Electroweak Baryogenesis in Supersymmetric Variants

Michael G. Schmidt
Institut für Theoretische Physik
Universität Heidelberg
Philosophenweg 16, D-69120 Heidelberg, Germany

We argue that the creation of a baryon asymmetry in the early universe is an intriguing case where several aspects of “Beyond” physics are needed. We then concentrate on baryogenesis in a strong first-order phase transition and discuss that supersymmetric variants of the electroweak theory (MSSM and some version of NMSSM) rather naturally provide the necessary ingredients. The charginos and the stops play a prominent role. We present CP-violating dispersion relations in the chargino sector and show results of a concrete model calculation for the asymmetry production based on quasi-classical Boltzmann transport equations and sphaleron transitions in the hot electroweak phase.

1. Introduction

The great progress in elementary particle physics in the last decades was based on high energy accelerator experiments and on the detection that all the newly found particles and their interactions can be described by a rather simple set of gauge theories – the Standard Model (SM) of elementary particle physics.

The Big Bang theory of cosmology predicts very high temperatures in the early universe. The elementary particles – their spectrum and interactions – become essential, and such processes in the hot plasma can be calculated in the SM. This led to a detailed picture (“the first three minutes”) one could only dream of not so long ago. Still there are some fundamental problems of standard cosmology related to initial conditions: starting the history of the universe with an epoch of exponential growth today is the only convincing solution – in particular since it also generates the right type of fluctuations seen in the background radiation and needed for galaxy formation. There is also the finding that a large amount of dark matter not present in the SM is needed to stabilize gravitationally the observed structures and to explain the timescale of galaxy formation. This requires physics beyond the SM.

In spite of the success of the SM there are also good reasons inside elementary particle physics to require extensions of the SM. We mention the problem of scales and the Grand Unification including gravity. Such “Beyond the SM” models may be very severely restricted by the requirement of a reasonable cosmology. The explosion in astrophysical observations based on telescopes, satellites, and fully computerized evaluation of a huge material made this point more and more relevant.

Up to now there is no direct evidence for “Beyond the SM” physics and thus we have to cultivate theoretical prejudices, building models, and to ask for observable consequences in accelerator experiments but also in astrophysics. Supersymmetry [1] is the most attractive theoretical idea in the last decades which has the chance to be tested in forthcoming experiments. It is de facto also a very natural ingredient of string theory – the modern version of a grand unification including gravity. Supersymmetry is theoretically appealing because it just slightly extends our ideas about geometry. It is also attractive phenomenologically: the chiral superfields combine quarks and leptons with interesting scalar fields, in particular the strongly coupled stops, and lead to a rich Higgs/Higgsino/gaugino spectrum. There are convincing dark matter candidates. It produces naturally potentials with flat directions and...
– not to forget the oldest pro-argument – the stabilization of a chosen fine-tuned scale. A light Higgs (114 GeV?) would be strongly in favor of supersymmetry. Unfortunately the choice of supersymmetric models is not unique and the present discussion – different from the early days of SUSY [2] – is mostly on the minimal (MSSM) version because even there the new parameters related to SUSY breaking are hard to fix.

Baryogenesis, the generation of an asymmetry between baryons and antibaryons in the early universe, is a very important chapter of cosmology. Even though this is a tiny effect \( \frac{n_B}{n_{\gamma}} \sim 10^{-10} \) the “tiny” rest of baryons left after pair-annihilation constitutes our world! Following Sakharov [3], it requires a few highly nontrivial effects to combine to produce such an asymmetry. (i) Baryon number B violation is very natural in grand unified models, but it also happens in the electroweak gauge theory via instantons or in high temperature physics via sphaleron thermal transitions where, however, (B-L) number is conserved. At very high temperatures the Higgs field does not have a quasi-classical vev, and this B+L-violating transition is unsuppressed. It can be estimated very accurately [4]. (ii) CP has to be violated. In the SM we have CP-violation in the KM-matrix, but this is a very small effect. In “Beyond” models like the MSSM there are more possibilities, but the size of such effects is still under discussion [5,14]. (iii) One needs nonequilibrium. In the very early universe at the (GUT/inflation) scale the expansion of the universe provides strong nonequilibrium, but later on at the electroweak scale \( T \sim 100 \) GeV one presumably needs a strong first-order phase transition (PT) which we know e. g. from the condensation of vapor.

