural implementations of a qubit, as a spin-1/2 system has a two-dimensional Hilbert space associated with its spin. The states $|0\rangle$ and $|1\rangle$ are then defined as eigenstates of one component of the angular momentum operator, say, the $z$ component. This of course already assumes Alice and Bob can both define $z$ directions in a consistent way. That is, they may locally define any axis to be the $z$ axis, but their arrangement should be such that the two axes do not rotate with respect to each other. More generally,
since angular momentum generates rotations in space, the definitions of the phase $\phi$ and the states $|0\rangle$ and $|1\rangle$ depend on directions in space. One way to make Alice’s and Bob’s definitions consistent is to use the fixed stars. This is then an example of a shared resource between Alice and Bob necessary for the ability to teleport reliably.

2. **Photon polarization.** Another popular representation is photon polarization. Although a photon is a spin-1 particle, it has only two spin (more accurately, helicity) degrees of freedom because it is massless. Alice and Bob can define their basis states to correspond to left-hand and right-hand circular polarization. These two polarization directions can be consistently defined *locally*, without a shared standard, because circular polarization states are helicity eigenstates and helicity is defined relative to the propagation direction of the photon. Since the helicity operator generates rotations of the polarization around the propagation direction, however, the phase of the state $|R(\phi)\rangle$ depends on the definition of the two spatial directions perpendicular to the propagation direction. Hence, here too, Alice and Bob need to use the fixed stars or a similar resource defining spatial direction.

3. **Photon number.** One can also encode a qubit in two number states of the electromagnetic field. Given a particular mode one can choose the states containing 0 and 1 photons to implement $|0\rangle$ and $|1\rangle$. In this case, these two states are eigenstates of the (free field) Hamiltonian with different eigenvalues. Since the Hamiltonian generates translations in time, Alice and Bob now need to share a resource that fixes time, in other words, they need synchronized clocks during the entire protocol. $^1$ With the time origin fixed, the phase difference between $|0\rangle$ and $|1\rangle$ at that time can be determined locally by measuring any operator that has nonvanishing matrix elements between $|0\rangle$ and $|1\rangle$, such as the electric or magnetic fields.

4. **Harmonic oscillator eigenstates.** A similar representation makes use of the two lowest vibrational levels of a material particle moving in a one-dimensional harmonic potential [6]. Again, since these states are different eigenstates of the Hamiltonian, the origin of time must be fixed. The residual phase difference at time zero can be defined locally by measuring position or momentum (both defined relative to the harmonic potential) or any other observable with nonvanishing matrix elements between $|0\rangle$ and $|1\rangle$.

One may also choose degenerate eigenstates of the 2-D (3-D) degenerate harmonic oscillator so that synchronized clocks are no longer necessary. However, the degeneracy is caused by a symmetry between 2 (3) spatial directions, so that again fixed stars must be used to distinguish the two (three).

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$^1$Note that even if Alice and Bob have perfect atomic clocks that they synchronized at some point in time, they will still need to keep checking that the clocks don’t drift apart due to relativistic or other effects.
5. **Atomic energy eigenstates.** Ground states in atoms or ions are an experimentally attractive type of implementation. If the atomic or ionic ground state has total angular momentum quantum number $J$, then two states may be chosen out of a multiplet of $2J + 1$ degenerate states. Typically, one chooses two eigenstates of the angular momentum along a particular direction, say $J_z$, with eigenvalues differing by one or two units. This is such that one can induce transitions between those two states by using two light beams with either opposite circular polarizations (when the states differ by two units of angular momentum), or with one circular and one appropriate linear polarization (when the states differ by one unit of angular momentum). This case, therefore, is, in this operational sense, equivalent to the previous case of photon polarization. More often, however, non-degenerate ground states are chosen from two different multiplets with different angular momenta, and in such a case one needs the additional resource of synchronized clocks.

6. **Charge and flux states.** Yet two other types of representations use superconductivity. In particular, one may use charge states (two different eigenstates with fixed charge) or magnetic flux states (in fact, states with a well-defined direction of a current) as qubits. These states are not degenerate, and so again synchronized clocks are needed. Since magnetic flux and charge (=electric flux) are complementary variables, the residual phase at time zero between two charge states can be measured locally by measuring a magnetic flux. Similarly, the residual phase between flux states can be measured by measuring charge.

7. **General implementations.** Ignoring practical difficulties one can in principle use any two orthogonal quantum states to implement a qubit. There are, however, some natural restrictions. First, the qubit Hilbert space should be spanned by two eigenstates of the system Hamiltonian, since otherwise the qubit would leave that space at later times. Preferably these two states should be degenerate, but if synchronized clocks are available then non-degenerate states can be used as well. If degenerate energy eigenstates are used, then the two basis states can be chosen to be eigenstates of a Hermitian operator that commutes with the Hamiltonian. In the examples above this operator was always an angular momentum operator, exploiting the rotational symmetry of the system. However, precisely this symmetry makes it necessary to share a standard defining direction. In general, the operator commuting with the Hamiltonian corresponds to a symmetry and to “break” this symmetry a reference is necessary. For example, another choice, at least in principle, could be to choose eigenstates of the momentum operator (corresponding to invariance under translations in space) as basic qubit states. The relative phase between two momentum states may then be defined in terms of absolute position measurements, so that Alice and Bob would need to know (at the very least) their mutual distance. Of course, proper
momentum eigenstates are not localized, so that strictly speaking they cannot even be used locally by Alice and Bob anyway.

8. Relation to Quantum Clock Synchronization These considerations indicate why quantum clock synchronization based on entanglement (see [7]) is not possible. Synchronized clocks are needed to establish an entangled state based on nondegenerate states in the first place. Also note that Alice and Bob cannot synchronize their clocks by using an entangled state based on degenerate angular momentum eigenstates (for which they only need to use the fixed stars) and only later performing local operations to lift that degeneracy. The latter operations obviously will introduce only phase differences that depend on the durations of local operations but not on global time differences. Finally, note that nonlocal entanglement between different types of systems (such as a spin-1/2 particle and a one-dimensional harmonic oscillator) can be obtained only if both standards corresponding to these representations are present.

3 Final remarks

Every quantum communication protocol implicitly assumes that at all times there is a well-defined isomorphism between each of the Hilbert spaces associated with the quantum systems involved in the protocol, even if those systems belong to different parties in different locations. In practice establishing or identifying this isomorphism may not be trivial. Certain protocols, such as quantum key distribution protocols, involve quantum channels through which the parties are allowed to send unlimited numbers of qubits. In such a case, one party can agree to send the other party a continuous stream of qubits prepared in a predetermined sequence of different pure states. The isomorphism may then be operationally defined and checked during the entire protocol by comparing the outcomes of appropriate measurements performed by the second party against the predetermined sequence.

For quantum teleportation, however, the use of such a channel is explicitly forbidden, and the isomorphism must be established in an independent way. We argued here that this is only possible if the parties share, for the entire duration of the protocol, an appropriate (classical) resource. The nature of this resource depends on the physical representation of the qubit. If different eigenstates of the system’s Hamiltonian are used as qubit basis states, then synchronized clocks are necessary. When degenerate eigenstates are used that are at the same time eigenstates of an operator commuting with the Hamiltonian (corresponding to a symmetry), then a shared classical resource breaking that symmetry is required. Most relevant physical implementations exploit rotational symmetry, and in those cases a resource defining spatial direction is necessary.
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References