Quantum teleportation, first proposed by Bennett et al. [1], is a disembodied transport of quantum states between subsystems through a classical communication channel requiring a shared entangled state. Several experiments have been implemented to demonstrate the teleportation [2]. Most of the studies have been confined to the teleportation of single-body quantum states: quantum teleportation of two-level states [1], N-dimensional states [3], and continuous variables [4]. Recently Lee and Kim considered the teleportation of bipartite entangled states through noisy quantum channels [5]. Imoto [9] to copy pure entangled states was studied by Koashi and Shi et al. [10] and Shi et al. [7] proposed schemes for the quantum teleportation of a two-qubit entangled state. Teleportation of some pure entangled states of both discrete and continuous variables is considered using a multipartite quantum channel by Gorbachev et al. [8]. A possibility to copy pure entangled states was studied by Koashi and Imoto [9].

In the teleportation schemes we need certain types of maximally entangled states. We consider the following entangled coherent state (ECS) [10]

$$|\alpha;\alpha\rangle_{12}^+ = \frac{1}{\sqrt{2(1 + e^{-4|\alpha|^2})}} (|\alpha\rangle_1|\alpha\rangle_2 + |\alpha\rangle_2|\alpha\rangle_1),$$

(1)

It is interesting to see that the state $|\alpha;\alpha\rangle_{12}^+$ is a maximally entangled state (MES), irrespective of the parameter $\alpha$ [11]. Here $|\alpha\rangle_i$ ($i = 1, 2$) are coherent states of system $i$. The ECS can be rewritten as the form

$$|\alpha;\alpha\rangle_{12}^- = \frac{1}{\sqrt{2}} (|\alpha\rangle_1^+|\alpha\rangle_2^- + |\alpha\rangle_1^-|\alpha\rangle_2^+)$$

(2)

in terms of the even and odd coherent states

$$|\alpha\rangle_i^\pm = \frac{1}{\sqrt{2(1 - e^\pm2|\alpha|^2})} (|\alpha\rangle_i \pm |\alpha\rangle_i).$$

(3)

Eq.(2) shows that the state $|\alpha;\alpha\rangle_{12}^-$ manifestly has one unit of entanglement. In the limit $\alpha \to 0$, the ECS reduces to the singlet-like state

$$|\Psi^\pm\rangle_{12} = \frac{1}{\sqrt{2}} (|0\rangle_1|1\rangle_2 \pm |1\rangle_1|0\rangle_2),$$

(4)

where $|0\rangle_i$ and $|1\rangle_i$ are photon number states.

van Enk and Hirota [11] have discussed how to teleport a Schrödinger cat state of the form

$$|\alpha\rangle_{\text{cat}} = N(e_+|\alpha\rangle + e_-|\alpha\rangle),$$

$$N = \left[|e_+|^2 + |e_-|^2 + 2e^{-2|\alpha|^2} \text{Re}(e_+e_-^*)\right]^{-1/2}$$

(5)

through a quantum channel described by the MES $|\alpha;\alpha\rangle_{12}^-$. Inspired by their teleportation scheme, in this paper, we consider the teleportation of the following ECS

$$|\Psi\rangle_{12} = N_{\Phi}(e_+|\alpha\rangle_1|\alpha\rangle_2 + e_-|\alpha\rangle_1 - |\alpha\rangle_1|\alpha\rangle_2),$$

$$N_{\Phi} = \left[|e_+|^2 + |e_-|^2 + 2e^{-4|\alpha|^2} \text{Re}(e_+e_-^*)\right]^{-1/2}$$

(6)

In the teleportation of entangled states, particularly two-qubit pure states, one can use two EPR pairs, a four-particle quantum channel, or a less expensive three-qubit GHZ state [8]. If we want to teleport the ECS $|\Psi\rangle_{12}$, at least we need a tripartite entangled state as a quantum channel. In a recent paper [12], we have given a tripartite entangled state as

$$|\sqrt{2}\alpha;\alpha\rangle_{345}^- = \frac{1}{\sqrt{2(1 - e^{-8|\alpha|^2})}} (|\sqrt{2}\alpha\rangle_3|\alpha\rangle_4|\alpha\rangle_5 - |\alpha\rangle_3|\alpha\rangle_4 - |\alpha\rangle_5),$$

(7)

where $|0\rangle_i$ and $|1\rangle_i$ are photon number states.

