Theory of Quantum Hall Effect: Effective Fractional Charge

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We report that the theory of Shrivastava (1986) is in full agreement with the experimental data of the quantum Hall effect. In particular, the data of Willett et al (1987), Eisenstein and Stormer (1990), Du et al (1983) and Yeh et al (1999) are in agreement with the theory of Shrivastava (1986). Laughlin's (1983) theory of fractional quantization of charge does not generate the experimentally observed series of effective charges in the quantum Hall effect. The need for the spin-orbit interaction is clear and hence the ground state must include this interaction. The correct state is a product of spin, orbit and Landau oscillator functions.

1 Introduction

We assume that in a semiconductor if the electric field is applied along the x -direction and a magnetic field along the z -direction, then there is a Hall voltage along the y direction. The Hall resistivity is given by

$$\rho_H(x, y) = -\frac{B_z}{nec} \quad (1)$$

where n is the concentration of electrons, e the charge and c is the velocity of light. It may be observed that all of the three dimensions are equally important at this stage. Since the magnetic field is directed along the z -direction, the current is along x direction and the electric voltage is along y direction, the electrons are confined in the $x-y$ plane and hence may exhibit two-dimensional behaviour. It was found by v.

Klitzing et al^{1,2} that the above resistance is quantized as

$$\rho(x, y) = \frac{h}{\nu e^2} \quad (2)$$

where ν is an integer. It can be interpreted as a Landau level filling factor for electrons in two-dimensions. For $\nu = 1$, the quantity

$$\frac{h}{e^2} = 25812.8 \quad \text{ohms} \quad (3)$$

is called “one Klitzing”. Later on, it was found by Tsui, Stormer and Gossard³ that ν need not be an integer. While measuring the transverse resistivity as a function of field, a plateau was found at $\nu = 1/3$. Later studies discovered many different values of the fraction, ν .

In this brief review, we describe our theory of the quantum Hall effect. In particular, effort is made to compare the theory with the experimental data.

2 Theory:

We consider the spin as well as the orbital motion so that

$$g_j j = g_s \vec{s} + g_l \vec{l} = \frac{1}{2}(g_l + g_s) \vec{j} + \frac{1}{2}(g_l - g_s)(\vec{l} - \vec{s}) . \quad (4)$$

The conduction electrons have $l = 0$. However, we consider also the bound electrons which have finite orbital angular momentum quantum number. Multiplying both sides of the above equation by $\vec{j} = \vec{l} + \vec{s}$ and taking eigenvalues, we find,

$$g_j j(j+1) = \frac{1}{2}(g_l + g_s)j(j+1) + \frac{1}{2}(g_l - g_s)[l(l+1) - s(s+1)] , \quad (5)$$

which upon substituting $s = \frac{1}{2}$ gives,

$$g_j = g_l \pm \frac{g_s - g_l}{2l+1} \quad (6)$$

for $j = l \pm \frac{1}{2}$. For $g_s = 2, g_l = 1$ we find

$$g_{\pm} = 1 \pm \frac{1}{2l+1} . \quad (7)$$

The cyclotron frequency is defined in terms of the magnetic field as,

$$\omega = \frac{eB}{mc} . \quad (8)$$

Corresponding to this frequency, the voltage along y direction is

$$\hbar\omega = eV_y . \quad (9)$$

From (8) and (9)

$$\hbar \frac{eB}{mc} = eV_y . \quad (10)$$

We multiply the above by e/h from both sides so that,

$$\frac{e^2 B}{2\pi mc} = \frac{e^2}{h} V_y \quad (11)$$

which describes the current in the x -direction so that it is proved that

$$\rho_{xy} = \frac{h}{e^2} \quad (12)$$

which is the same as (2) for $\nu = 1$ because of the Ohm's law. We take into account, the gyromagnetic ratio so that (11) may be written as,

$$I_x = \frac{1}{2}g \frac{e^2 B}{2\pi mc} = \frac{1}{2} \frac{ge^2 V_y}{h} . \quad (13)$$

For $l = 0, g = 2$,

$$I_x = \frac{e^2}{h} V_y \quad (14)$$

which describes the correctly quantized current for $\nu = 1$. From (13) $\nu = \frac{1}{2}g_{\pm}$ which gives the filling factor, one value for $+$ sign and the other value for $-$ sign in (7). For $l = 0$, we obtain $\frac{1}{2}g_+ = 1$ and $\frac{1}{2}g_- = 0$, for $l = 1$, we get $\frac{1}{2}g_+ = 2/3$ and $\frac{1}{2}g_- = 1/3$. These values of $\nu = \frac{1}{2}g_{\pm}$ are given in Table 1 from the theory of Shrivastava,⁴ which is also described in refs. 5 and 6.

