Radiative B Decays in the Standard Model and Beyond

Paolo Gambino

CERN, Theory Division, CH–1211 Geneve 23, Switzerland.

Abstract

The rare decay mode $B \to X_s \gamma$ provides one of the most important probes of new physics. I review its theoretical prediction in the Standard Model and illustrate the impact of new physics with a few examples.

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The end of the LEP decade has not left us idle to wait for LHC. New, very important results are expected from the B factories, where we hope to find signals of physics beyond the Standard Model (SM) before the advent of LHC. Over the years we have learned to identify in the difficult hadronic environment some observable which is reasonably free of hadronic uncertainties and can be predicted with good accuracy. A notable example are the inclusive radiative decays of the B meson. They are the subject of my talk. FCNC decays occur only at loop-level in the SM and thus provide an excellent indirect probe of new physics and of the masses and couplings of the virtual particles participating.

Although Alberto has never worked on B physics, I should recall that the first realization of the importance of radiative corrections in $B \to X_s \gamma$ came from Alberto’s group at NYU [1]. Since then, a lot of work has been done both to improve the SM prediction and to explore the significance of this decay for new physics searches. In the following I will try to summarize what we understand of this decay mode and to explain why it is such a formidable test for many scenarios of new physics.

It seems apt to start with the experimental situation. There are at present three measurements [2] of the branching ratio for the inclusive decay $B \to X_s \gamma$ (BR$_\gamma$ in the following). Cleo and Belle find

$$\text{BR}_\gamma(\text{Cleo}) = [3.15 \pm 0.35 \pm 0.32 \pm 0.26] \times 10^{-4},$$

$$\text{BR}_\gamma(\text{Belle}) = [3.34 \pm 0.50 \pm 0.35 \pm 0.27] \times 10^{-4},$$

where the first uncertainty is statistical, the second systematic and the third will be explained in a moment. The Aleph measurement has larger errors. The world average is $\text{BR}_\gamma = 3.20 \pm 0.42$ with an error of about 13%. Improved results from Cleo and the B factories are expected soon.

The decay $B \to X_s \gamma$ is dominated by an effective short-distance transition $b_R \to s_L \gamma$ induced in the SM by the diagrams in Fig. 1. Neglecting the strange quark mass, a free $b$

![Feynman diagrams](image)

Figure 1: Feynman diagrams contributing to the $b_R \to s_L \gamma$ transition in the $R_\xi$ gauge. The external photon couples to any of the internal lines and crosses represent mass insertions.

quark at rest decays emitting a photon of fixed energy $E_\gamma = m_b/2$. Since the signature of the decay is a high energy photon, potential background is highly suppressed. However, the strong interactions and the binding of the quark in the B meson smear the delta distribution into a smooth curve (see Fig. 2), whose shape depends on a universal non-perturbative shape function. This function reflects the Fermi motion of the $b$-quark inside
the meson [3]. A measurement of the photon spectrum would be by itself extremely useful

\[
\text{Figure 2: Typical photon energy spectrum for } B \rightarrow X_s \gamma.
\]

for our understanding of non-perturbative contributions. For example, it could help in reducing the theoretical uncertainties in the measurement of $V_{ub}$ [4].

For decreasing $E_\gamma$, background becomes a significant problem. What in fact experiments do is to consider only those events characterized by photons with energy above a certain threshold, presently of 2.1 GeV [2]. As can be seen in Fig. 2, this is still far from a totally inclusive measurement, but close enough for us to fill the gap by modelling in some way the shape function and assigning an error for this non-perturbative extrapolation. This is the source of the third error in Eqs. (1.2).

But why do we want an inclusive measurement after all? The crucial advantage of inclusive rates over exclusive ones is that (in general) precise theoretical predictions with small non-perturbative corrections are possible only for sufficiently inclusive quantities. Indeed, the Operator Product Expansion (OPE) used in the context of the Heavy Quark Effective Theory (HQET) allows us to compute in a model independent way non-perturbative contributions to the inclusive BR, as power series in $\Lambda_{QCD}/m_b$. Moreover, the first correction to the free quark decay appears only at $O(1/m_b^2)$ [5]. The rate should therefore be sufficiently inclusive in order to average over enough hadronic states and to be insensitive to the shape of the photon spectrum of Fig. 2.

