Constraints on explicit CP violation from the Brookhaven muon $g-2$ experiment

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(Received 27 February 2001; published 8 June 2001)

We use the recently derived CP phase dependent analytic results for the supersymmetric electroweak correction to $g_\mu-2$ to constrain the explicit CP phases in softly broken supersymmetry using the new physics effect seen in the $g-2$ Brookhaven measurement. It is shown that the BNL data strongly constrain the CP violating phase θ_μ (the phase of the Higgs mixing parameter μ) and ξ_2 [the phase of the $SU(2)$ gaugino mass $\tilde{m}_2$] and as much as 60–90 % of the region in the $\xi_2-\theta_\mu$ plane is eliminated over a significant region of the MSSM parameter space by the BNL constraint. The region of CP phases not excluded by the BNL experiment allows for large phases and for a satisfaction of the EDM constraints via the cancellation mechanism. We find several models with large CP violation which satisfy the EDM constraint via the cancellation mechanism and produce an a_μ^{SUSY} consistent with the new physics signal seen by the Brookhaven experiment. The sparticle spectrum of these models lies within reach of the planned accelerator experiments.

DOI: 10.1103/PhysRevD.64.016010

PACS number(s): 11.30.Er, 12.60.Jv

I. INTRODUCTION

Recently the Brookhaven experiment E821 made a precise determination of the muon anomaly $a_\mu = (g_\mu - 2)/2$ and found a deviation from the standard model result at the 2.6σ level. The experiment finds [1]

$$a_\mu^{exp} - a_\mu^{SM} = 43(16) \times 10^{-10}. \quad (1)$$

A correction to a_μ is expected in supersymmetric models [2] and a realistic analysis of the correction to a_μ was given in the supergravity unified model with gravity mediated breaking of supersymmetry in Refs. [3,4]. Specifically in the analysis of Ref. [4] it was pointed out that the supersymmetric electroweak correction to a_μ could be as large or larger than the standard model electroweak correction [5,6]. The fact that the supersymmetric electroweak contribution to a_μ can be large is supported by several later works which include constraints of the unification of the gauge coupling constants using the high precision CERN e^+e^- collider LEP data, radiative breaking of the electroweak symmetry, experimental bounds on sparticle masses and relic density constraints on neutralino dark matter assuming R parity conservation [7,8]. Soon after the release of data [1] on the observation of a difference between the experimental value and the theoretical prediction of a_μ in the standard model several analyses appeared [9–17] both within the supersymmetric framework [9–15] as well as in non-supersymmetric

scenarios [16,17]. Specifically the work of Ref. [9] used a 2σ error corridor on a_μ^{SUSY} where $a_\mu^{SUSY} = a_\mu^{exp} - a_\mu^{SM}$ to constrain SUSY, i.e.,

$$10.6 \times 10^{-10} < a_\mu^{SUSY} < 76.2 \times 10^{-10}, \quad (2)$$

where the error corridor also includes uncertainties due to hadronic error [18] in the theoretical predictions. The analysis of Ref. [9] showed the interesting result that the BNL data imply that the sparticles have upper limits which lie within reach of the planned accelerator experiments. Thus, for example, one finds that within the minimal supergravity (SUGRA) model [19] the BNL constraint implies that the lighter chargino mass $m_{\chi_1^\pm} \leq 650$ GeV, $m_{1/2} \leq 800$ GeV and $m_0 \leq 1.5$ TeV ($\tan\beta \leq 55$) consistent with the fine tuning criteria [20]. These mass ranges are within reach of the CERN Large Hadron Collider (LHC) and a part of the parameter space may also be accessible at run II of the Fermilab Tevatron. Further, it was shown that the BNL data imply that $\text{sgn}(\mu)$ is positive using the standard sign convention [21]. This result is consistent with the experimental $b \rightarrow s + \gamma$ constraint that eliminates much of the parameter space for the case when $\mu < 0$ [22]. It was also discussed in Ref. [9] that the effects from extra dimensions [23] on $g_\mu - 2$ are typically very small and do not pose a serious background to the supersymmetric electroweak contribution to $g_\mu - 2$.