Table 1: The calculated values of ν from ref. 4.

l	$\frac{g_-}{2} = \frac{l}{(2l+1)}$	$\frac{g_+}{2} = \frac{(l+1)}{(2l+1)}$
0	0	1
1	1/3	2/3
2	2/5	3/5
3	3/7	4/7
4	4/9	5/9
5	5/11	6/11
6	6/13	7/13

The Landau levels are introduced by projecting the above to two dimensions. We shall consider only the Landau level quantum number, n so that $\nu = n(\frac{1}{2}g_{\pm})$. Thus we can multiply the above fractions by an integer n whenever needed to interpret the data.

3 Supporting Data

We prove that this is the correct theory by means of comparing the above values with the experimental data. The values of ρ_{xx} and ρ_{xy} for a single interface of GaAs/AlGaAs have been measured by Willett et al⁷ at a temperature of 150 mK and at high fields at 85 mK. These values are shown in Fig.1. The fractions occur in two sets, the values 2/5, 3/7, 4/9, 5/11, 6/13 etc., shown in the experimental data are exactly the same as predicted by column 2 of table 1. Similarly, the values 2/3, 3/5, 4/7, 5/7, 6/11, etc., seen in the experimental data are exactly the same as predicted by column 3 of table 1. When we multiply the values by n we can interpret all of the experimentally measured values by use of table 1. The two columns of table 1 belong to two Kramers conjugates one belonging to $+\frac{1}{2}$ and the other to $-\frac{1}{2}$ spin and the experimental data also displays them in two groups. Thus this grouping aspect of the theory is also in agreement with the data.

Fig.1: The experimental data of Willett et al ⁷ is shown. The series, on the right hand side of 1/2 as well as that on the left hand side are exactly the same as those predicted by Shrivastava.⁴

Accurate measurements of the quantized Hall resistance upto a magnetic field of 30 Tesla have been performed by Eisenstein and Stormer⁸. The dip in ρ_{xx} at $\nu = 1/3$ is clearly seen in this measurement displayed here in Fig. 2. The value of 1/3 is in agreement with that predicted in Table 1. The entire series of fractions along with grouping in the experimental data is exactly the same as predicted. It was shown in ref. 4 that the cyclotron frequency is determined by the Bohr magneton,

Fig.2: The experimental data of the Hall resistance as a function of magnetic field as measured by Eisenstein and Stormer.⁸ All of the fractions between 15 and 30 T are exactly the same as those predicted by Shrivastava.⁴

$$\hbar\omega_c = g\mu_B B \quad (15)$$

where $\mu_B = e\hbar/2mc$. Therefore, the correction $(\frac{1}{2})g_{\pm}$ can be considered as a correction to the charge. Therefore, we can write the effective charge as

$$e_{eff} = (\frac{1}{2})ge = \nu e.$$

Therefore for $\nu = 1/3$, the effective charge becomes $(1/3)e$ which is in agreement with the ideas of a fractional charge.

In Fig.3 we show the measurements of the fractions by Du et al⁹ at a temperature of 40 mK. The series predicted by table 1 are in full agreement with the data. All of the values on the left as well as on the right hand side of $1/2$ are in full agreement with our series. From (7), one of the series is

strongest ones are at $8/3$, $5/2$ and $7/3$. Since the effective charge uses the Landau level quantum number, it is not possible to distinguish large charge from a large Landau level quantum number. The series, $l/(2l+1)$ is the particle-hole conjugate of $(l+1)/(2l+1)$. For $l = 7$ two values, $7/15$ and $8/15$ are predicted and $l = \infty$ value is $1/2$. When the same particle occurs in different levels its charge remains unchanged. We can multiply the values by $n = 5$ so that the predicted values of $1/2$, $7/15$ and $8/15$ become $5/2$, $7/3$ and $8/3$. These predicted values are exactly the same as those observed experimentally by Eisenstein et al¹⁰ and shown in Fig. 4. Thus $7/3$ is the particle-hole conjugate of $8/3$ as seen in Table 2 for $n = 5$. The states at $7/3$, $8/3$ and $5/2$ are also observed experimentally by Pan et al¹² to which our interpretation applies satisfactorily.