The definition of inclusive radiative decay comes under strain when we proceed beyond leading order in QCD and start considering gluon radiation. In the presence of a soft gluon and a soft photon we cannot distinguish between $B \rightarrow X_s \gamma$ and $B \rightarrow X_s g$, which a priori originate from different primary hard processes. Therefore, we should not consider decays with very soft photons — which are anyway undetectable at present facilities — and set a minimum $E^\text{min}_\gamma > 0$ in order to minimize the error related to the extrapolation in Fig. 2. Conventionally, the inclusive BR, is defined in the literature [3] with $E^\text{min}_\gamma = 0.05 m_b$. 
Although here I will follow this convention, from a practical point of view we see in Fig. 2 that $E_{\gamma}^{\text{min}} \approx 1.8 \text{ GeV}$ would be sufficient.

In summary, to good approximation we can express $\text{BR}_{\gamma}$ as

$$\text{BR}_{\gamma} \sim \left( \frac{m_b \Lambda_{\text{QCD}}}{m_c^2} \right) + O \left( \frac{m_b \Lambda_{\text{QCD}}}{m_c^2} \right),$$

where the spectator model describes the $B$ meson decay as a free $b$ quark decay and the last term comes from charm loop effects in the diagram of Fig. 1(a) and has been shown to be small [6]. Altogether, the known non-perturbative corrections amount to about 2% [5].

The three scales involved in the process are

$$\Lambda_{\text{QCD}} \ll m_b \ll M_W, M_t.$$ (4)

We have used the first inequality to derive Eq. (3). Let us now concentrate on the perturbative corrections. They clearly involve large logarithms of the form $L = \ln m_b/M_W$ which need to be resummed. We can now use the second scale hierarchy in Eq. (4) to expand at first order in $1/M_W^2$ and to resum the large perturbative corrections. The OPE selects the relevant dimension five and six operators so that an effective Hamiltonian can be written as

$$H_{\text{eff}}^{\Delta B=1} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i C_i(\mu) \hat{O}_i(\mu),$$ (5)

where the most important operators are

$$O_2 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L), \quad O_7 = \frac{e}{4\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, \quad O_8 = \frac{g_s}{4\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} t^a b_R G_{\mu\nu}^a.$$ (6)

The implementation of the above effective Hamiltonian consists of three steps: (i) From the calculation of the loops in Fig. 1 we know the Wilson coefficients $C_i(\mu)$ at the high scale set by the masses of the particles inside the loop. The coefficients $C_i(M_W)$ encode all the information on the heavy degrees of freedom of the theory. (ii) The coefficients $C_i$ must be evolved down to the scale $\mu_b \approx m_b$ characteristic of the decay by renormalization group methods; to this end it is necessary to know the QCD anomalous dimension matrix of the operators $O_i$. (iii) Finally, the relevant matrix elements of $H_{\text{eff}}^{\Delta B=1}(\mu_b)$ must be calculated. Each of the above steps can be performed at a given order in $\alpha_s$. At present, the SM calculation has been performed at the NLO level where we have control of all the next-to-leading QCD logarithms $O(\alpha_s^2 L^{n-1})$ [7–10].

In this general framework, the decay $B \to X_s \gamma$ presents two peculiar features: (a) the mixing between magnetic dipole operators $O_{7,8}$ and the fourth fermion operator $O_2$ occurs first at two-loop; therefore a NLO analysis has required a three-loop calculation [9]. (b) QCD corrections enhance $\text{BR}_{\gamma}$ by a factor $\approx 3$ [1]. The big effect of leading and (to lesser extent) of subleading logarithmic corrections is a source of concern. In particular, what is the uncertainty due to unknown higher order corrections?
A preliminary question is: why are the QCD corrections so large in the first place? The branching fraction for inclusive radiative decays is usually written as

\[ \text{BR}_\gamma = \text{BR}_{SL} \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha}{\pi f \left( \frac{m_b}{m_t} \right)} |C_7(\mu_b)|^2, \]

(7)

