BEAMING IN GAMMA-RAY BURSTS:
EVIDENCE FOR A STANDARD ENERGY RESERVOIR

D. A. Frail∗, S. R. Kulkarni† ‡, R. Sari,§ S. G. Djorgovski†, J. S. Bloom†, T. J. Galama†, D. E. Reichart†, E. Berger†, F. A. Harrison†, P. A. Price†¶, S. A. Yost†, A. Diercks†, R. W. Goodrich∥, F. Chaffee∥

∗ National Radio Astronomy Observatory, P.O. Box 0, Socorro, NM 87801, USA
† California Institute of Technology, Palomar Observatory 105-24, Pasadena, CA 91125, USA
‡ Massachusetts Institute of Technology, Physics Dept., Cambridge, MA 02139, USA
§ California Institute of Technology, Theoretical Astrophysics 130-33, Pasadena, CA 91125, USA
¶ School of Astronomy & Astrophysics, Mount Stromlo Observatory, Cotter Road, Weston, ACT, 2611, Australia
∥ W. M. Keck Observatory, Kamuela, HI 96743, USA

Gamma-ray bursts (GRBs) are the most brilliant objects in the Universe but efforts to estimate the total energy released in the explosion – a crucial physical quantity – have been stymied by their unknown geometry: spheres or cones. We report on a comprehensive analysis of GRB afterglows and derive their conical opening angles. We find that the gamma-ray energy release, corrected for geometry, is narrowly clustered around $5 \times 10^{50}$ erg. We draw three conclusions. First, the central engines of GRBs release energies that are comparable to ordinary supernovae, suggesting a connection. Second, the wide variation in fluence and luminosity of GRBs is due entirely to a distribution of opening angles. Third, only a small fraction of GRBs are visible to a given observer and the true GRB rate is at least a factor of 500 times larger than the observed rate.

This paper has been submitted to Nature. You are free to use the results here for the purpose of your research. In accordance with the editorial policy of Nature, we request that you not discuss this result in the popular press. If you have any question or need clarifications please contact Dale Frail, dfrail@nrao.edu or Shri Kulkarni, srk@astro.caltech.edu.
Observations of GRBs are well described by the fireball model, in which an explosive flow of relativistic matter (ejecta) is released from a central source. The collision of fast-moving ejecta with slower moving ejecta result in bursts of gamma rays. Shortly thereafter, the ejecta starts shocking and sweeping up significant amounts of circumburst matter. The shocked gas, hereafter called the blast wave, powers long-lived and broad-band (X-ray, optical and radio) emission – the so-called afterglow emission.

The afterglow emission appears to be primarily synchrotron radiation. As with other astrophysical shocks, the shocked electrons are accelerated to relativistic energies, forming a power-law distribution, \( dN/dE_e \propto E_e^{-p} \); here, \( E_e \) is the energy of the electron and \( p \), the index of the power law. In the presence of magnetic fields, the electrons radiate synchrotron emission with a flux \( f(t, \nu) \propto t^\alpha \nu^\beta \), where the spectral index \( (\beta) \) and the temporal index \( (\alpha) \) are related to \( p \) and the dynamics of the blast wave. Broad-band observations have repeatedly confirmed the expectations of this simple picture.

The two outstanding issues in this field are (a) determining the progenitors of GRBs and (b) understanding the physics of the central engine. The focus of this Article is the latter topic, specifically the energetics of these mysterious sources.

Observationally it is known that the fluence (defined as the received energy per unit area) of the broad-band afterglow phase is always (and usually much) smaller than that of the gamma-ray burst fluence. This then motivates the use of the isotropic equivalent gamma-ray energy, \( E_{\text{iso}}(\gamma) = 4\pi F_\gamma d_L^2 (1+z)^{-1} \) as a surrogate for the energy released by the central engine. Here, \( F_\gamma \) is the fluence of the burst; \( z \) is the redshift; and \( d_L \) is the luminosity distance.

However, \( E_{\text{iso}}(\gamma) \) could grossly overstate the true gamma-ray energy release \( (E_\gamma) \) if the explosion is not spherical. Indeed, jets are present in almost all accretion-driven phenomena, e.g., young stellar objects, neutron star binaries, and quasars. Likewise, there is excellent observational evidence for GRB fireballs with conical geometry. Henceforth, following standard usage, we will interchangeably use the term “jet” for conical blast waves.

