Strong constraint on large extra dimensions from cosmology

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Introduction — In the past few years there has been an enormous interest in the possibility that the presence of large extra dimensions can explain the hierarchy problem [1–4], the fact that the energy scale for gravitation (the Planck scale $\sim 10^{19}$ GeV) is so much larger than that for the standard model (100 GeV). The idea is that the standard model fields are located on a 3+1 dimensional brane embedded in a higher dimensional bulk, where only gravity is allowed to propagate.

This already puts stringent constraints on the size of the extra dimensions. Newtons law should definitely hold for any scale which has so far been observed. At present the best experiments have probed scales down to about 1 mm. Thus, if there are extra dimensions, they can only appear at a scale smaller than that. For simplicity we make the assumption that the $n$ new dimensions form an $n$-torus of the same radius $R_0$ in each direction $^*$. If there are such extra dimensions, the Planck scale of the full higher dimensional space, $M_{P,n+4}$, can be related to the normal Planck scale, $M_{P,4}$, by use of Gauss’ law [1]

$$M_{P,4}^2 = R^n M_{P,n+4}^{n+2},$$

and if $R$ is large then $M_{P,n+4}$ can be much smaller than $M_P$. If this scenario is to solve the hierarchy problem then $M_{P,n+4}$ must be close to the electroweak scale ($M_{P,n+4} \lesssim 10 - 100$ TeV), otherwise the hierarchy problem reappears. This already excludes $n = 1$, because $M_{P,n+4} \simeq 100$ TeV corresponds to $R \simeq 10^8$ cm. However, $n \geq 2$ is still possible, and particularly for $n = 2$ there is the intriguing perspective that the extra dimensions could be accessible to experiments probing gravity at scales smaller than 1 mm. In the remainder of the letter we use $M$ instead of $M_{P,n+4}$ to simplify notation.

$^*$This assumption has been made in practically all works on the subject, however see Ref. [5] for a different model.

So far, the strongest constraints come from the observation of the neutrino emission of SN1987A [6–8]. In the standard model, a Type II supernova emits energy almost solely in the form of neutrinos. Furthermore, the observed neutrino signal fits very well with the theoretical prediction. If extra dimensions are present, then the usual 4D graviton is complemented by a tower of Kaluza-Klein states, corresponding to the new available phase space in the bulk. Emission of these KK states can potentially cool the proto-neutron star too fast to be compatible with observations. So far, this has lead to the tight bound that $R \lesssim 0.66$ mm ($M \gtrsim 31$ TeV) for $n = 2$ and $R \lesssim 0.8$ nm ($M \gtrsim 2.75$ TeV) for $n = 3$ [8].

The other obvious place in astrophysics to look for these extra dimensions is cosmology [2,9,10] (see also [11]). In the present letter we go through the possible cosmological effects from the presence of large extra dimensions. We solve the Boltzmann equation for the production of KK modes, both during the radiation dominated epoch and during the reheating phase preceding it. We show that unless the maximum temperature reached during reheating is very low, the constraints from cosmology are much stronger than the supernova bounds.

Boltzmann equations — The fundamental equation governing the evolution of all species in the expanding universe is the Boltzmann equation [13], $L[f] = C[f]$, where $L = \partial f/\partial t - \mu H \partial f/\partial \mu$ is the Liouville operator and $C$ is the collision operator describing all possible interactions. $f$ is the distribution function for the given particle species. In the present case, there are two terms contributing to the collision operator: production and decay. There are several possible production channels [2]: gravi-Compton scattering, pair annihilation and bremsstrahlung. In a supernova, nucleon-nucleon bremsstrahlung is by far the dominant mechanism because of the very high nucleon density. In Ref. [10] this was assumed to be the case also in the early universe. However, this assumption turns out to be wrong, the

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Density is very high [7]. This integration yields the result

\[ n_m = \sum_{i=v,e,\gamma} \langle \sigma v \rangle n_i^2 - 3Hn_m - \Gamma_{\text{decay},m}, \]  

(2)

where \( m \) is the mass of the KK state. For the relatively low mass modes we look at, the decay lifetime is very long [3]. Therefore decays can be completely neglected at early times and the production phase can be separated from the decay phase. The production equation is then given by [9]

\[ \dot{n}_m = -3Hn_m + \frac{11m^5T}{128\pi^3M_P^4} K_1(m/T), \]  

(3)

where \( K_1(x) \) is a modified Bessel function of the second kind and we have assumed that \( m_e = 0 \). This assumption has very little influence on the results.