The analysis carried out in Ref. [9] was under the assumption of CP conservation where the phases of all the soft SUSY parameters are set to zero. However, in general the soft breaking parameters can be complex and their presence brings in new sources of CP violation over and above the one in the Cabibbo-Kobayashi-Maskawa (CKM) matrix of the standard model. The normal size of the phases is $O(1)$

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FIG. 1. A plot of the allowed region (shaded) in the $\xi_2 - \theta_\mu$ plane allowed by the constraint of Eq. (2) when $m_0 = 100$, $m_{1/2} = 246$, $\tan \beta = 20$, $A_0 = 1$, $\xi_1 = .3$, and $\alpha_{A_0} = .5$ where all masses are in GeV.

which creates a problem in that phases of this size typically lead to the electric dipole moment for the electron and for the neutron which are in excess of the experimental limits [24]. Some of the possible ways to avert this disaster consist of assuming small phases [25], assuming that the sparticle spectrum is heavy [26] and more recently the possibility that there are cancellations [27] which allows for large phases and a not too heavy sparticle mass spectrum. For the last scenario one will have then large CP violating phases which would affect a variety of low energy physics, such as sparticle masses and decays, Higgs mixing, proton decay, B physics and baryogenesis. Thus the inclusion of CP phases is an important new ingredient in SUSY phenomenology. Now a_μ^{SUSY} also depends on the CP phases and thus the experimental constraints on a_μ^{SUSY} can be translated into constraints on the CP phases.

A full analysis of the effects of CP phases on $g_\mu - 2$ in the minimal $N=1$ supergravity and in minimal supersymmetric standard model (MSSM) was given in Ref. [28]. Remarkably it is found that the CP phases strongly affect a_μ^{SUSY} in that they can change both its sign and its magnitude. It is this fact, i.e., that the a_μ^{SUSY} is a very sensitive function of the CP phases that leads us to utilize the current BNL data to constrain the CP phases. The phases that enter most dominantly in the $g_\mu - 2$ analysis are θ_μ and ξ_2 and one finds that the BNL constraint eliminates a big chunk of the parameter space in the $\xi_2 - \theta_\mu$ plane. The domains al-

lowed and disallowed by the BNL data depend sensitively on m_0 , $m_{1/2}$ and $\tan \beta$ and less sensitively on other parameters (here m_0 is the universal soft breaking mass for the scalar fields, $m_{1/2}$ is the universal gaugino mass, and $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$ where H_2 gives mass to the up quark and H_1 gives mass to the down quark and the lepton). Often as much 60–90 % of the area in the $\xi_2 - \theta_\mu$ plane is excluded by the BNL constraint. In the limit of vanishing phases the allowed region reduces to the constraint $\mu > 0$ which was deduced in the earlier analysis [9] under the constraint of CP conservation. Of course, not all the parameter space allowed by the $g_\mu - 2$ constraint is allowed by the electric dipole moment (EDM) constraints. However, we find that these constraints can be mutually consistent. Thus we give examples of models where the phases are large, i.e., $O(1)$, the EDM constrains are satisfied and the contribution to a_μ^{SUSY} is consistent with the new physics signal seen in the precise BNL measurement. The outline of the rest of the paper is as follows: In Sec. II we give some of the basic formulas which enter into the $g_\mu - 2$ analysis with CP violation. In Sec. III we give a discussion of the numerical results obtained by imposing the constraint of Eq. (2). Conclusions are given in Sec. IV.