As a result of relativistic beaming (“abberation”), an observer can see only a limited portion of the blast wave with angular size \( \sim \Gamma^{-1} \), where \( \Gamma \) is the bulk Lorentz factor of the blast wave. This relativistic beaming implies that there is no observable distinction between a spherically expanding blast wave and a conical blast wave (whose opening angle we denote by \( \theta_j \)) until the blast wave...
has slowed down to $\Gamma < \theta_j^{-1}$. However, gamma-ray emission is expected to occur when $\Gamma$ is large, $\Gamma \gtrsim 100$, and thus unless the opening angles are very small, $\theta_j < 0.01$, a conical GRB will not light up the full celestial sphere, but only the so-called beaming fraction $f_b = (1 - \cos \theta_j)$; we note that for $\theta_j \ll 1$, $f_b \approx \theta_j^2/2$.

There are two consequences of relativistic beaming: (1) the true GRB rate is $f_b^{-1}$ times larger than the observed GRB rate and (2) the true gamma-ray energy released $E_\gamma$ is smaller than $E_{\text{iso}}(\gamma)$ by the same factor, i.e., $E_\gamma = f_b \times E_{\text{iso}}(\gamma)$.

In contrast to the situation during the gamma-ray burst phase, during the afterglow phase $\Gamma$ ranges from $\sim 10$ hours after a burst to trans-relativistic ($\Gamma \sim 1$) and even non-relativistic values days to months after the burst. Thus at some point during the lifetime of the afterglow $\Gamma$ falls below $\theta_j^{-1}$ which has a clear observational signature. Thus multi-wavelength (X-ray, optical and radio) afterglow observations offer us an elegant way to measure $\theta_j$. Here, we report an extensive analysis of $\theta_j$ values for all well-studied GRBs. From these values of $\theta_j$ we are able to infer the true $\gamma$-ray energy release of the central engines of GRBs.

The organization of the paper is as follows. First we summarize the physics of conical afterglows followed by extraction of $\theta_j$ values from afterglow observations. We then find the surprising result that $E_\gamma$ is tightly clustered around $5 \times 10^{50}$ erg. We end with a discussion of the ramifications of this result.

**Conical Afterglows**

The temporal evolution of the afterglow emission is directly related to the dynamics of the blast wave which in turn is influenced both by the circumburst medium and the geometry of the explosion. The afterglow emission from a conical blast wave\(^7\)\(^\text{--}\)\(^11\) differs from that of a spherical blast wave in two distinct ways. First, the observer will start noticing a deficit of emitting material when $\Gamma < \theta_j^{-1}$. The magnitude of this deficit, relative to that of a spherical fireball, is proportional to the ratio of the area of the emitting surface for a conical blast wave ($\propto \theta_j^2$) to that of a spherically emitting blast wave ($\propto \Gamma^{-2}$). This deficit results in the afterglow emission declining more rapidly, relative to a spherical case, or a “break” in the power law decay, $\Delta \alpha = 3/4$.

The second effect that also becomes important when $\Gamma \ll \theta_j^{-1}$ is the spreading of the jet in the lateral dimension.\(^10\)\(^,\)\(^11\) The ejecta now encounter more surrounding matter and decelerate faster than in the spherical case. This results in an overall steepening ($\Delta \alpha = 1.2\text{--}1.5$) of the afterglow.
emission. In the case of a laterally spreading jet, the lightcurve evolves with $\alpha \approx p$. We note $p \sim 2.2 - 2.4$ appears to fit all well-studied afterglows (e.g., ref. 11, 12). This value is also favored by shock acceleration models.\(^{13}\)