Production during the radiation dominated epoch — The universe enters the radiation dominated epoch at some temperature \( T \), which we shall refer to as the reheating temperature, \( T_{\text{RH}} \). In this case the present day number density can be found by integrating the Boltzmann equation

\[ n_0(m) \simeq \frac{19}{64\pi^3} \frac{\dot{\rho}_{\text{s},\text{RH}} T^3}{M_P} \frac{m}{M_P} e^{-\Gamma_{\text{decay},m} t_0} \int_{m/T_{\text{RH}}}^{\infty} q^3 K_1(q) dq. \]  

(4)

This equation applies to the number density for one mode with mass \( m \). However, if we are interested in the total present day contribution to the mass density from all modes, then we need to sum over all modes. This sum can be replaced by an integral over \( dm \) because the mode density is very high [7]. This integration yields the result

\[ \rho_{0,\text{thermal}} \simeq 1.9 \times 10^{-22} S_{n-1} \text{GeV}^4 \left( \frac{T_{\text{RH}}}{M} \right)^{n+2} \times \int_0^\infty dz z^{n+1} e^{-\Gamma_{\text{decay},m} t_0} \int_0^{\infty} dq q^3 K_1(q), \]  

(5)

where \( S_{n-1} = 2\pi^{n/2}/\Gamma(n/2) \). In Fig. 1 we show the constraints on \( M \) from demanding that \( \rho_0 \leq \rho_{\text{crit}} \), for the case of \( n = 2 \).

FIG. 1. The lower bound on \( M \) as a function of \( T_{\text{RH}} \), when only modes produced during the radiation dominated epoch are considered. The calculations assume that \( h = 0.75 \).

From this it is evident that for \( T_{\text{RH}} \geq 3 \text{ MeV} \) the bound is tighter than what is found from SN1987A. However, it was shown in Refs. [14,15] that \( T_{\text{RH}} = 0.7 \text{ MeV} \) can still be compatible with BBN, so if \( T_{\text{RH}} \) is sufficiently low the overproduction of KK states can be avoided.

Production during reheating — In the above treatment it was assumed that the universe enters the radiation dominated epoch instantaneously at the reheating temperature. However, this is not the case for any physically acceptable scenario. Plausibly, the universe enters the radiation epoch after some reheating by the decay of a massive scalar field (or by some other means of entropy production). The only reasonable alternative is that the radiation dominated epoch extended to much higher temperatures (of the order \( M \)). Here, we look at the “standard” case where reheating occurs from the decay of the inflaton field (for further discussion of inflation in scenarios with large extra dimensions, see for instance Ref. [16] and references therein).

What happens is that the universe starts reheating when the inflaton enters the oscillating regime. The important parameters are the density, \( \rho_{\phi,i} \), of the inflaton when reheating begins and the decay rate of the inflaton, \( \Gamma_{\phi} \). The Boltzmann equations for this system have been solved numerous times (see e.g. Refs. [13,17]). The result is that the temperature of the produced radiation immediately increases to a maximum value which depends on \( \rho_{\phi,i} \). After this, there is a period of continual entropy production during which the universe is matter dominated by the \( \phi \) field and \( T \propto t^{-1/4} \) (as opposed
At the time $t \simeq T_\phi^{-1}$ the inflaton decays rapidly and the universe becomes radiation dominated. Using this, it is easy to calculate the number of KK modes produced during the reheating phase. $T_\phi$ is directly related to $T_{RH}$ by $T_{RH} \simeq 0.5 \sqrt{\frac{m_{Pl}}{\rho_{\phi,i}}}$ \cite{15}, but the additional parameter $\alpha_i$ is introduced in the analysis. However, instead of this we use the parameter $\alpha \equiv T_{MAX}/T_{RH}$, where $T_{MAX}$ is the maximum temperature reached during reheating (the relation between $\alpha_i$ and $T_{MAX}$ is given in Ref. \cite{18}). From this, one gets an expression completely equivalent to Eq. (5)

$$\rho_{0,RH} \simeq 1.9 \times 10^{-22} S_{n-1} \text{GeV}^4 \left( \frac{T_{RH}}{M} \right)^{n+2} \times \int_0^\infty 2dz \; z^{n-1} e^{-T_{\text{decay,m}}/T} \int_T^\infty dq q^3 K_1(q),$$

(6)

FIG. 2. The lower bound on $M/\text{TeV}$ as a function of $T_{RH}$ and $\alpha$, from demanding that $\rho_{0,\text{thermal}} + \rho_{0,RH} \leq \rho_{\text{crit}}$. The value $h = 0.75$ has been used. Contours are for $M/\text{TeV} \geq 10, 20, 50, 100$.

Fig. 2 shows the lower bound on $M$ for $n = 2$ as a function of $T_{RH}$ and $\alpha$. If $T_{RH} = 0.7$ MeV, the bound on $M$ is stronger than that from SN1987A as soon as $T_{MAX} \gtrsim 5$ MeV. This very clearly shows that the number of modes produced during reheating can easily be the dominant contribution. The reason is that the production rate depends steeply on the mass of the graviton and the temperature. For kinematical reasons, only modes with $T \sim m$ are produced. Thus the modes are non-relativistic from the outset, and their energy density contribution is not diluted by the entropy production.