II. $G_\mu - 2$ WITH CP VIOLATING PHASES

a_μ^{SUSY} at the one loop level arises from the chargino exchange and from the neutralino exchange. The chargino con-

FIG. 2. A plot of the chargino contribution $a_\mu^{X^-}$ (dotted line), neutralino contribution $a_\mu^{X^0}$ (dashed line) and the total a_μ^{SUSY} (solid line) as a function of ξ_2 in the range $-\pi \leq \xi_2 \leq \pi$ when $\theta_\mu + \xi_2 = -1$, $m_0 = 100$, $m_{1/2} = 246$, $\tan \beta = 20$, $A_0 = 1$, $\xi_1 = .4$, $\alpha_{A_0} = .5$, where all masses are in GeV. The small fluctuation of the chargino contribution from exact constancy is due to small rounding off errors in the numerical integration program.

tribution is typically the dominant one although the neutralino contribution can become very significant in certain regions of the parameter space. In fact the neutralino exchange contribution is central in determining the boundary of the allowed and the disallowed region in the plane of the CP violating phases on which a_μ^{SUSY} sensitively depends. To define notation and explain the main features of the analysis we exhibit below the CP phase dependent chargino contribution and the reader is referred to Ref. [28] for the full analytic analysis including the neutralino exchange contribution. The chargino mass matrix with CP phases is given by

$$M_C = \begin{pmatrix} |\tilde{m}_2| e^{i\xi_2} & \sqrt{2} m_W \sin \beta \\ \sqrt{2} m_W \cos \beta & |\mu| e^{i\theta_\mu} \end{pmatrix} \quad (3)$$

where we have absorbed the phases of the Higgs sector by field redefinitions. In Eq. (3) θ_μ is the phase of the Higgs mixing parameter μ , ξ_2 is the phase of the SU(2) gaugino mass $\tilde{m}_2$. The chargino mass matrix can be diagonalized by the biunitary transformation $U^* M_C V^{-1} = \text{diag}(\tilde{m}_{\chi_1^+}, \tilde{m}_{\chi_2^+})$ where U and V are unitary matrices. The chargino contribution is given by [28]

$$a_\mu^{X^+} = a_\mu^{1X^+} + a_\mu^{2X^+} \quad (4)$$

where

FIG. 3. A plot of the allowed region (shaded) in the $\xi_2 - \theta_\mu$ plane allowed by the constraint of Eq. (2) with all the same parameters as in Fig. 1 except that $\tan \beta = 5$.

FIG. 4. A plot of the allowed region (shaded) in the $\xi_2 - \theta_\mu$ plane allowed by the constraint of Eq. (2) with all the same parameters as in Fig. 1 except that $m_{1/2} = 527$ GeV.

$$a_\mu^{1\chi^+} = \frac{m_\mu \alpha_{EM}}{4\pi \sin^2 \theta_W} \sum_{i=1}^2 \frac{1}{M_{\chi_i^+}} \text{Re}(\kappa_\mu U_{i2}^* V_{i1}^*) F_3 \left(\frac{M_\nu^2}{M_{\chi_i^+}^2} \right). \quad (5)$$

and

$$a_\mu^{2\chi^+} = \frac{m_\mu^2 \alpha_{EM}}{24\pi \sin^2 \theta_W} \sum_{i=1}^2 \frac{1}{M_{\chi_i^+}^2} (|\kappa_\mu U_{i2}^*|^2 + |V_{i1}|^2) F_4 \left(\frac{M_\nu^2}{M_{\chi_i^+}^2} \right). \quad (6)$$

Here $F_3(x) = (x-1)^{-3}(3x^2 - 4x + 1 - 2x^2 \ln x)$, $F_4(x) = (x-1)^{-4}(2x^3 + 3x^2 - 6x + 1 - 6x^2 \ln x)$ and $\kappa_\mu = m_\mu / \sqrt{2} M_W \cos \beta$. As discussed in Ref. [28] the entire phase dependence of the chargino contribution to a_μ resides in the combination $\theta_\mu + \xi_2$. The neutralino contribution, however, depends on additional phases and one can choose these additional combinations to be $\theta_\mu + \xi_1$ and $\xi_1 - \alpha_{A_\mu}$ where ξ_1 is the phase of the U(1) gaugino mass $\tilde{m}_1$ and α_{A_μ} is the phase of the trilinear soft SUSY breaking term A_μ at the electroweak scale.