The first claim of a jet was made for the radio afterglow of GRB 970508, which showed deviations from the predictions of a simple spherical adiabatic model.\(^8\) However, it was the spectacular isotropic energy release\(^{14}\) of GRB 990123 – approaching the rest mass of a neutron star – which emphasized the possible importance of jets in GRBs. A case for a jet in the afterglow of this burst was made on the basis of a sharp break ($\Delta \alpha \geq 0.7$)\(^{15}\) in the optical afterglow and upper limits in the radio.\(^{16}\) The clearest evidence for a jet is a sharp break over a broad range of frequencies and such a signature was seen in the lightcurves of GRB 990510 at optical\(^{17,18}\) and radio\(^{18}\) wavelengths and was found to be consistent with the X-ray\(^{19}\) light curve. Furthermore, the detection of polarization\(^{20,21}\) from this event gave further credence to the jet hypothesis: the non-spherical geometry leads to polarized signal, from which the geometry of the jet can be inferred.\(^{22,23}\)

More recently, the identification of jets has shifted from single frequency measurements to global model fitting of joint optical, radio and X-ray datasets (e.g., ref. 5, 24). This approach has the advantage that by simultaneously fitting all the data, the final outcome is less sensitive to deviations in small subsets of the data. In addition, since the character of the achromatic break is different above and below the peak of the synchrotron spectrum,\(^{11}\) broad-band measurements give more robust determinations of the jet parameters. This approach was crucial in distinguishing the jet break for GRB 000301C\(^5\) whose decaying lightcurves exhibited unusual variability,\(^{25}\) now attributed to microlensing.\(^{26}\)

**Determination of the Jet Opening Angles**

We use the formulation of Sari, Piran & Halpern\(^{11}\) to convert the measured jet break times $t_j$ to opening angles of the conical blast wave:

$$\theta_j = 0.057 \left( \frac{t_j}{1 \text{ day}} \right)^{3/8} \left( \frac{1 + z}{2} \right)^{-3/8} \left( \frac{E_{\text{iso}}(\gamma)}{10^{53} \text{ erg}} \right)^{-1/8} \left( \frac{\eta_{\gamma}}{0.2} \right)^{1/8} \left( \frac{n}{0.1 \text{ cm}^{-3}} \right)^{1/8}, \quad (1)$$

where $\eta_{\gamma}$ is the efficiency of the fireball in converting the energy in the ejecta into $\gamma$ rays, and $n$ is the mean circumburst density. In Table 1 we present a complete sample of all GRBs with known redshifts as of December 2000. The determinations of $t_j$ are of varying quality. The best events are those for which it is possible to globally model the broad-band data within the physical framework of
the relativistic jet model (e.g., GRB 000301C, GRB 990510). For some bursts \( t_j \) is inferred from only one band (e.g., GRB 990705) and in some cases with additional constraints from radio observations (e.g., GRB 990123). Finally, there are some events with only upper (e.g., GRB 991208) or lower limits on \( t_j \) (e.g., GRB 971214), for which only upper or lower limits of \( \theta_j \) can be placed, respectively.

We obtain a range in \( \theta_j \) corresponding to the wide range in \( t_j \) values in Table 1 (from \( \lesssim 1 \) d to 30 d). The derived jet angles vary from 3° to more than 25° with a strong concentration near 4° (Figure 1). It is reasonable to ask whether the observed distribution in Figure 1 suffers from selection effects. To begin we note that out of the 21 known optical afterglows, the light curves of only two GRBs – GRB 980326 (ref. 27) and GRB 980519 (ref. 28) – show rapid decline implying \( t_j \lesssim 1 \) d. Likewise, out of a sample of 10 bright X-ray afterglows observed with the BeppoSAX satellite there is no evidence for a significant break within 8 to 48 hours after a burst,\(^29\) suggesting that \( t_j \gtrsim 1 \) d for these events. If we increase the sample to include the 28 GRBs detected by BeppoSAX for which follow-up searches (typically 8–12 hr after the burst) were made for an X-ray afterglow we find only one unambiguous case where no afterglow was detected (GRB 990217; ref. 30). There are a further six cases where a hitherto uncataloged X-ray source was detected in the GRB error circle. In every case the X-ray source is a plausible afterglow but lacking multi-wavelength confirmation, the afterglow identification remains uncertain, e.g., GRB 970111, ref. 31. From these statistics we conclude that steep decays, \( t_j \lesssim 1 \) d, and therefore very narrow opening angles, \( \theta_j < 3° \), are required for less than ten percent of the BeppoSAX GRB sample.