Table. 2: Derivation of $7/3$ and $8/3$ states for $n = 5$

l	$l/(2l+1)$	$(l+1)/(2l+1)$	$nl/(2l+1)$	$n(l+1)/(2l+1)$
∞	$1/2$	$1/2$	$5/2$	$5/2$
7	$7/15$	$8/15$	$7/3$	$8/3$

It may be noted that $5/2$ obtained by this method which is the value of $1/2$ multiplied by $n = 5$ is indistinguishable from the second level filled upto $1/2$ which is $2 + \frac{1}{2}$. Since all values are emerging in a coherent manner, the $n = 5$ interpretation is correct.

The first fraction discovered was $1/3$. Later on other fractions were seen. For a long time only odd denominators were reported which shows that even denominators are weak. For some time odd denominator rule was established but such a rule is not correct because even denominators occur as ~~well as even numerators~~ with odd denominators are found. Later on even denominators were reported. Therefore, it is observed that the fractions corresponding to even denominators are weak compared with those of odd denominators. There is a limiting

Fig. 4: The ρ_{xx} measured by Pan et al¹² is shown in the upper graph and those of Eisenstein et al¹⁰ [cond-mat/9909238] are shown in the lower graph. The interpretation of 8/3 and 7/3 is described in the text and 5/2 is the $n = 5$ of $n/2$, also given in the text.

value of the series $l/(2l+1)$ which for $l \gg 1$ gives $1/2$ which has an even denominator. Because of the limit, there is a Fermi surface at $1/2$. When higher Landau levels are populated the Fermi surface will have to be shifted from $1/2$ to $n/2$. Thus $1/2, 2/2, 3/2, 4/2, 5/2, 6/2, 7/2$ are predicted which are all observed by Yeh et al¹³ as shown in Fig. 5. Thus at least 14 fractions given in Table 1 and 7 fractions

in the limit $l \rightarrow \infty$ which adds upto 21 are correctly predicted by ref. 4. Besides $n\nu$ are also predicted. Therefore, for $n=2$ the predicted number of fractions becomes 42, etc. which are predicted correctly. Yoshioka¹⁴, has pointed out that some of the fractions claimed by the experimentalists are incorrect and have to be ignored. Hence the theory of ref. 4 is correct.

Fig. 5: Experimental data of Yeh et al¹³ showing $n/2$ states exactly as predicted by Shrivastava's theory.

In Fig. 6, we show the experimental measurement by Stormer. The fractions found are $1/3$, $2/5$, $3/7$, $4/9$ and $1/2$ which are correctly predicted by our Table 1. The values $2/3$, $3/5$, $4/7$ and 1 are all the same as predicted by us. The values $4/3$ and $5/3$ are the $n(1/3)$ which are also predicted. The values n multiplied by 1 and predicted as observed. The value 2 multiplied by $2/5$ is seen. Thus all of the observed values are predicted.

It has been reported by Yeh et al¹³ that the effective mass and g factor of some of the fractions are equal to those of others. We

have shown⁵ that the effective masses can be equal only when the two quasiparticles are particle-hole conjugates. The particle-hole conjugates should obey the following relation,

Fig. 6: Experimental data of Stormer from ref. 39 is exactly the same as predicted by us in ref. 4.

$$\nu_p + \nu_h = 1 . \quad (18)$$

For a given value of l , the two fractions given in table 1 always obey this relation and hence the fractions of the second column have the same masses as those in the corresponding row of the third column. For example $1/3 + 2/3 = 1, 2/5 + 3/5 = 1$, etc.