It is a MES in the sense that the amount of entanglement between system 3 and systems 4,5 is exactly one ebit. The bipartite entanglement can be characterized by one measure of entanglement, the concurrence [13]. The concurrence between system 3 and systems 4,5 is denoted by $C_{3(45)}$, and similarly those between system 4 (5) and systems 3,5 (3,4) are denoted by $C_{4(35)}$ and $C_{5(34)}$, They are obtained as [12]

$$C_{3(45)} = 1,$$

$$C_{4(35)} = C_{5(34)} = \frac{\sqrt{1 - e^{-4|\alpha|^2}(1 - e^{-12|\alpha|^2})}}{1 - e^{-8|\alpha|^2}},$$

(8)

We see that the entanglement between system 4 (5) and systems 3,5 (3,4) is not maximally entangled except the limit case $|\alpha| \to \infty$, while the system 3 with systems 4,5...
is then given by systems 1, 2 and 3 are at Alice's side and systems 4 and 5 are separated from the remaining systems. Now the system 1 is isolated from the remaining systems. This transformation plays an important role in our teleportation scheme. Now Alice wish to teleport the ECS $|\Phi\rangle_{12}$ to the remote partner Bob. They share the MES $|\sqrt{2\alpha};\alpha\rangle_{345}^{-}$. The systems 1, 2, and 3 are at Alice's side and systems 4 and 5 are at Bob's side. The initial state of the whole system is then given by

$$|\Psi\rangle_{12345} = |\Phi\rangle_{12}|\sqrt{2\alpha};\alpha\rangle_{345}^{-}$$

(11)

We first apply the transformation $B_{21} = P_{1}B_{21}P_{1}$ to the initial state. From Eq. (10), the state after the transformation becomes a direct product of the vacuum state $|0\rangle_{1}$ with the unnormalized state

$$|\Psi'\rangle_{2345} = \epsilon_{+}(|\sqrt{2\alpha}_{2}|\sqrt{2\alpha}_{3}|\alpha\rangle_{4} \otimes |\alpha\rangle_{5}$$

- $|\sqrt{2\alpha}_{2}| - \sqrt{2\alpha}_{3}| - \alpha\rangle_{4} \otimes | - \alpha\rangle_{5}$

+ $\epsilon_{-}(| - \sqrt{2\alpha}_{2}|\sqrt{2\alpha}_{3}|\alpha\rangle_{4} \otimes |\alpha\rangle_{5}$

- $| - \sqrt{2\alpha}_{2}| - \sqrt{2\alpha}_{3}| - \alpha\rangle_{4} \otimes | - \alpha\rangle_{5}.$

(12)

Now the system 1 is separated from the remaining systems. Then apply the transformation $B_{23}$, we obtain

$$|\Psi''\rangle_{2345} = B_{23}|\Psi'\rangle_{2345}$$

$$= \epsilon_{+}(2\alpha_{2}|0\rangle_{3}|\alpha\rangle_{4} \otimes |\alpha\rangle_{5}$$

- $|0\rangle_{2}|2\alpha_{3}| - \alpha\rangle_{4} \otimes | - \alpha\rangle_{5}$

- $\epsilon_{-}(2\alpha_{2}| - \alpha\rangle_{3}| - \alpha\rangle_{4} \otimes | - \alpha\rangle_{5}$

- $|0\rangle_{2}|2\alpha_{3}| - \alpha\rangle_{4} \otimes |\alpha\rangle_{5}.$

(13)

After these two transformations, Alice performs a two-mode number measurement on the mode 2 and 3. The probability of finding $n$ and $m$ photons in mode 2 and 3 is given by

$$P(n, m) = |2(n|m)|\Psi''\rangle_{2345}^{2}. \quad (14)$$

The probability is zero if both $n$ and $m$ are not zero, i.e., one of the two integers must be zero in order to have nonzero probability. Let us suppose $n \neq 0$ and $m = 0$. In this case the state on Bob's side collapses into

$$|\Phi'\rangle_{45} = N_{\Phi}(|\epsilon_{+}|\alpha\rangle_{4}|\alpha\rangle_{5} - \epsilon_{-}(-1)^{n}| - \alpha\rangle_{4} | - \alpha\rangle_{5},$$

$$N_{\Phi} = \left[|\epsilon_{+}|^{2} + |\epsilon_{-}|^{2} - 2e^{-4|\alpha|^{2}}(-1)^{n}\Re(\epsilon_{+}\epsilon_{-}^{*})\right]^{-1/2}$$