Such a PT between a symmetric phase and a “Higgs” phase, where the Higgs field gets its quasi-classical vev, is naively predicted at the electroweak scale in the early universe if one uses thermal perturbation theory with a positive Higgs thermal (mass)\(^2\) (“hard thermal loops”). Thus the (Sakharov) necessary conditions for baryogenesis could be fulfilled in the SM [6]. If a GUT interaction conserves B-L, no B-L would be created during pre/reheating after inflation, and

\[ (H^*_1 H_2)^{3/2} \]

Figure 1. \( (H^*_1 H_2)^{3/2} \)-contribution of the stop in one loop order.

B+L would be washed out in the (quasi) equilibrium period before the electroweak PT. The last chance to create an asymmetry would then be during this PT. However, in the last few years it became clear [7] that there is no PT at all for the SM with a Higgs mass \( m_H \) larger than the \( W \)-mass and thus in the experimentally not excluded range above \( \sim 110 \) GeV.

One then has to return to other sources of a baryon asymmetry. In particular “leptogenesis” [8] in SO(10) models with a B-L-violating interaction is rather popular because it allows for some (loose) connection to neutrino-mass generation. But one can also discuss other directions, e.g. Affleck-Dine condensate instability and Q-Balls [9] (squark droplets), or electogenesis [10] with an out-of equilibrium right-handed electron. Staying with the electroweak PT one has to discuss non-standard variants – and this we will do in the following. Quite generally we can conclude that any attempt to explain the baryon asymmetry requires non-standard model ingredients. Thus baryogenesis is a very interesting laboratory for such models.

2. Supersymmetric Modifications of the SM and the Electroweak Phase Transition

In order to increase the strength of the electroweak PT, one has to provide a larger \( \varphi^3 \) term in the effective high temperature Higgs ac-
Figure 2. Sketch of our procedure to determine the weak scale parameters from the GUT parameters.

(I) In the MSSM a “light” stop, superpartner of the r.h. top, with a big Yukawa coupling to a Higgs gives such a $-\langle \varphi^* \varphi \rangle^{3/2}$ potential (fig. 1). “Light” means that its (mass)$^2$ in the symmetric phase given by

$$m_3^2 = m_{3B}^2 + m_T^2$$

where $m_{3B}^2$ is the SUSY breaking scalar (mass)$^2$ and $m_T^2 \sim (gT)^2$ the thermal mass, is small – this is for negative $m_{3B}^2$! At zero temperature one has $m_{stop}^2 = m_{3B}^2 + m_{top}^2$, i.e. its mass is below that of the top. One can discuss the Higgs/stop system perturbatively [11,12] and on the lattice [13] starting from a 3-dimensional effective action obtained by integrating out the massive and non-\(\beta\) parameters.

(II) In supersymmetric models including a gauge singlet superfield (“next to minimal”, NMSSM) a “$\varphi^3$”-type term is already present on the tree level. We consider a superpotential [20,17,18]

$$W = \lambda SH_1 H_2 + \frac{k}{3} S^3 + \mu H_1 H_2 + rS$$

with soft SUSY breaking scalar masses, gaugino masses and with A-terms

$$\mathcal{L}_A = \lambda A_3 SH_1 H_2 + \frac{k}{3} A_4 S^3$$

The first term in (3) is the term $\lambda^* S^* H_1 H_2^*$ from (2) e.g. are such “$\varphi^3$”-terms if the potential leads to vevs of the singlet $< S >$ comparable in size with $< H_1, H_2 >$, the electroweak scale.