4 Half-filled Landau level

In the expressions (16) and (17), we take the $l = \infty$ limit so that

$$\begin{aligned} \lim_{l \rightarrow \infty} \nu_+ &= 2/2l = \frac{1}{2}(+) \\ \lim_{l \rightarrow \infty} \nu_- &= \frac{1}{2}(-) . \end{aligned} \quad (19)$$

Since these are obtained from different series we use an additional index A for $+$ and B for $-$. In this way $1/2$ comes from the right hand side as well as from the left hand side such as in Fig. 1. Various fractions from the ν_- series start from $1/3$ at the highest magnetic field and move to lower fields (right to left) as the value of l increases. Similarly, the fractions from the series ν_+ start from a fixed value $2/3$ and move to higher fields (left to right) as l increases. Both the series merge in the centre of $1/3$ and $2/3$. The remaining pattern of fractions at which there are minima in ρ_{xx} are generated by multiplying $\nu_{\pm}$ by n , the Landau level quantum number. The series represented by ν_+ and ν_- are also conjugate states representing two spin orientations. Therefore, one of the $1/2$ values is like a hole, $\frac{1}{2}(B)$ and the other half is like an electron $\frac{1}{2}(A)$. Since the electron and the hole are separated by a distance, this state is compressible. When we multiply this result by n , the Landau level quantum number, we obtain,

$$\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \frac{6}{2}, \frac{7}{2}, \frac{8}{2}, \text{ etc.}$$

which are in agreement with the experimental data. The predicted values $n/2$ are exactly as arranged in the data of Yeh et al¹³. Our result that the state is compressible is in agreement with the theoretical work of Halperin et al.^{15,16} We now determine the wave vector dependance of the response function of the $1/2$ filled Landau state.

We write the Fourier transform of the Coulomb interaction as

$$v(q) = \frac{4\pi e^2}{q^2} \quad (20)$$

so that the relation between the dielectric function, $\epsilon(q, \omega)$ and the response function becomes,

$$\frac{1}{\epsilon(q, \omega)} = 1 + v(q)\chi(q, \omega) . \quad (21)$$

The well known dielectric constant has an anomaly at $2k_F$ which is called as the Kohn anomaly.^{17,18} The dielectric constant showing loga-

arithmic divergence is given by

$$\epsilon(q, \omega) = 1 + \frac{k_s^2}{2q^2} \left\{ 1 + \frac{k_F}{q} \left(1 - \frac{q^2}{4k_F^2} \right) \ln \frac{q + 2k_F}{q - 2k_F} \right\} \quad (22)$$

where k_F is the Fermi wave vector and k_s is the screening.

We write eq.(19) as,

$$\chi(q, \omega) = \left(\frac{1}{\epsilon(q, \omega)} - 1 \right) \frac{q^2}{4\pi e^2} \quad (23)$$

which does not vary as q^2 because of the wave vector dependence of the dielectric constant. The q^2 dependence is associated with the incompressibility but clearly the response function given by (23) is compressible. For

$$\epsilon(q, 0) = 1 + \frac{k_s^2}{q^2} \quad (24)$$

$$\chi(q, 0) = \left(\frac{q^2}{q^2 + k_s^2} - 1 \right) \frac{q^2}{4\pi e^2} \quad (25)$$

For small $q(q^2 \ll k_s^2)$,

$$\begin{aligned} \chi(q, 0) &= \left(\frac{q^2}{k_s^2} - 1 \right) \frac{q^2}{4\pi e^2} \\ &\simeq \frac{q^4}{4\pi e^2 k_s^2} - \frac{q^2}{4\pi e^2} \end{aligned} \quad (26)$$

$$(27)$$

there is a correction term which varies as q^4 . For large q

$$\chi(q, 0) = \left[\left\{ 1 + \frac{k_s^2}{q^2} \right\}^{-1} - 1 \right] \frac{q^2}{4\pi e^2} \simeq -\frac{k_s^2}{4\pi e^2} \quad (28)$$

which is independent of q . Thus the response function varies as q^2 at small q with a q^4 term as q increases and becomes constant at large q .

According to eq.(19) there are two types of liquids, one is the + or A type and the other a - or B type. Therefore, we suggest a

two-component theory for the half filled Landau level. We define the induced density

$$\delta n_A(r, t) = n_A(r, t) - \bar{n}_A \quad (29)$$

for $\frac{1}{2}(+)$ and

$$\delta n_B(r, t) = n_B(r, t) - \bar{n}_B \quad (30)$$

for $\frac{1}{2}(-)$ type quasiparticles. Here $\bar{n}_A$ and $\bar{n}_B$ correspond to homogeneous values and $v_A(q, \omega)$ and $v_B(q, \omega)$ are the Fourier transforms of $v_A(r, t)$ and $v_B(r, t)$ respectively. In the generalized RPA, we write