(15)

For the case $n = 0$ and $m \neq 0$, the state on Bob's side collapses into

$$|\Phi''\rangle_{45} = N_{\Phi'}(|\epsilon_{+}|\epsilon_{-}|\alpha\rangle_{4} - \epsilon_{-}(-1)^{m}|\alpha\rangle_{4}|\alpha\rangle_{5},$$

$$N_{\Phi'} = \left[|\epsilon_{+}|^{2} + |\epsilon_{-}|^{2} - 2e^{-4|\alpha|^{2}}(-1)^{m}\Re(\epsilon_{+}\epsilon_{-}^{*})\right]^{-1/2}$$

(16)

Now Alice sends a classical information to Bob and Bob makes a local transformation $(-1)^{n}a_{4}^{+}a_{4}a_{5}^{+}a_{5}$ on his state $|\Phi''\rangle_{45}$. The local transformation is a multiplication of two $\pi$ phase shifters of mode 4 and 5 and the resultant state after the transformation is just $|\Phi'\rangle_{45}$. We see that provided $n$ is odd, the teleportation scheme works perfectly. However for even $n$, the transformation for perfect teleportation is

$$|\alpha\rangle_{4}|\alpha\rangle_{5} \rightarrow |\alpha\rangle_{4}|\alpha\rangle_{5},$$

$$| - \alpha\rangle_{4} | - \alpha\rangle_{5} \rightarrow | - \alpha\rangle_{4} | - \alpha\rangle_{5},$$

(17)

which in general not a unitary transformation except the limit case $|\alpha| = 0$. From Eqs. (13) and (14), the probabilities $P(0, n)$ and $P(n, 0)$ for odd $n$ are obtained as

$$P(0, n) = P(n, 0) = \frac{e^{-4|\alpha|^{2}}|2\alpha|^{2n}}{2n!(1 - e^{-8|\alpha|^{2}})}.$$ 

(18)

Then the probability of success is given by

$$P_{\text{odd}} = \sum_{n=0}^{\infty} e^{-4|\alpha|^{2}}|2\alpha|^{2n} = \frac{1}{2}. \quad (19)$$

Here we have used a simple fact $\sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$. Note that the successful probability is independent on both $\alpha$ and $\epsilon_{\pm}$. The teleportation scheme we have proposed is not optimal. However indeed it gives the nonzero probability $1/2$ independent of $\alpha$.

There is another problem left that if we can produce the tripartite maximally entangled coherent state which plays the role of the quantum channel. If we can not, the scheme does not work. Fortunately we can create this MES in an easy way. We first prepare the systems 3, 4, and 5 in the state $|2\alpha_{3}|0\rangle_{4}|0\rangle_{5}$. Then apply the transformation $B_{45}B_{34}$, we obtain

$$B_{45}B_{34}|2\alpha_{3}|0\rangle_{4}|0\rangle_{5}$$

$$= B_{45}(|\sqrt{2\alpha}_{3}|0\rangle_{4}|0\rangle_{5}$$

2
The perfect teleportation is obtained for even ϵ, which indicates that the state for fixed α.

Bply a transformation generated similarly as the state Bob’s state collapses into n of the state |Φ⟩ = (|√2α⟩3|α⟩4 ⊗ |α⟩5

\[ |-\sqrt{2}a⟩_3| - \sqrt{2}a⟩_4 \otimes |0⟩_5⟩/\sqrt{2(1-e^{-8|\alpha|^2})} \]

\[ = (|\sqrt{2}a⟩_3|α⟩_4 \otimes |α⟩_5 \]

\[ = |-\sqrt{2}a⟩_3| - \alpha⟩_4 \otimes | - \alpha⟩_5⟩/\sqrt{2(1-e^{-8|\alpha|^2})}, \quad (20) \]

which is just the MES |√2α⟩3|α⟩4⟩45 in the teleportation scheme. In a short summary we can let the initial state of the composite system be |Φ⟩123|0⟩4|0⟩5. Then apply a transformation B31B32B33B34 to the initial state of the composite system, and make the two-mode number measurement to implement the teleportation.