III. DISCUSSION OF RESULTS

We want to analyze the effect of the constraint of Eq. (2) on the CP phases on which a_μ^{SUSY} sensitively depends. We

shall work in the region of the parameter space where sparticle masses are of moderate size and CP phases are $O(1)$. In this region one can manufacture an a_μ^{SUSY} of the size of the new physics effect indicated by the BNL experiment. However, since the sparticle masses are moderate size and the phases are $O(1)$ we need the cancellation mechanism to achieve consistency with the EDM constraints. For the purpose of the analysis we shall use the parameter space discussed in Ref. [29] which consists of the parameters m_0 , $m_{1/2}$, A_0 , $\tan \beta$, θ_μ , ξ_1 , ξ_2 and ξ_3 where ξ_1 is phase of the U(1) gaugino mass $\tilde{m}_1$, and ξ_3 is the phase of SU(3) gaugino mass $\tilde{m}_3$. The electro-weak sector of the model does not involve the SUSY QCD phase ξ_3 which, however, enters in the EDM analysis of the neutron. The neutralino exchange contribution depends also on ξ_1 and α_{A_0} in addition to its dependence on θ_μ and ξ_2 . However, the dependence of the sum of both contributions on ξ_1 and α_{A_0} is weak. Thus mainly the phases strongly constrained by the Brookhaven experiment are ξ_2 and θ_μ .

In Fig. 1 we display the allowed parameter space in the $\xi_2 - \theta_\mu$ plane in the range $-\pi \leq \xi_2 \leq \pi$ and $-\pi \leq \theta_\mu \leq \pi$ for the specific input values of m_0 , $m_{1/2}$, $\tan \beta$, A_0 , α_{A_0} and ξ_1 as given in the caption of Fig. 1. As discussed in Sec. II, the chargino exchange contribution to a_μ^{SUSY} is a function only of the combination $\theta_\mu + \xi_2$. This means that in the part of the parameter space where the chargino contribution is dominant

FIG. 5. A plot of the allowed region (shaded) in the $\xi_2 - \theta_\mu$ plane allowed by the constraint of Eq. (2) with all the same parameters as in Fig. 1 except that $m_0 = 400$ GeV.

a value of $\theta_\mu + \xi_2$ allowed by the BNL constraint will generate a 135° line in the $\xi_2 - \theta_\mu$ plane. Similarly a range of allowed values of $\theta_\mu + \xi_2$ will generate an area at 135° incline and we see that is approximately true in Fig. 1. Now, of course, if the chargino contribution was the sole contribution in a_μ^{SUSY} Fig. 1 would consist of only parallel lines at 135° incline within the allowed range of $\theta_\mu + \xi_2$ consistent with Eq. (2). However, a_μ^{SUSY} also contains the neutralino exchange contribution which is strongly dependent on ξ_2 and θ_μ individually even when $\theta_\mu + \xi_2$ is fixed. The strong dependence of the neutralino contribution on ξ_2 when $\theta_\mu + \xi_2$ is fixed is shown in Fig. 2. Because of this the sum of the chargino and the neutralino contributions does not possess the simple dependence on θ_μ and ξ_2 in the sum form. Thus in Fig. 1 the boundaries at 135° are not exactly straight lines since near the boundary the neutralino contribution can move a_μ^{SUSY} in or out of the allowed range admitting or eliminating that point in the parameter space of the admissible set. Further, since the neutralino contribution violates the simple dependence on $\theta_\mu + \xi_2$ it destroys the translational invariance of a_μ^{SUSY} on θ_μ (with $\theta_\mu + \xi_2$ fixed). We see this violation in Fig. 1 from the asymmetrical endings of the allowed region, i.e., the lower right hand and the upper left hand of the admissible region are not mirror reflections of each other. In addition to θ_μ and ξ_2 , the parameters $\tan\beta$, $m_{1/2}$ and m_0 also have a strong effect on a_μ . We study the effect of changes in these below.