The superpotential (2) is not $Z_3$-symmetric any more like its first two terms. Thus there are no problems with domain walls [19] in case this $Z_3$ is spontaneously broken. We also can find easily parameter sets where the particle spectrum is not in contradiction with experiments and the size of the singlet $S$ field is of the order of the electroweak scale in the relevant region of the potential in $S, H_1, H_2$. In the original version [21] one always obtains $< S > \gg < H_1, H_2 >$. However, we had to introduce a $\mu$-term again and
thus we have finetuning problems as in the MSSM [22]. (Indeed to avoid these was one of the reasons to introduce a singlet!). Besides this there is the danger of quadratically exploding singlet tadpoles [23,24]. Such tadpole diagrams require three ingredients: (i) a singlet field, (ii) renormalizable interactions, (iii) soft SUSY breaking terms. One can try to forbid the dangerous non-renormalizable operators, e.g. in models [24,25] with gauged $R$-symmetry or duality symmetry, both broken at some superheavy scale. This of course still leaves us with the $\mu$-finetuning problem. Another way to evade the tadpole divergencies restricts the SUSY breaking terms thus avoiding destabilization via the tadpole: Gauge-mediated SUSY breaking (GMSB) in the context of singlet models does not have domain wall problems. A $\mu$-parameter is generated by radiative corrections and the singlet vev [26]. However, one of the properties of GMSB models seems to be the strong suppression of $A$-terms, important for the creation of a $\phi^3$-term as we just argued before.

The model (2), (3) different from elegant models in the old days [2] contains quite a few parameters even if one introduces universal SUSY breaking scalar $m^2_0$ and gaugino masses $M_0$ at the GUT scale. One can connect the parameters at the GUT and the electroweak scale via a set of renormalization-group equations. Indeed we found [17,18] a way to select cases where $M_z, \tan \beta$ and $<S>$ are fixed from the outset. The remaining parameters are $\lambda, k$ in (2) and universal $m^2_0, M_0, A_0$ (fig. 2). The parameter set is restricted by the postulate of a stable electroweak minimum and by an experimental restriction to chargino masses $\geq 90$ GeV. We then found a strong first-order PT ($v(T_{crit.})/T_{crit.} \geq 1$) for light CP even Higgs masses as large as 115 GeV (and even higher) (fig. 3).

In both models I and II CP-violation is much less restricted than in the SM. In the MSSM one can have explicit CP-violating phases in the $\mu$ and $A_{top}$ parameter [5,14], in the NMSSM there are even more possibilities. They are restricted by the experimental bound on the neutron electric dipole moment. In the NMSSM [27] – not in the MSSM [27,28]– we found also the very appealing possibility of a spontaneous CP violation in the $H_1, H_2, S$ system considering the effective field equations at the high temperature of the PT in the phase transition region (“bubble wall”). This CP violation can be strong without being limited by experimental bounds.

If there is a first-order PT, one can discuss analytically and numerically the shape of the critical bubble, calculate the transition probability and the nucleation temperature (“one bubble/universe”). The Higgs phase bubble expands, and due to the friction of the hot plasma, it approaches a stationary expansion with a “wall” velocity $v_w$. This is the most interesting period where the baryon asymmetry is supposed to be generated: There is a strong baryon number violation due to the “hot” sphaleron transition in the symmetric phase in front of the bubble wall; there is CP violation in the bubble wall region and there is strong equilibrium due to the wall sweeping through the hot plasma and changing the masses of many particles since it is a Higgs field configuration. Of course these are just necessary conditions which are well fulfilled: They have to be bundled into a concrete scenario of baryon asymmetry formation.