We would like to investigate further what the successful probability is if we use a nonmaximally entangled state as a quantum channel in the teleportation scheme. We choose the state as

\[ |\sqrt{2α}⟩_3|α⟩_4⟩_5 = \frac{1}{\sqrt{2(1+e^{-8|\alpha|^2})}}(|\sqrt{2α}⟩_3|α⟩_4 \otimes |α⟩_5 \]

\[ + |-\sqrt{2}a⟩_3| - \alpha⟩_4 \otimes | - \alpha⟩_5⟩, \quad (21) \]

which acts as a quantum channel. The corresponding concurrences are given by [12]

\[ C'_3(45) = \tanh(4|\alpha|^2), \]

\[ C'_4(35) = C'_5(34) = \frac{\sqrt{(1-e^{-4|\alpha|^2})(1-e^{-12|\alpha|^2})}}{1+e^{-8|\alpha|^2}}, \]

which indicates that the state |√2α⟩3|α⟩4⟩45 is not maximally entangled except the limit case |α| → ∞. From Eqs. (8) and (22), we have the inequality

\[ C_3(45) ≥ C'_3(45), \quad C_4(35) ≥ C'_4(35), \quad C_5(34) ≥ C'_5(34), \]

for fixed α. That is to say, the amount of entanglement of the state |√2α⟩3|α⟩4⟩45 is always larger than or equal to that of |√2α⟩3|α⟩4⟩45. Noting the equation

\[ |\sqrt{2α}⟩_3|α⟩_4⟩_5 = B_{45}B_{34}|2α⟩_3⟩_4|0⟩_5⟩, \]

we know that the entangled state |√2α⟩3|α⟩4⟩45 can be generated similarly as the state |√2α⟩3|α⟩4⟩45.

Following the same steps as before, after Alice measures n photons in mode 2 and 0 photons in mode 3, Bob’s state collapses into

\[ |Ψ⟩ = N_{Φ}(c^+|α⟩_4⟩_5 + ε(-1)^n| - \alpha⟩_4 \otimes | - \alpha⟩_5), \]

\[ N_{Φ} = \left[ |ε^+|^2 + |ε^-|^2 + 2e^{-4|\alpha|^2}(-1)^n \text{Re}(ε^+ε^-)^2 \right]^{-1/2} \]

\[ (24) \]

The perfect teleportation is obtained for even n. The successful probability is given by

\[ P_{even} = \sum_{even \ n > 0} P(0, n) + P(n, 0) = \frac{(1-e^{-4|\alpha|^2})^2}{2(1+e^{-8|\alpha|^2})}, \]

which is independent on the parameters ε, however dependent on the parameter |α|. In the limit |α| → ∞, the successful probability becomes 1/2. In this limit case, the amount of entanglement in the state |√2α⟩3|α⟩4⟩45 is one ebit and the probability can be 1/2. Except this case, the successful probability is always less than 1/2 as the corresponding quantum channel is not a MES.

Our teleportation scheme can be generalized to the multipartite cases. For the sake of simplicity we only consider the tripartite case. Consider an unnormalized tripartite ECS,

\[ |ψ⟩_{123} = ϵ_1|√2α⟩_1|α⟩_2|α⟩_3 + ε_2 - |√2α⟩_1| - α⟩_2 | - α⟩_3. \]

\[ (25) \]

To teleport the tripartite state |Ψ⟩_{123}, we may need the four-particle entangled state (unnormalized)

\[ |2α⟩_3|√2α⟩_5|α⟩_6|α⟩_7 - | - 2α⟩_3| - √2α⟩_5| - α⟩_6 | - α⟩_7, \]

\[ (26) \]

which is maximally entangled between the system 4 and systems 5, 6, 7 [12]. First we disentangle the system 3 from systems 1 and 2 by applying the operator B_{31}B_{32} to the state |Ψ⟩_{123}. We obtain

\[ B_{31}B_{32}|Ψ⟩_{123} = |0⟩_1|0⟩_2(ε_1|2α⟩_3 + ε_2 - |2α⟩_3). \]

\[ (27) \]

Then we make the two-mode number measurement on mode 3 and 4 and a classical communication from Alice to Bob to finish the teleportation process. The successful probability is also 1/2. The quantum channel described by the four-particle entangled state can be obtained as

\[ |2α⟩_3|√2α⟩_5|α⟩_6|α⟩_7 - |2α⟩_3|√2α⟩_5|α⟩_6|α⟩_7. \]

\[ (28) \]

It is straightforward to generalize the teleportation scheme to teleport the multipartite (more than three) entangled ECS of the form |Ψ⟩_{123}.