There is another method to infer the existence of a population of GRBs with extremely narrow opening angles. The beaming fraction during the afterglow phase is \( \max(\theta_j^2/2, \Gamma^{-2}/2) \). Thus, while narrow-angle GRBs will be rare, their X-ray, optical, and radio afterglows which are emitted at increasingly smaller \( \Gamma \) are accordingly less rare.\(^7\) However the current limits\(^{32,33}\) of these “burst-less” afterglows do not place further significant constraints on \( \theta_j \).

GRBs with large opening angles do not suffer from severe beaming but it is not easy to measure \( t_j \) for such bursts. For large \( t_j \) the afterglow emission is weak and (at optical wavelengths) the host galaxy starts dominating.\(^{34,35}\) Thus optical observations and X-ray are unlikely to yield \( t_j \). Fortunately, radio observations can and do play a crucial role, due to the long lifetime of the afterglow in this regime. This was the case for four out of five wide-angle jets identified in Table 1. One notable
example is GRB 970508 where a jet model\textsuperscript{36} of the radio data was found to be consistent with an analysis of the optical light curves.\textsuperscript{10}

**The Luminosities and Energies of GRB Central Engines**

In Figure 2 we plot the measured fluence versus the inferred inverse beaming factor. There appears to be a correlation in the sense that the bursts with the largest fluence have the narrowest opening angles. This trend was noted earlier\textsuperscript{11} albeit based on a few afterglows. The correlation is improved when the fluences are all scaled to the same redshift (unity), which effectively renders it to a correlation between $E_{\text{iso}}(\gamma)$ and $f_b^{-1}$. The physical meaning of this trend is better appreciated from Figure 3 where we find that $E_\gamma$, the true energy released in gamma rays, is clustered around $5 \times 10^{50}$ erg, with a 1$\sigma$ multiplicative factor of only two.

Figures 2 and 3 suggest the following simple scenario: the central engines of GRBs produce approximately a similar amount of energy, and a significant part, about $10^{51}$ erg, escapes as gamma-rays (Figure 3). However, for reasons not presently understood, there exists a wide range of jet opening angles. If so, GRBs with the narrowest opening angles would be brighter and consequently produce the correlation seen in Figure 2.

The narrowness of the $E_\gamma$ distribution is surprising and has several immediate implications. While it is not unreasonable to expect that the central engines produce a similar amount of energy, $E_0$, in each explosion, there is little reason to expect that they will produce similar gamma-ray outputs. Since the true total energy $E_0 \equiv E_\gamma/\eta_\gamma \propto n^{1/4}\eta_\gamma^{-3/4}$ (this follows from Equation 1), the narrowness in the distribution of $E_\gamma$ places restrictions on the dispersion of $n$ and $\eta_\gamma$.

If $\eta_\gamma$ is high (close to unity) then a small dispersion in $\eta_\gamma$ is naturally assured. Indeed, a number of recent papers\textsuperscript{37–39} have argued that internal shocks under certain conditions are very efficient at producing gamma rays ($\eta_\gamma \gtrsim 0.2$). Furthermore, Guetta et al.\textsuperscript{38} argue that the very conditions that are needed to make internal shocks efficient (a large dispersion in the distribution of the ejecta’s Lorentz factors) also produce the characteristic clustering of spectral break energies of GRBs in the range 0.1–1 MeV.

Given a distribution in $E_\gamma$ with a full width of a factor of four (see Figure 3), the dispersion in $n$ ($\propto E_\gamma^4$) has to be less than two orders of magnitude. At first this may seem to be a weak constraint on the possible progenitors. However, the progenitors discussed to date lie either in intergalactic space or the halos of galaxies (ns-ns coalescence, $n \sim 10^{-6}$ cm$^{-3}$ and $n \sim 10^{-4}$ cm$^{-3}$) or in a typical disk
interstellar medium (ISM) or dense ISM (collapsar; \( n \sim 1 \text{ cm}^{-3} \) and \( n \sim 10^2 \text{ cm}^{-3} \), respectively). Therefore our results limit the diversity of GRB environments, and specifically requires that the long-duration class of GRB events happen in only one of these environments. Furthermore, we note that winds of massive stars would produce a density of a few atoms \( \text{cm}^{-3} \) for \( t_j \gtrsim 1 \text{ d} \). In scenarios where there are two types of GRBs,\(^{40}\) the ones that do not go off in stellar-wind-stratified media must reside within the disk of their host galaxy rather than in galaxy halos or the intergalactic medium. Indeed, the distribution of GRBs within their host galaxies is consistent with a disk population.\(^{41}\) Likewise, broad-band modeling of GRB afterglows\(^{5,24,36}\) give estimates of gas densities consistent with disks, justifying our normalization of \( n \) in Equation 1. We conclude that the progenitors of long duration GRBs likely come from one type of progenitor.