3. Baryogenesis

We consider the following model [18,28,30–33] for the generation of a baryon asymmetry (fig. 4): The bubble wall of the first-order PT proceeds with stationary velocity $v_w$; the particles in the hot plasma which is in thermal equilib-
rium interact with the Higgs fields $H_{1,2}(z)$ of the wall, where $z$ is the direction perpendicular to the wall, i.e. they change mass, and thus nonequilibrium is created. In case of CP violation, particles and their CP conjugate antiparticles have different dispersion relations. This creates an asymmetry – still not the baryon asymmetry – which is transported in the region in front of the bubble wall where the interactions in the hot plasma transform this asymmetry e.g. of charginos and anticharginos into an asymmetry between left-handed quarks and their antiparticles. The latter then creates a baryon asymmetry through thermal sphaleron transitions in the hot symmetric phase. These transitions are not in equilibrium, otherwise the asymmetry would be reduced again. After some time the Higgs-phase behind the bubble wall takes over. The usual Higgs-phase sphaleron should be ineffective now in order to freeze out the baryon asymmetry just generated. This is why we need a strong first-order PT to freeze out the baryon asymmetry just generated. After mixing the lightest charginos with mass matrix

$$(i\tilde{W}^-, \tilde{h}_1^-)\left( \begin{array}{c} M_2 \\ g_2(H_1^0)^* \end{array} \right) \begin{array}{c} g_2(H_2^0)^* + \lambda S \\ \mu + \lambda S \end{array} \left( i\tilde{W}^+ \right) \tilde{h}_2^+ )$$

is most important. After mixing the lightest charginos with mass $m(z)e^{\theta(z)}$ and their CP conjugates we have up to order $\hbar$ dispersion relations [18,31]

$$E = (\tilde{p}^2 + m^2(z))^{1/2}$$

$$\pm \frac{1}{2} \left( \theta'(z) + \delta'(z) \sin^2 b - \gamma' \sin^2 a \right) m^2 / (\tilde{p}^2 + m^2)$$

Here the prime mass is the derivative perpendicular to the wall, $\delta$ and $\gamma$ are phases appearing in the diagonalization matrices, $\theta$ is the phase of the diagonalized Dirac mass and $\sin^2 a, \sin^2 b$ contain the parameters of the chargino system (4). $\tilde{p}$ in (5) is the kinetic momentum related to the group velocity $\frac{\partial E}{\partial \tilde{p}_{can}}$. It differs from the canonical momentum $\tilde{p}_{can}$ because of the CP-violating terms. This is the physical momentum which should enter quasi-classical transport equations [31]. (5) is symmetric under the exchange of $H_1$ and $H_2$. Because of the derivatives $\theta', \gamma', \delta'$ CP violation only becomes relevant in the wall region. The particles with such quasiclassical dispersion relations – here we mention the charginos and their superpartners and the particles they mainly interact with (stop/top) – are now used in classical Boltzmann transport equations. The light quarks only enter via a strong $(SU(3)_c)$ sphaleron interaction. The derivative terms in (5) then constitute the only source terms in the Boltzmann equations for the difference between particles and their CP conjugates needed for the asymmetry. The time derivatives and $z$-derivatives in the Boltzmann equations can be substituted by a $\tilde{z} = z - \nu_w t$ derivative in the stationary case.

The equations are solved in the fluid approximation (for bosons/fermions)

$$f_\mu(x, \tilde{p}, t) = 1 / \{ e^{\beta(E_i - \nu_i \tilde{p}_z - \mu_i)} + 1 \}$$

where as mentioned before we have included as small perturbations from the equilibrium distribution only the chemical potentials $\mu_i$ and (for consistency) flux velocities in the plasma. We expand in these small quantities (linear response) as well as in the wall velocity to first order. Taking momenta of the transport equations and eliminating $\nu_i$, one arrives at a set of diffusion equations which can be solved partly analytically, partly numerically.

If one writes Boltzmann equations just for particles [34,35], the main source term is related to the $z$-dependent mass due to the wall, and CP-violating effects are not important. A balance between the pressure of the bubble and the friction due to the plasma leads to a self-consistent wall velocity $\nu_w$. Indeed, in our recent evaluation it turns out that the stop plays an impor-
Figure 5. Wall velocity in dependence on the parameter $\tan \beta (T = 0)$ for $m_Q = 2\text{TeV}$, $A_t = \mu = 0$, and $m_A = 400\text{GeV}$ for $m_{\tilde{T}}^2 = m_{\tilde{B}}^2 = -60^2, 0, 60^2\text{GeV}^2$. Lower bunch of graphs for $\delta' = 0$, upper for $\delta' \neq 0$ (exact linear response), see ref. [35].