where the normalization to the semileptonic BR removes large uncertainties from the bottom mass and the CKM elements. The phase space factor \( f \left( \frac{m_b}{m_t} \right) \approx 0.53 \) is related to the semileptonic decay. Up to negligible u contributions \( C_7(M_W) = K_t - K_c \), where I have split top and charm contributions to the diagrams in Fig. 1. In the absence of QCD, \( K_c = 0.639 \) and \( K_t = 0.444 \), which leads to \( \text{BR}_\gamma(0\text{th order}) = 1 \times 10^{-4} \). As anticipated, this result is very far from the experimental value. Notice that GIM cancellations are still operative despite the very heavy top quark, a fact that makes the calculation very sensitive to (perturbatively) small changes. Let me now rescale the top contribution by a factor \( r = \frac{m_b}{m_t} \approx 0.61 \). Using \( C_7 = rK_t - K_c \), I get \( \text{BR}_\gamma = 3.5 \times 10^{-4} \), which is very close to the complete NLO analysis and to the experimental value! There are good reasons to believe that this is \textit{not} a numerical coincidence \[11\]: the charm contribution is quite stable under radiative corrections while most of their effect on the top contribution can be absorbed by normalizing the bottom mass in the operator \( O_7 \) at the weak scale. This is true both at LO and at NLO and is partly explained by observing that the physical amplitude for \( b \rightarrow s\gamma \) requires a helicity flip which brings the factor \( m_b \) in the operator \( O_7 \). This factor \( m_b \) is not necessarily identical in the charm and top loops. In particular, in Fig. 1(c) the bottom yukawa providing \( m_b \) is naturally normalized at a scale \( O(M_W) \); its contribution turns out to be dominant in \( K_t \), leading to the rescaling factor \( r \). But \( r \) is governed by the b mass anomalous dimension (known to four loops) and this allows us to estimate the effect of higher perturbative orders. It turns out to be quite small \[11\]. The uncertainty from higher order QCD effects can also be estimated from scale dependence of the NLO results and is \( O(5\%) \) \[3,7-9\].

QCD corrections are not the only relevant higher order effects that have been studied for \( B \rightarrow X_s\gamma \). In the framework of a QED + QCD effective theory, electroweak effects can be naturally divided into (i) purely QED effects and (ii) proper electroweak corrections. As far as QED effects are concerned, it is crucial to use the correct normalization of the fine structure constant \( \alpha \) in Eq. (7). Although we former students of Alberto should have known better, we had to be reminded \[12\] that the correct normalization of electric charge for a real photon is \( \alpha(q^2 = 0) \). In \[12\], Bill and Andrzej also considered all the leading QED logarithms, arising in the radiative decay as well as in the semileptonic one \[13\]. Their interplay with QCD corrections has been studied in \[3,14\], resumming the \( O(\alpha Q_u Q_d/\alpha_s) \) logs. Relative to QCD corrections, these QED effects are in fact \( O(\alpha Q_u Q_d/\alpha_s) \) and quite small.

Since heavy weak bosons decouple, the proper electroweak corrections affect only the Wilson coefficients. They are not logarithmically enhanced, but can be more important
than purely QED effects. A way to approach them would be to consider the complete $O(\alpha(\alpha_sL)^n)$ effects, but the dominant part is likely to be captured by an expansion around $\sin \theta_W = 0$ supplemented by $O(g^2 s_W^2 M_t^2)$ contributions [15]. The dominant terms decrease $C_7(M_W)$ by about $(2.5 \pm 0.5)\%$ for a light Higgs. However, the interplay with QCD effects is not trivial and the eventual suppression of the branching ratio is only $(2.0 \pm 0.3)\%$.

Including all the known QCD, electroweak, and non-perturbative effects and on the basis of up-to-date input parameters, the inclusive rate is [15]

$$\text{BR}_\gamma = (3.29 \pm 0.21 \pm 0.21) \times 10^{-4},$$  \hspace{1cm} (8)

where the errors come from a conservative analysis of residual scale dependence and from the uncertainty in the input parameters (most importantly the charm mass and the semileptonic branching ratio). Notice that Eq. (8) is in excellent agreement with the present experimental world average $\text{BR}_\gamma = 3.20 \pm 0.42$. As shown by a detailed analysis of theoretical uncertainties [11], improving considerably the 9% accuracy of Eq. (8) is going to be very difficult. This accuracy may not be sufficient for high precision tests of the SM, but is enough to put very strong constraints on new physics.

Let us now turn to the potential effects of new physics in $B \to X_s \gamma$. In the absence of new light degrees of freedom, new physics will manifest itself through (i) new contributions to the coefficients of the same operators involved in Eq. (5) and (ii) new operators absent in the SM, like operators with different chirality. Examples of new contributions involve the exchange of a charged Higgs instead of a pseudo-Goldstone boson in Fig. 1(b,c), and the diagrams obtained by replacing in the same figures the top with a chargino and the Higgs with a squark. Therefore, new physics contributions are at the same level as the SM ones. The excellent agreement of the SM prediction with experiment constrains new physics models very strongly because excludes large effects and favors those scenarios where new physics naturally decouples from this process.

Even though the overall new physics effect must be relatively small, we have seen how sensitive is $\text{BR}_\gamma$ to tiny contributions. A careful consideration of radiative corrections turns then out to be important in at least three cases: (i) when a new large scale $M_{\text{new}} \gg M_W, m_t$ arises, (ii) when there are new large effects appearing only beyond LO — like for instance in the Minimal Supersymmetric SM (MSSM) with large $\tan \beta$; (iii) when subtle cancellations at LO are disrupted by NLO effects. In all these cases we need a calculation at the same level as the SM one. At present, complete NLO calculations are available for a general Two-Higgs-Doublet-Model (2HDM) model [8, 16], for some restricted SUSY scenarios [17-19], and for left-right symmetric models [19].