Notice that this result is different from the result found by Giudice et al. \cite{18}, that the final abundance depends only on $T_{RH}$, and not on $T_{MAX}$. The reason is that there is a dense spectrum of modes with different masses. The abundance of each mode obeys the relations found in Ref. \cite{18}, i.e. that the final abundance for any mode with $m \lesssim 1$ few $\times T_{MAX}$ is independent of $T_{MAX}$. However, for higher $T_{MAX}$, many more modes are excited, and the end result is that the total production of KK modes increases strongly with increasing $T_{MAX}$.

Constraints from the diffuse gamma background— Apart from the production mechanisms there is also the possibility that the massive KK states decay into particles on the brane. The decay rate for different branches has been calculated by Han, Lykken and Zhang \cite{3}. In our case, where we do not look at very high temperatures, decays can proceed only into relatively light particles, $KK \rightarrow \gamma, \nu, e$. The rates for these processes are comparable, and are approximately given by $\tau_{\gamma,\nu,e} \simeq 6 \times 10^{15} \text{yr} \; M_{\text{MeV}}$. Thus, with the temperatures discussed here, the decay lifetime is longer than the age of the universe. Nevertheless, there can still be visible effects, especially from the decay contribution to the diffuse gamma background. It was shown by Hall and Smith \cite{9} that this leads to a very stringent constraint on $M$, even with $T_{RH} = 1$ MeV. The modes produced during reheating have higher mass and therefore much higher decay rates. This means that even tighter constraints can be put on $T_{MAX}$.

For $n = 2$, the contribution to the diffuse gamma background from KK decays is given by

$$\frac{dn}{dE} \simeq 345 M_{\text{GeV}}^{-4} T_{RH}^3 \left( \frac{E}{10^{10} \text{yr}} \right)^{6} \text{cm}^{-2} \text{s}^{-1} \text{ster}^{-1} \times \beta^{1/2} \left[ \int_\beta^\infty dzz^{7/2} E(z) \int_z^\infty q^3 K_1(q) dq + \int_\beta^\infty dzz^{7/2} E(z) \int_z^\infty q^3 K_1(q) dq \right],$$

(7)

where $\beta = E/T_{RH}$ and $E(z) = \exp(-3.3 \times 10^{-7} z^{3/2} T_{RH}^3 \text{MeV})$. Observationally, the diffuse gamma background in the MeV range has been measured by the EGRET \cite{19,21} (30 – $10^4$ MeV) and COMPTEL \cite{20,21} (0.8 – 30 MeV) experiments. The flux measured by EGRET is approximately $\frac{dn}{dE} = 2.3 \times 10^{-3} (E/\text{MeV})^{-2.07} \text{MeV}^{-1} \text{cm}^{-2} \text{s}^{-1} \text{ster}^{-1}$, and the flux measured by COMPTEL is $\frac{dn}{dE} = 6.4 \times 10^{-3} (E/\text{MeV})^{-2.3} \text{MeV}^{-1} \text{cm}^{-2} \text{s}^{-1} \text{ster}^{-1}$. Demanding that $\frac{dn}{dE}_{\text{obs}} \leq \frac{dn}{dE}_{\text{obs}}$ translates into the lower bound on $M$ shown in Fig. 3 as a function of $T_{RH}$ and $\alpha$. Note that for the relatively low masses we study here, constraints from light nuclei abundances \cite{22} are not important.

Discussion — We have discussed in detail how KK modes are produced in the early universe. It was shown that the reheating temperature should be very low in order to avoid overproduction of gravitons. Very interestingly, we found that KK modes can be produced
in abundance during reheating. Furthermore their energy density is not significantly diluted so that these KK modes can easily make up the dominant contribution. For $n = 2$ the robust bound from demanding that $\rho_{KK,0} \leq \rho_{\text{crit}}$ yields a stronger lower bound on $M$ than that from SN1987A already if the maximum temperature during reheating is $T_{\text{MAX}} \gtrsim 7$ MeV. The observed diffuse gamma background yields a much stronger bound which is $M \gtrsim 2000$ TeV ($R \lesssim 1.5 \times 10^{-7}$ mm) already for $T_{\text{MAX}} \simeq 5$ MeV (this assumes $T_{RH} = 0.7$ MeV which is the lowest possible bound compatible with BBN).

Any model that combines large extra dimensions with inflation plus reheating will have to address the problem that both the reheating temperature and the maximum temperature during reheating should be very low in order to avoid overproduction of gravitons in the early universe. This can have serious implications for phenomena such as baryogenesis and WIMP production. Models that rely on $T_{\text{MAX}} \gg T_{RH}$ (such as Refs. [17,18]) in order to accomplish production of very massive particles during reheating, while maintaining a very low $T_{RH}$ face serious constraints. Note that in the present treatment we have assumed that the inflaton only decays to matter on the brane. If gravitons are also produced at reheating, the bounds are tightened.

We finish by discussing briefly the few possibilities for avoiding the very stringent bound obtained above. In our analysis we assumed a toroidal geometry for the extra dimensions. However, other choices of geometry lead to different spectra of KK modes. As shown in Ref. [5], compact hyperbolic manifolds can lead to spectra with a lightest mode with $m \gtrsim 10$ GeV and large energy spac-