Finally, the narrowness of the \( E_\gamma \) distribution requires that the brightness of the \( \gamma \)-ray beam be roughly uniform from the center to the edge. This is contrary to models\(^{42}\) in which large intensity variations within the conical blast wave are invoked in order to explain the wide dispersion of peak luminosities. We find that most of the dispersion in the luminosity is due to the diversity in opening angles.

The mean value of \( E_\gamma \) is \( 5 \times 10^{50} \text{ erg} \) (Figure 3). If we accept the conclusions of Guetta et al.\(^{38}\) (see above), then \( \eta_\gamma \sim 0.2 \) and we then derive \( E_0 \sim 3 \times 10^{51} \text{ erg} \). Of course, \( E_0 \) is sensitive, in addition to the adopted value of \( \eta_\gamma \), to the overall scaling, i.e., the numerical coefficient of Equation 1. For example, the estimate of Rhoads,\(^{10}\) based on a different assumption for the sideways expansion speed, has a coefficient smaller by a factor of six than our Equation 1.

Fortunately, GRB 970508 allows us to directly determine the energy scale. The radio afterglow of this GRB lasted long enough (400 d) that the blast wave was non-relativistic, thereby allowing determination of the total energy\(^{36}\) independent of relativistic beaming. Table 1 shows that this burst has one of the lowest energies, although it is only \( 1\sigma \) away from the mean (if the energy distribution is assumed to be log normal). The agreement between these two entirely different approaches is remarkably good and gives some support to our choice of the numerical coefficient and normalization of Equation 1.

Freedman & Waxman\(^{12}\) and Kumar\(^{43}\) have suggested an elegant way to estimate the energy in the afterglow phase based on X-ray observations. This method yields the \( \epsilon_\epsilon \epsilon_\alpha \) where \( \epsilon_\alpha \) is the energy of the blast wave per steradian and \( \epsilon_\epsilon \) is the fraction of energy in the shocked electrons. This
estimate is independent of the ambient density. If $\epsilon_e$ is high and relatively constant (analogous to the situation with $\eta_\gamma$) then $E_0$ can be estimated provided $f_b$ is known. It is of interest to note that the ratio $\epsilon_a/E_{iso}(\gamma)$ is nearly constant,$^{12}$ suggesting that likely both $\epsilon_e$ and $\eta_\gamma$ are narrowly distributed.

Applying our determinations of $f_b$ to the sample of ref. 12 (a total of six common GRBs) we obtain $E_a = (2.7 \pm 1.4) \times 10^{50}$ erg. Within the limitations of the small sample, the distribution appears to be clustered and the results are in agreement with our findings (Figure 3). The principal advantage of our method is that the events are always identified in the $\gamma$-ray band, whereas X-ray observations are available for only a minority of cases. Furthermore, the X-ray afterglow technique ignores the effects of inverse Compton scattering effects,$^{44}$ and is therefore sensitive to the poorly known strengths of magnetic fields in strong shocks.

**GRBs and SNe**

Above we find that the mean total energy of GRBs is $E_0 \sim 3 \times 10^{51}$ erg. This energy is only slightly larger than the typical $10^{51}$ erg of electromagnetic and kinetic energy yield of ordinary supernovae (Ia, Ibc, II). This reduced energy budget raises the possibility that GRBs are the result of the formation of neutron stars,$^{45}$ albeit with special properties,$^{46}$ and does not necessarily require black holes. The mystery about GRBs is no longer in understanding their supposedly extraordinary energy budget but in explaining why the ejecta of GRBs have such a high Lorentz factor.