The case of the 2HDM is particularly instructive. The only additional contribution with respect to the SM comes from charged Higgs boson loops. There are two models, type I and type II, which differ by the way fermions couple to the Higgs doublets. In type II model, also realized in the MSSM, the charged Higgs loops always increase $\text{BR}_\gamma$ and the decoupling occurs slowly. So $\text{BR}_\gamma$ provides strong lower bounds on the mass of the
charged Higgs boson, $M_H$, whose dependence on $\tan \beta$ saturates for $\tan \beta \approx 5$. This bound is much better than the one from direct searches at LEP2 ($M_H > 77.4$ GeV) and than the indirect bound from a number of other processes (see Fig. 3). for instance, $R_b$ is relevant only for very small $\tan \beta$, while rare $B \to \tau \nu$ decays constrain $M_H$ only for very large $\tan \beta$. The $B \to X_s \gamma$ bound depends sensitively on small effects, and in particular on the way one combines errors. I find $M_H > 286$ GeV by combining theoretical and experimental errors in quadrature (gauss in the figure). If instead I calculate the theoretical error by scanning all relevant scales between 1/2 and 2 times their central values and then I add this theoretical error linearly to the overall experimental error, the result is $M_H > 258$ GeV. Stretching the range of scales between 1/3 and 3 times their central values and using the conservative approach the bound is 230 GeV. These values are based on Cleo results only and should be interpreted as 95% CL bounds: the inclusion of Belle measurement makes the bounds slightly stronger. NLO effects have shifted the bound up by about 30% [8]. In Model I no bound on $M_H$ can be obtained from $B \to X_s \gamma$, as the charged Higgs loops tend to decrease $BR_\gamma$ [8] and decouple for large $\tan \beta$; the most important bound in that case comes from $R_b$. More general 2HDMs are considered in [16].

Most of the lower bounds on $M_H$ considered in Fig. 3 are evaded in supersymmetric models, as new contributions may interfere destructively with the charged Higgs loops. Indeed, in the limit of unbroken supersymmetry it is known that $BR_\gamma = 0$. For light superpartners some of the cancellations which operate with exact SUSY are still partially effective [20]. It is worth reminding, however, that the unconstrained MSSM with a supersymmetry breaking scale $\Lambda \sim 1$ TeV has a very large number of free parameters and
no natural suppression on FCNC (see e.g. [21]). In addition to the superpartner loops (mediated by $\chi^\pm, \tilde{q}, H^\pm$), these are originated by new flavor violation in the sfermion mass matrices, leading to gluino and neutralino mediated FCNC. As the data strongly limit new sources of FCNC, the sum of these two mechanisms must be small.

The case in which all flavor transitions occur in the charged current sector and are controlled only by the CKM matrix is called minimal flavor violation (MFV). This is an approximation scheme justified in several models where supersymmetry breaking is flavor diagonal. As MFV removes many free parameters, it is more predictive than the general MSSM. It is realized, for example, in gauge–mediation models and generally provides a good approximation in minimal SUGRA models. This scenario is particularly constrained by the present experimental information [17, 18, 22]. In the following, I will limit myself to the framework of MFV; a more general scenario, as far as $B \to X_s \gamma$ is concerned, is studied in [23].

There are two cases when supersymmetric contributions can be potentially large: when at least some of the superpartners are light and for large $\tan\beta$. For reasons of space, I will now concentrate on this second scenario. $\tan\beta$ is the ratio of the v.e.v. of the two Higgs doublets, which in the MSSM couple to up and down fermion separately. Large $\tan\beta$ scenarios are favored in SO(10) Grand Unification schemes, because they lead to Yukawa couplings for the top ($y_t \sim m_t/(\sin\beta M_W)$) and bottom ($y_b \sim m_b/(\cos\beta M_W)$) quarks of comparable magnitude. Recently, they have gained more interest because the present lower bound on the lightest neutral Higgs ($M_h \gtrsim 110$ GeV) favors large values of $\tan\beta$ and heavy squarks.