$$\begin{aligned} \delta n_A(q, \omega)[1 - \chi_o^A(q, \omega)\psi_{AA}(q)] &= \chi_o^A(q, \omega)v_A(q, \omega) + \chi_o^A(q, \omega)\psi_{AB}(q) \\ &\times \langle \delta n_B(q, \omega) \rangle \\ \delta n_B(q, \omega)[1 - \chi_o^B(q, \omega)\psi_{BB}(q)] &= \chi_o^B(q, \omega)v_B(q, \omega) + \chi_o^B(q, \omega)\psi_{BA}(q) \\ &\times \langle \delta n_A(q, \omega) \rangle \end{aligned} \quad (31)$$

where $\psi_{AA}, \psi_{BB}, \psi_{BA}$ and ψ_{AB} represent the effective static interaction between particles belonging to components A and B with $1/2$ field Landau level as in eq.(19). $\chi_o^A(q, \omega)$ is the noninteracting response function of the A component and $\chi_o^B(q, \omega)$ is that of the B component. The above two relations may be combined with $\psi_{AB} = \psi_{BA}$ so that,

$$\begin{aligned} \delta n_A(q, \omega) &= \chi_o^A(q, \omega)[1 - \chi_o^B(q, \omega)\psi_{BB}(q)]v_A(q, \omega)/\Delta(q, \omega) \\ &\quad + \chi_o^A(q, \omega)\chi_o^B(q, \omega)\psi_{BA}(q)v_B(q, \omega)/\Delta(q, \omega) \\ \Delta(q, \omega) &= \{1 - \chi_o^A(q, \omega)\psi_{AA}(q)\}\{1 - \chi_o^B(q, \omega)\psi_{BB}(q)\} \\ &\quad - \chi_o^A(q, \omega)\chi_o^B(q, \omega)\psi_{AB}(q)\psi_{BA}(q). \end{aligned} \quad (32)$$

The equation for $\delta n_B(q, \omega)$ can be written from the above by interchanging A and B . From (28) and (31) we can write the response function as a symmetric matrix with matrix elements,

$$\begin{aligned} \chi_{AA}(q, \omega) &= \chi_o^A(q, \omega)[1 - \chi_o^B(q, \omega)\psi_{BB}(q)]/\Delta(q, \omega) \\ \chi_{AB}(q, \omega) &= \chi_o^A(q, \omega)\chi_o^B(q, \omega)\psi_{AB}(q)/\Delta(q, \omega) \\ \chi_{BB}(q, \omega) &= \chi_o^B(q, \omega)[1 - \chi_o^A(q, \omega)\psi_{AA}(q)]/\Delta(q, \omega) \end{aligned} \quad (33)$$

Thus the wave vector dependence of the response function in (23) is continued in the matrix elements and small corrections occur over the unperturbed values. There is enhancement in the response function at $q = 2k_F$. The half filled Landau level consists of A and B type liquids which are coupled by spin-orbit interaction. For small wave vectors there is a correction in the response function varying as the fourth power of the wave vector over and above quadratic behaviour. For large wave vectors, the response function is independent of the wave vector. The compressibility arises from the filling structure of the Landau levels.

Halperin et al^{15,16} have reported that the quasiparticles at the half filled Landau levels are compressible and their response function varies linearly as a function of wave vector going to a finite value as the wave vector approaches zero, $\chi(q) = \text{const.} + q/(2\pi e^2)$. In our theory, eq.(26-27), the wave vector dependence is more complicated but does not contradict Halperin.

5 Diamagnetic points.

We assume that magnetic induction is independent of time so that $dB/dt = 0$. According to a Maxwell equation,

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (34)$$

so that $\nabla \times E = 0$ gives $E = 0$. Since the current $j = \left(\frac{1}{\rho}\right) E$, the value $E = 0$ is consistent with $\rho = 0$. In quantum Hall effect when $\rho_{xx} = 0$ there is a large magnetic field along the z direction. Therefore, we replace B_z by $B_z - B_o$ so that

$$\frac{d}{dt}(B_z - B_o) = 0 \quad (35)$$

which means that the Maxwell equation is satisfied with no field inside the semiconductor only when the magnetic field is shifted by B_o .