We note however that there are at least two possible exceptions to the tight clustering of jet energy. (1) If SN 1998bw is associated with a GRB$^{47,48}$ then $E_{iso}(\gamma) \sim 7 \times 10^{47}$ erg. However, Kulkarni et al.$^{48}$ have argued that the extraordinarily bright radio emission from this SN requires $\gtrsim 10^{50}$ erg of the explosion energy to be in the form of mildly relativistic ejecta ($\Gamma \sim$ few). (2) Bloom et al.$^{49}$ identify the late time red bump in the rapidly decaying event GRB 980326 ($t_j < 0.55$ d; ref. 27) with an underlying SN. If so, the inferred redshift $z \sim 1$ and $E_\gamma < 7 \times 10^{49}$ erg. Unfortunately, the radio observations are not sensitive enough to place meaningful constraints on the amount of energy in mildly relativistic ejecta. In both cases, the true energy release could be closer to $E_0$ but this energy could be primarily in mildly relativistic ejecta. Careful observations (especially X-ray and radio) of SNe may uncover significant numbers of such “failed” GRBs.

**Beaming Fraction and the GRB Rate**

Since conical fireballs are visible to only a fraction, $f_b$, of observers, the true GRB rate, $R_t = \langle f_b^{-1} \rangle R_{obs}$, where $R_{obs}$ is the observed GRB rate and $\langle f_b^{-1} \rangle$ is the harmonic mean of the
beaming fractions. We find $\langle f_b^{-1} \rangle \sim 500$ (see caption to Figure 1). The formal uncertainty in this estimate is only 16% but systematic uncertainties related our choice of the numerical coefficient and normalization of Equation 1. make this estimate accurate to a factor of two.

Estimates$^{50-52}$ of the local observed rate of GRBs give values of $R_{\text{obs}}(z = 0)$ ranging from 0.2 to 0.7 Gpc$^{-3}$ yr$^{-1}$. The rate is uncertain because it is not known how the GRB rate evolves with redshift. We adopt a value $R_{\text{obs}}(z = 0) = 0.5$ Gpc$^{-3}$ yr$^{-1}$ as in Ref. 51. The true rate is $R_t(z = 0) \sim 250$ Gpc$^{-3}$ yr$^{-1}$, which should be compared with the estimated rate$^{53}$ of neutron star coalescence, $R_c(z = 0) \sim 80$ Gpc$^{-3}$ yr$^{-1}$ and the estimated rate$^{53}$ of type Ibc SN, $R_{\text{Ibc}} \sim 6 \times 10^4$ Gpc$^{-3}$ yr$^{-1}$. Clearly, the collapsar scenario is capable of easily supplying a sufficient number of progenitors (including failed GRBs). Within the uncertainties of the estimates, the coalescence scenario is also (barely) capable of providing sufficient progenitors.

Assumptions, Uncertainties and Caveats

Our derivation of the jet opening angle is based on Equation 1 which makes two implicit assumptions. First, we assume that GRBs explode in a constant density medium and that any sharp break in the afterglow ($\Delta \alpha > 0.75$) is attributed to a combination of the observer viewing beyond the edge of the conical jet and sideways expansion. Second, we assume that the conical blast wave maintains a fixed opening angle right from the GRB phase until $\Gamma$ approaches $\theta_j^{-1}$. The latter assumption imposes strict conditions on the working of the central engine. Specifically, the ejecta have to be approximately uniform across the entire opening angle in the gamma-ray phase and the bulk of the explosive energy in the afterglow phase must have a single bulk Lorentz factor.

The origin of the observed break is currently a matter of considerable theoretical debate.$^{54-56}$ The uncertainty is driven by the as yet unclear hydrodynamics of sideways expansion. Some authors$^{54,57}$ argue that transition is very smooth, and is completed in one decade of the break time for a constant density circumburst medium but takes two decades for the $n(r) \propto r^{-2}$ circumburst medium. Others$^{11}$ argue that several uncertainties in these calculations make this conclusion premature.

Furthermore, the analysis of the afterglow lightcurves does not yield $\phi$, the angle between the line of sight to the observer and the principal axis of the jet. In general one may expect some dependence of $t_j$ on $\phi$ and thus $E_\gamma$ distribution should broaden even if $E_0$, $\eta_\gamma$ and $n$ were constants.