To study the effect of the dependence on $\tan\beta$ we carry out an analysis in Fig. 3 similar to that of Fig. 1 but with $\tan\beta = 5$ and with all other parameters fixed at their values in Fig. 1. Now as pointed out in Refs. [7,8] a_μ^{SUSY} has a strong dependence on $\tan\beta$. As shown in the first paper of Ref. [8] this dependence arises from the chiral interference term in the chargino exchange contribution which is proportional to $\tan\beta$ for large $\tan\beta$. Thus a reduction in the value of $\tan\beta$ reduces the magnitude of a_μ^{SUSY} and its relative smallness results in a smaller range in $\theta_\mu + \xi_2$ around $\theta_\mu + \xi_2 = 0$ consistent with Eq. (2). Further, since the magnitude of a_μ^{SUSY} is smaller and closer to the lower limit of Eq. (2) it is more sensitive to the neutralino exchange contributions which can move it out of the allowed region more easily reducing the allowed region in the $\xi_2 - \theta_\mu$ plane, which is what we see in Fig. 3.

Next we study the effect of changing the value of $m_{1/2}$. An increase in the value of $m_{1/2}$ increases the chargino mass and the neutralino mass which reduces the magnitude of a_μ^{SUSY} . The reduction in the magnitude of a_μ^{SUSY} leads to a smaller range for $\theta_\mu + \xi_2$ which is what we see in Fig. 4 relative to Fig. 1. Finally we look at the effect of the variation of m_0 on a_μ^{SUSY} . Now similar to the effect of increasing $m_{1/2}$, an increase in the value of m_0 decreases the overall magnitude of a_μ^{SUSY} bringing it closer to the BNL lower limit as given by Eq. (2). As in the analysis of Fig. 3 and Fig. 4 the

FIG. 6. A plot of the allowed region (shaded) in the $\xi_2 - \theta_\mu$ plane allowed by the constraint of Eq. (2) with all the same parameters as in Fig. 1 except that $m_0 = 600$ GeV.

fact that the overall magnitude of a_μ^{SUSY} is smaller means that changes in the phase can more easily move its value out of the BNL admissible domain thus reducing the allowed range of $\theta_\mu + \xi_2$. This is what we see in Fig. 5 where $m_0 = 400$ GeV. The same argument would indicate that a further increase in the value of m_0 should further decrease the allowed range of $\theta_\mu + \xi_2$ which is what we find in Fig. 6 where $m_0 = 600$ GeV. We note that a_μ^{SUSY} is more sensitive to changes in $m_{1/2}$ than in m_0 . This was seen already in the analysis of Ref. [9] where the upper limit of m_0 was found to be significantly larger than the upper limit on $m_{1/2}$. The above explains why the allowed area in Fig. 4 is smaller than in Figs. 5 and 6. This is so because in Fig. 4 $m_{1/2}$ is close to its upper limit while in Figs. 4 and 5 m_0 is significantly lower than its upper limit. We notice also that the allowed regions in Figs. 5 and 6 consist of complete straight lines which

means that the neutralino contribution role here is suppressed severely by increasing m_0 .

Now not all the parameter space admissible by the $g_\mu - 2$ constraint in Figs. 1–6 is admissible by the constraints on the electron and on the neutron EDM. To satisfy the EDM constraints via the cancellation mechanism we have to utilize also the parameter ξ_3 along with θ_μ , ξ_2 and other soft parameters. (We include the two loop effects of Ref. [30] in the analysis.) In Table I we exhibit five points (a)–(e) that lie in each of the allowed regions of Fig. 1 and Figs. 3–6, i.e., point (a) lies in the allowed region of Fig. 1, point (b) lies in the allowed region of Fig. 3, etc., which satisfy the EDM constraints and the corresponding value of a_μ^{SUSY} lies in the BNL range of Eq. (2). Thus Table I gives five models which have large CP violating phases, their contributions to the EDM of the electron and of the neutron lie within experi-

TABLE I. Cases where the EDM and the $g - 2$ experiments are satisfied.