Using the Maxwell equation, we can write,

$$\nabla \times (E_x - E_o) = 0 \quad (36)$$

so that $E_x - E_o = 0$. The Ohm's law is

$$\rho_{xx}j = E_x - E_o \quad (37)$$

which is consistent with $\rho_{xx} = 0$ for $E_x - E_o = 0$. This means that $\rho_{xx} = 0$ points are consistent with shifts in both the electric as well as the magnetic field. The magnetic field of 8.38 T may be written as energy $H^2/8\pi = 1.59 \times 10^{20}$ eV/cm³ which for a volume of $80 \mu\text{m} \times 80 \mu\text{m} \times 15$ nm, where the dimensions of the quantum well are used, is equal to 0.1598 meV. Thus the field shift is associated with an energy of the order of 0.1598 meV. As the x -component of the resistivity approaches zero, the energy of about 0.1598 meV is emitted.

6 Laughlin's wave function and ground state¹⁹⁻²¹

Laughlin's prototype wave function for the state with fractional charge is described by

$$\psi_m(z_1, z_2, \dots, z_n) = \prod_{j < k}^N (z_j - z_k)^m \exp \left(-\frac{1}{4\pi l^2} \sum_j^N |z_j|^2 \right) \quad (38)$$

where m is an odd integer, in this case 3 and $z_j = x_j + iy_j$ is the complex coordinate. For large N , the electron density is $\rho = 1/2\pi m l^2$ where l is the magnetic length, $l = (\hbar c/eB)^{1/2}$. The repulsive Coulomb interactions by themselves can not produce a state with quasiparticle charge $1/3$. Therefore, it is first assumed on the basis of experimental data that the quasi particle charge is $1/3$ and then the wave function is written down. Similarly, for Coulombian interactions the wave function for the quasi particle charge $1/5$ may be written but the correct series

can not be obtained. Lam and Girvin²² find that ψ_m is unstable to crystallization for $m > 7$. Haldane and Rezayi²³ and Morf and Halperin²⁴ have calculated the electron charge densities and the creation energies associated with the wave functions. By means of eq.(15) and the subsequent discussion, we have no difficulty in producing an effective charge of $1/3$ except that mapping has to be done from Laughlin's wave functions of fractional charge of $1/3$ to the spin, orbit and Landau state, $\psi_m \rightarrow \psi_s \psi_L |n\rangle$. Laughlin's charge is independent of spin so that there is complete decoupling between spin and charge. Thus charge can be fixed without any knowledge of spin. Besides, Laughlin's wave function can not get equal mass for unequal charge. On the other hand, the theory of ref. 4 which agrees with the data fixes the charge as well as the spin. Wilczek^{25,26} has given the ideas of attaching magnetic flux tubes to particle and Jain²⁷ has suggested a composite fermion model with even number of flux quanta attached to each electron. The success of Jain's paper is the result of the correct series, but there is a prior dated paper⁴. The idea of attaching flux tubes to particles²⁵⁻²⁷ is not in agreement with the experimental data and as we have seen above, both the electric as well as the magnetic fields have to be shifted. Accordingly, both the vector potential as well as the scalar potential of the electromagnetic field requires corrections, which are called Chern-Simons fields.^{28,29} There are several experiments which show that the effective fractional charge is observable.³⁰⁻³⁵

7 Conclusions

The fractional charges found in the quantum Hall effect can be well explained by the angular momentum theory of this author.⁴ Usually, the electrons are in the conduction band and hence the orbital angular momentum quantum number is zero, $l = 0$. However, when finite values of l are considered, we can define an effective fractional charge. Laughlin's theory of fractional charge defines the wave function and the

energies. When the energy computations are performed by others^{23,24} good agreement is found with Laughlin's calculations. However, Laughlin's wave functions do not use the spin or the orbit and hence there is spin-charge decoupling. Our theory of ref. 4 makes use of the spin and orbit to predict the effective charge and the various Hall steps correctly and in agreement with the experimental data. We also study the wave vector dependence of the response function from which we conclude that the quasi particles are compressible. Störmer³⁷ and Tsui³⁸ have obviously performed very ingenious measurements of the Hall resistance, and Laughlin has fractionalized the charge. In our theory, the fractions add up to unity as in eq.(18) and exhibited in the pictorial form in Fig.7.

Fig. 7. Pictorial representation of Laughlin's fractional charge and Shrivastava's adding up to unity. In our theory every fragment has spin whereas in Laughlin's theory only charge is fractionalized.

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