In the teleportation scheme described above the probability of success is 1/2 and independent of α. The it is interesting to consider the limit α → 0. The concurrence C3(45) is equal to one for the tripartite state |√2α⟩3|α⟩4⟩5, irrespective of α. This state can be rewritten as

\[ |ψ⟩_{345} = \frac{1}{\sqrt{2}}(|√2α⟩_3|α⟩_4⟩_5 + |√2α⟩_3|α⟩_4⟩_5), \]

\[ (29) \]

which directly leads to

\[ |ψ⟩_{345} = \lim_{α → 0} |√2α⟩_3|α⟩_4⟩_5 \]

\[ = \frac{1}{\sqrt{2}}(|1⟩_3|0⟩_4⟩_5 + |0⟩_3|Ψ^+⟩_4). \]

\[ (30) \]

Here the state |ψ⟩_{345} is as a quantum channel to teleport the state
Then the initial state of the whole system becomes

\[ |\psi^{'}\rangle_{12345} = |\phi\rangle_{12}|\psi\rangle_{345}. \]  

(32)

It is easy to check that

\[ B_{12}|10\rangle_{12} = |\Psi^{+}\rangle_{12}, \]
\[ B_{12}|01\rangle_{12} = |\Psi^{-}\rangle_{12}, \]

(33)

where \( |\Psi^{-}\rangle_{12} = \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle) \). As the operator \( B_{12} \)

\[ B_{12}|\Psi^{+}\rangle_{12} = |10\rangle_{12}, \]
\[ B_{12}|\Psi^{-}\rangle_{12} = |01\rangle_{12}. \]

(34)

Then applying the operator \( B_{13}B_{12} \) to the initial state, we obtain

\[ B_{13}B_{12}|\psi^{'}\rangle_{12345} = \frac{1}{2\sqrt{|a|^{2} + |b|^{2}}}|0\rangle_{2} \]
\[ \sum_{a}a_{a}^{\dagger}a_{b}^{\dagger}a_{a}a_{b}|0\rangle_{2}, \]
\[ +|10\rangle_{13}(a_{0}a_{45} + b\Psi^{+}_{45}) \]
\[ +|01\rangle_{13}(-a_{0}a_{45} + b\Psi^{+}_{45}) \]
\[ +a_{0}(0)_{13}|\Psi^{+}_{45} \]
\[ +b \sqrt{2}(0)|\Psi^{-}_{13} - |02\rangle_{13})|00\rangle_{45}. \]

(35)

Here we have used Eqs. (33) and (34). We see the system 2 decouples from the other systems. The teleportation scheme works perfectly if the resultant state of the two-mode number measurement is \(|10\rangle_{13}. \) If the resultant state is \(|01\rangle_{13}, \) then Alice needs to communicate classically with Bob and Bob makes a local transformation \((-1)^{a_{a}^{\dagger}a_{b}^{\dagger}a_{a}a_{b}} \) to finish the teleportation. Again the probability of success is 1/2. The similar setup as this teleportation scheme has been proposed to perform optical state truncation [14], optical simulation of quantum logic [15] and the quantum teleportation [16].

In conclusion we have proposed a simple scheme to teleport both the bipartite and multipartite ECS with the successful probability 1/2 independent of \( \alpha. \) As a crucial ingredient in the scheme, the quantum channel described by the multipartite ECS, can be readily made by only linear optical devices such as beam splitters and phase shifters. Thus we provide a way to achieve all linear optical teleportation of quantum states [15,16]. The probability of success in our scheme is 1/2 due to the use of only linear operations and the absence of photon-photon interaction [17]. The measurement we used is the two-mode number measurement. Both the measurement and the preparation of the quantum channel can be implemented in the experiments by the present techniques. We expect that the present scheme can be used in the experiments to demonstrate the quantum teleportation of the entangled states.