(case) ξ_2, θ_μ, ξ_3 (radian)	d_e, d_n (ecm)	a_μ^{SUSY}
(a) $-.63, .3, .37$	$-4.2 \times 10^{-27}, -5.3 \times 10^{-26}$	47.0×10^{-10}
(b) $-.85, .4, .37$	$4.2 \times 10^{-27}, 4.8 \times 10^{-26}$	10.8×10^{-10}
(c) $-.8, .2, 1.3$	$4.0 \times 10^{-27}, 5.4 \times 10^{-26}$	12.2×10^{-10}
(d) $-.32, .3, -.28$	$-1.2 \times 10^{-27}, 3.3 \times 10^{-26}$	20.1×10^{-10}
(e) $-.5, .49, -.5$	$1.8 \times 10^{-27}, -6.6 \times 10^{-27}$	12.7×10^{-10}

TABLE II. Sparticle masses (in GeV) for cases (a)–(e) in Table I.

(case)	$\chi_1^0, \chi_2^0, \chi_3^0, \chi_4^0$	χ_1^+, χ_2^+	$\tilde{\mu}_1, \tilde{\mu}_2$	$\tilde{u}_1, \tilde{u}_2$
(a)	98.2, 186.9, 389.8, 403.5	190.2, 405.6	145.0, 209.6	628.5, 647.4
(b)	97.2, 184.0, 408.3, 426.6	187.0, 426.6	144.6, 209.1	628.6, 647.5
(c)	213.8, 421.0, 845.8, 852.2	429.3, 853.2	232.0, 397.2	1335.3, 1378.0
(d)	98.1, 186.0, 378.4, 393.4	189.2, 395.3	413.5, 440.3	738.3, 754.4
(e)	98.3, 187.1, 393.7, 407.3	190.4, 409.3	609.1, 627.6	863.2, 877.0

mental limits using the cancellation mechanism and they produce a SUSY contribution to $g_\mu - 2$ consistent with the signal observed by the BNL experiment. The sparticle spectrum corresponding to cases (a)–(e) of Table I is shown in Table II. One finds that in all the cases the sparticle spectrum is low enough that some if not all of the sparticles must become visible at the LHC, and in some cases the sparticle spectrum is low enough to even lie within reach of run II of the Tevatron. We note that using points of Table I one can generate trajectories using the scaling technique given in Ref. [31] where the EDM constraints are satisfied and thus one can use this technique to produce many more models of the type discussed above.

IV. CONCLUSION

In this paper we have used the new physics signal seen by the Brookhaven $g - 2$ experiment and the recently derived CP phase dependent analytic results on the supersymmetric electroweak correction to $g_\mu - 2$ to put limits on the explicit CP violating phases that arise from the soft SUSY breaking sector of MSSM. Using a 2σ error corridor around the observed effect we find that the BNL constraint excludes a

large region in the $\xi_2 - \theta_\mu$ plane. The amount of the region excluded depends sensitively of $\tan\beta$, m_0 , and $m_{1/2}$ and less sensitively on the remaining parameters. In most of the parameter space the excluded region is as much as 60–90 % of the total area, i.e., the area mapped by $-\pi \leq \xi_2 \leq \pi$, $-\pi \leq \theta_\mu \leq \pi$. In the limit when the phases vanish the allowed region limits to the case $\mu > 0$ deduced in the previous analysis using CP conservation [9]. We also show that the regions allowed by the BNL constraint contain points where the cancellation mechanism operates and provide examples of models with large CP violation consistent with the experimental EDM limits of the electron and the neutron and with SUSY contributions to a_μ consistent with the new physics signal seen by the BNL experiment. These models also possess sparticle spectra which lie within reach of collider experiments planned for the near future.

ACKNOWLEDGMENTS

One of us (T.I.) wishes to thank Professor Ali Azzam for encouragement. This research was supported in part by the NSF Grant PHY-9901057.

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