A significant role in reducing the wall velocity compared to the SM (fig. 5) to values of the order 0.01. This helps technically in the expansion, but more importantly it turns out to strengthen the final baryon asymmetry, as we will see now.

The final baryon asymmetry is obtained as [31, 18]

$$\eta_B = n_{B/S} = \frac{1.35\Gamma_{WS}}{2\pi^2 g^* v_W T} \int_0^\infty d\bar{z} \mu_{BL}(\bar{z}) e^{-\nu \bar{z}} \quad (7)$$

with $\nu = \frac{3}{2} \Gamma_{WS}/2v_W$, and where $\Gamma_{WS}$ is the electroweak sphaleron rate in the hot phase, $g^*$ the effective number of degrees of freedom; the integration is over the $z$-region in front of the wall and $\mu_{BL}$ is the chemical potential for left-handed baryonic matter minus its CP conjugate.

In fig. (6) we present a typical result [33] for the baryon asymmetry in the MSSM due to the symmetric combination of Higgsino sources – only this one is nonvanishing using the dispersion relations for quasiclassical particles with kinetic momentum. (The antisymmetric combinations proportional to $(\tan \beta)'$ as well as stop sources vanish in this approach. They are, however, present in the diffusion equations obtained in the spirit of quantum-Boltzmann equations [32,15] or if one uses canonical momenta in the WKB approach [18].) Thus one needs rather strong CP violation – fig. (6) is for the case of maximal CP violation – in order to get close to the observed baryon asymmetry. In the NMSSM such a strong CP violation naturally appears in the singlet sector due to “transitional” spontaneous CP violation.

4. Summary

Baryogenesis is a very challenging problem at the borderline between cosmology and particle physics because of its highly nontrivial necessary ingredient. Independent of concrete models its explanation will always involve physics beyond the SM! We have discussed supersymmetric models leading to a strong first-order PT. For this the MSSM requires a Higgs mass below 110 GeV-115 GeV (depending on its parameters) and thus is at the borderline to be ruled out by experiments, the NMSSM in the version presented here is more flexible, but again it is testable in accelerator experiments; like in all SUSY models one needs a rather lowlying Higgs. We consider this to be an advantage compared to models in the far away GUT region.

Acknowledgement

I would like to thank S. Huber and P. John for enjoyable collaboration on the subjects of this talk and N. Polonski and S. Weinstock for discussions.

REFERENCES

1. J. Wess, J. Bagger, “Supersymmetry and Supergravity” (Princeton University Press) and refs. therein. As I recollect, this is the text from which most of us learnt about supersymmetry.


Figure 6. $H_1 + H_2$ contributions to the baryon asymmetry dependent on $v_w$ for different values of the wall thickness $L_w = 20/T, 15/T, 10/T$ (from below) and $|\mu| = |M_2| = 150 \text{ GeV}$, $\arg(\mu M_2) = \pi/2$ and $\delta \beta = 0.01$. $\eta$ is given in units of $2 \times 10^{-11}$ (observational bound).


\[
\begin{array}{c|c}
M_{\text{GUT}} & M_Z \\
\begin{array}{c}
y_t^0 \\
\lambda_0 \\
k_0
\end{array} & \begin{array}{c}
y_t \\
\lambda \\
k
\end{array} \\
M_0 & m^2_{H_2}, A_\lambda \\
\lambda_0 & m^2_{H_2}, A_k \\
m^2_0 & m^2_S \\
\begin{array}{c}
\mu_0 \\
r_0 \\
B_0
\end{array} & \begin{array}{c}
\mu \\
r \\
B
\end{array} \\
\n\n\end{array}
\]
\[\nabla V = 0 \quad \text{tan}\beta \leftrightarrow y_t \\
M_Z \quad <S>
\]
$x = -200 \text{ GeV}$
$tan \beta = -10$
$k = 0.5$
$\lambda = 0.05$
$m_0 = 200 \text{ GeV}$