However, the narrowness of the $E_\gamma$ distribution shown in Figure 3 provides empirical support for our assumptions. Furthermore, as noted earlier, where high quality observations are available (e.g.,
GRB 990510 and GRB 000301C) the breaks are found to be quite sharp and the inferred ambient density is $\sim 1 \text{ cm}^{-3}$.

Finally, several other mechanisms have been proposed to produce steep declines in the afterglow light curves: 

(i) a sudden drop in the external density,$^{55}$

(ii) transition from relativistic to non-relativistic regime,$^{58}$

due to expansion in a dense circumburst medium, and

(iii) a break in the power-law distribution of radiating electrons.$^{59}$

These models have not been systematically compared against well studied afterglows and thus remain at a level of suggestions. Model (ii) can be rejected because the expected centimeter wave attenuation due to free-free absorption is not seen. Model (iii) is unable$^{59}$ to explain the broad-band achromatic breaks and will therefore fail to account for the early time low frequency emission. We conclude that at this stage the simple jet model which we have adopted provides a consistent and adequate description of the observations.

Our understanding of gamma-ray bursts has increased dramatically over the past four years. For nearly three decades these objects were considered so enigmatic that expectations of their distance ranged from local to cosmological scales. In the BATSE era, prior to the discovery of the afterglow phenomenon, the standard assumption was that GRBs possessed fixed peak luminosities. As more and more redshifts were obtained, the isotropic equivalent energy record increased, eventually reaching the rest mass energy of neutron stars. The standard candle hypothesis was consequently abandoned. It is remarkable that with a more detailed understanding of the afterglow we are able to infer the energy release in these bursts and find that GRBs are “standard candles” in some sense.

We have deduced the distribution of the opening angles of GRB jets and empirically uncovered a key clue, namely the total energy release and its approximate constancy, but we are still left with three significant mysteries. First, we do not know what physical mechanism results in such a wide variation in the opening angles of the jets. Second, the similarity of the energy release in GRBs and ordinary supernovae is puzzling. This coincidence is all the more remarkable considering the diversity of the progenitors and likely differing collapse mechanisms in these various classes of explosions. Third, we do not understand why in GRBs, the explosion energy couples only to $10^{-5} M_\odot$ of the exploding star and thereby produce ejecta with high Lorentz factor. Fortunately, new missions (HETE-2 and SWIFT) with their vastly increased GRB localization rates will provide empirical data which may help solve these mysteries.