In the limit of large $\tan\beta$, supersymmetric contributions to $B \to X_s \gamma$ scale like $\tan\beta$, as can be seen from

\[
\begin{array}{ccc}
& x & \\
\hline
\tilde{b}_R & \tilde{t}_L & \tilde{t}_R \\
\hline
\tilde{t}_L & \tilde{h}^+ & \tilde{t}_L \\
\end{array}
\sim \frac{m_b}{\cos\beta M_W} \sim \frac{m_b}{M_Z^2 \tan\beta},
\]

where the cross represents the v.e.v. of the “wrong” Higgs field and the photon couples to any internal line, like in Fig. 1. Therefore, the primary effect of large $\tan\beta$ in our process is to slow down the decoupling of heavy sparticles. The sign of the Higgs mixing parameter $\mu$ determines whether the interference of the supersymmetric contributions with the SM ones is constructive ($\mu < 0$) or destructive ($\mu > 0$). Because of the large value of $\tan\beta$, one should worry about potentially large higher orders, like $O(\alpha_s, \tan^2\beta)$ effects. Moreover, one should not decouple the heavy susy particles at the same scale as the SM ones. Both these effects have been considered in [18] (see also [24]). Concerning $O(\alpha_s, \tan^2\beta)$ effects, the main phenomenon is the modification of the relation between mass and Yukawa coupling of the bottom [25]

\[
y_b = \frac{m_b}{\sqrt{2} M_W \cos\beta} (1 + \epsilon_b \tan\beta); \quad \epsilon_b = \frac{\alpha_s}{\sqrt{2} \pi} \frac{\mu}{m_{\tilde{b}}} f(m_{\tilde{b}}, m_{\tilde{t}}).
\]
Figure 4: Branching ratio for $B \rightarrow X_s \gamma$ in a minimal SUGRA scenario with $m_0 = 600$ GeV, $m_{1/2} = 400$ GeV, $A_0 = 0$, and $\mu > 0$ (left) and $\mu < 0$ (right) as a function of $\tan \beta$. The solid and dashed lines represent two choices of the scale at which susy particles are decoupled, while the dotted line represents the results of the calculation with LO supersymmetric contributions.

Here $m_{\tilde{g}}, m_{\tilde{b}_l}$ are the gluino and sbottom masses and I have considered only QCD effects. Eq. (9) originates from graphs analogous to the one above with $b_L$ instead of $s_L$. It induces two-loop $\alpha_s \tan^2 \beta$ effects in the calculation of $BR_{\gamma}$ when $m_b$ is used to express the bottom Yukawa coupling. The results of [18] are exemplified in Fig. 4 by a typical minimal SUGRA scenario characterized by squark, charged higgs and gluino masses clustered between 700 GeV and 1 TeV, with the charginos somewhat lighter. We see that the case $\mu < 0$ is excluded by experiment. The case of $\mu > 0$ is characterized by destructive interference between SM and chargino contributions. In this situation specifically supersymmetric higher order effects are significant: for $\tan \beta = 40$ BRs is enhanced by $50\%$.

Notice that $\epsilon_b$ in Eq. (9) does not vanish when all the superpartners become heavy. Of course, squark loops are anyway proportional to $1/M_H^2$, so that the complete amplitude vanishes as $M_{\text{susy}} \rightarrow \infty$. However, for $M_{\text{susy}} \rightarrow \infty$, the charged Higgs does not become necessarily heavy and its couplings to quarks are modified in a way analogous to Eq. (9), by $\alpha_s \tan \beta$ contributions which can be parametrized by a universal $\epsilon \approx \epsilon_b$. Hence, observable effects can be present even for very heavy superpartners! This does not violate the decoupling theorem: it just means that the effective charged Higgs theory obtained by decoupling gluino and squarks does not correspond to what is usually called Model II [18]. In this situation the above bounds on the charged Higgs mass are evaded. The $M_H$ bounds of Fig. 3 are modified as shown in Fig. 5, depending on the value of $\epsilon$. Notice that for $\tan \beta = 20$ values of $M_H$ as low as 150 GeV are allowed for $\epsilon = 1\%$.

In summary, radiative B decays represent one of the most important probes of new physics and a major testing ground for the SM. We have seen that the measurements of $B \rightarrow X_s \gamma$ agree well with the SM and place severe constraints on some new physics
scenarios. Higher order effects are crucial both in the SM and in new physics models. Among the new physics scenarios that have been investigated at NLO, I have shown how a very strong bound on the mass of the charged Higgs ($M_{H^+} > 250$ GeV) can be found in type II 2HDM and briefly illustrated the case of the MSSM with large $\tan\beta$.

It's been a great joy to be back at NYU with Alberto, his former students, his friends. I wish Alberto many more years of fun doing physics. Many thanks to Massimo for organizing this pleasant meeting. I am indebted to G. Degrassi, G. Giudice, P.A. Grassi, S. Mele, and M. Misiak for helpful discussions and communications.

References