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Figure 1. The observed distribution of jet opening angles along with a model fit (line). We assume that the observed differential distribution of beaming factors can be represented by two power laws: $p_{\text{obs}}(f_b) = (f_b/f_0)^{\alpha+1}$ for $f_b < f_0$ and $p_{\text{obs}}(f_b) = (f_b/f_0)^{\beta+1}$ for $f_b > f_0$. Since for every observed burst there are $f_b^{-1}$ that are not observed, the true distribution is $p_{\text{true}}(f_b) = f_b^{-1} p_{\text{obs}}(f_b)$. Fitting to the data, we find the following: $\alpha$ is poorly constrained; $\beta = -2.77_{-0.30}^{+0.24}$; $\log f_0 = -2.91_{-0.06}^{+0.07}$. Thus, the true differential probability distribution (under the small angle approximation, $f_b \propto \theta_j^2$) is given by $p_{\text{true}}(\theta_j) \propto \theta_j^{-4.54}$ with the observed distribution being $p_{\text{obs}} \propto \theta_j^{-2.54}$. The distribution $p_{\text{true}}(f_b)$ allows us to estimate the true correction factor, $\langle f_b^{-1} \rangle$ that has to be applied to the observed GRB rate in order to obtain the true GRB rate. We find $\langle f_b^{-1} \rangle = f_0^{-1}[(\beta - 1)/\beta] \sim 520 \pm 85$. 
Figure 2. The gamma-ray fluence $F_\gamma$ (in units of erg cm$^{-2}$) plotted as a function of the inverse beaming fraction $f_b^{-1}$ (where $f_b = 1 - \cos \theta_j \approx \theta_j^2/2$, and $\theta_j$ is the opening angle of the jet). A correlation is apparent in the sense that GRBs that have narrower jet opening angles are brighter (high fluence) than those that do not. A linear fit to these data (open squares) gives a relatively large rms scatter of a factor of 3.3. The correlation is improved when the fluences are all scaled to the same unity redshift (crosses), thereby removing the distance dependence. The rms scatter (dashed lines) of these points around a linear fit (solid line) is reduced to only a factor of 2.3. This factor is marked by dashed lines around the linear fit. The wide variation in observed fluence, more than two orders of magnitude, appears to be mainly due to different beaming angles.
Figure 3. The distribution of the apparent isotropic $\gamma$-ray burst energy of GRBs with known redshifts (top) versus the geometry-corrected energy for those GRBs whose afterglows exhibit the signature of a non-isotropic outflow (bottom). The mean isotropic equivalent energy $\langle E_{\text{iso}}(\gamma) \rangle$ for 17 GRBs is $110 \times 10^{51}$ erg with a 1-$\sigma$ spreading of a multiplicative factor of 6.2. In estimating the mean geometry-corrected energy $\langle E_{\gamma} \rangle$ we applied the Bayesian inference formalism and modified to handle datasets containing upper and lower limits. Arrows are plotted for five GRBs to indicate upper or lower limits to the geometry-corrected energy. The value of $\langle \log E_{\gamma} \rangle$ is $50.71 \pm 0.10$ (1$\sigma$) or equivalently, the mean geometry-corrected energy $\langle E_{\gamma} \rangle$ for 15 GRBs is $0.5 \times 10^{51}$ erg. The standard deviation in $\log E_{\gamma}$ is $0.31^{+0.09}_{-0.06}$, or a 1-$\sigma$ spread corresponding to a multiplicative factor of 2.0.
Table 1. Jet Break Times and Energetics. The gamma-ray fluences ($F_\gamma$), given in units of $10^{-6}$ erg cm$^{-2}$, are from a diverse collection of instruments. The best determinations of energy fluence are from the Burst and Transient Experiment (BATSE) on the Compton Gamma-Ray Observatory (CGRO). Most of the GRBs (10), prior to the de-orbit of CGRO on 2000 May 26, are BATSE bursts. In these cases we used fits to BATSE data which were integrated over the energy range from 20 to 2000 keV. For the remainder of the events we used the fluence as determined from the Gamma-Ray Burst Monitor on the BeppoSAX satellite (40-700 keV), or fluences (25-1000 keV or 25-100 keV) from the Interplanetary Network of satellites (Ulysses, KONUS, and NEAR). The luminosity distance ($d_L$) is given in units of $10^{28}$ cm. It was calculated from the observed redshift ($z$), and adopting cosmological parameters of $H_0=65$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_M=0.3$, and $\Lambda_\Omega=0.7$. Other realistic cosmologies were tried but they did not fundamentally change our conclusions. The isotropic $\gamma$-ray energies ($E_{\text{iso}}(\gamma)$), given in units of $10^{51}$ erg, have been “k-corrected” such that all energy estimates are referenced to the same 20-2000 keV co-moving bandpass. Although these order-of-unity corrections affect individual determinations of $E_{\text{iso}}(\gamma)$, they do not affect our results derived from the sample as a whole. The jet break times ($t_j$), given in days, are taken from the literature. The notes and the references in the table indicate how $t_j$ was determined. The strongest evidence for collimated outflows come from GRBs with achromatic breaks in their broad-band light curves (B). In most cases such multi-frequency datasets are not available, so there is a second class of events with breaks determined primarily from radio (R), optical (O), or X-ray (X) data. We include here a number of events for which no break was observed, yielding only lower limits of $t_j$. For some GRBs the steep decline of the light curve, indicating a jet geometry, is already fully manifest at the time of the first measurement. In these cases (D) we have only an upper limit on $t_j$. The final group of GRBs are those for which $t_j$ cannot be determined (N), owing to complications in the light curve such as the presence of a supernova signature (i.e., GRB 970828), or the lack of sufficient data. The beaming-corrected gamma-ray energy ($E_\gamma$), given in units of $10^{51}$ erg, was calculated by applying the geometric correction factor $f_b$ to $E_{\text{iso}}(\gamma)$.

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