An updated review is presented of our understanding of color confinement. Lattice results on condensation of magnetic charges are discussed. The role of vortices is analysed.

1. Introduction

Confinement of color in QCD means absence of colored particles in asymptotic states. Very stringent experimental evidence exists for confinement. In the standard cosmological model the expectation for the relative abundance of quarks to nucleons $n_q/n_p$ is, in the absence of confinement\cite{1}, $\frac{n_q}{n_p} \approx 10^{-12}$. Millikan like experiments trying to detect fractionally charged particles do not see any and give an upper limit\cite{2} $\frac{n_q}{n_p} \leq 10^{-27}$ corresponding to the analysis of $\sim 1$ g of matter.

A factor $10^{-15}$ is too small to be explained in terms of small parameters. Confinement is most likely an absolute property, to be explained in terms of symmetry, like superconductivity. This will be the first prejudice that we adopt in our analysis.

The second prejudice will be that the mechanism of confinement has to be independent of the number of colors $N_c$ ($N_c = 3$ in QCD) and of the number $N_f$ of light flavours ($N_f < N_c$). This assumption is based on the idea that the limit $N_c \to \infty$, at $\lambda = g^2 N_c$ fixed of $SU(N_c)$ gauge theory is smooth\cite{1}: $1/N_c$ is a good expansion parameter, and $1/3$ does not differ too much from 0. Evidence for that comes from lattice simulations, which confirm the explanation of the $\eta'$ mass and the solution of the $U(1)$ problem in terms of topological susceptibility\cite{4,5}. Moreover the quenched approximation (no quark loops) proves to be correct within 10%, supporting the idea that quarks loops are non leading in $1/N_c$.

2. Duality: order and disorder\cite{6,7}

Duality is a deep concept in statistical mechanics and quantum field theory. It applies to systems admitting field configuration with non trivial topology. Such systems can be given two complementary and equivalent descriptions:

a) a direct description in terms of the fundamental fields $\Phi$, which works in the weak coupling regime ($g \ll 1$), or ordered phase. The symmetry of the ground state is described by the vev of the fields $\langle \Phi \rangle$ which are called order parameters. Topological configurations $\mu$ are non local in this description.
b) A dual description, which works in the strong coupling regime ($g > 1$), or disordered phase. The topological excitations $\mu$ are local, their vev’s $\langle \mu \rangle$ (disorder parameters) identify the symmetry of the ground state and the original fields $\Phi$ are now nonlocal excitations. In the dual description the coupling constant $g_D$ is related to $g$ by the relation $g_D \approx 1/g$. Duality maps the strong coupling regime of the direct description into the weak coupling of the dual and viceversa.

Examples of systems with dual behaviour are the 2d Ising model[7], the first where the idea of duality was tested; supersymmetric QCD with $N = 2$, in which $\Phi$ and $\mu$ are fields with electric and magnetic charge[8]; liquid $He_4$ where the dual excitations are vortices[9], Heisenberg magnet[11], where the dual excitations are Weiss domains; $U(1)$ compact gauge theory, where $\mu$ are monopoles[11,12].

Finite temperature QCD is the euclidean version on an imaginary time interval $0, 1/T$, with $T$ the temperature; due to asymptotic freedom high temperature (quark gluon plasma) corresponds to weak coupling (ordered phase), low temperature (confined phase) to strong coupling (disordered phase). To explain confinement in terms of symmetry we have to understand strong coupling symmetry, or $\langle \mu \rangle$ the disorder parameters in the language of duality.

3. Topological configurations in QCD

The natural topology in 3d is related to the mapping of the sphere $S_2$ at spatial infinity on a group. A mapping $S_2 \rightarrow SU(2)/Z_2$ has monopoles as topological configurations $\mu$[13]. For $SU(N)$ the corresponding mapping is $S_2 \rightarrow SU(N)/SU(p) \otimes SU(N - p) \otimes U(1)$

If $\mu$ carries magnetic charge $\langle \mu \rangle \neq 0$ signals condensation of magnetic charges, or dual superconductivity. Confinement follows by dual Meissner effect, the chromoelectric field between quarks being confined into Abrikosov flux tubes, with energy proportional to the distance[14].

An alternative approach is inspired by a 2+1 dimensional version of QCD[15]: the space is 2d and the mapping is from $S_1$, the circle at spatial infinity. The natural topological configuration is a vortex, and vortices are expected to condense in the disordered phase. It is not clear how to translate this phenomenon in the realistic case of 3 + 1 dimensions. Instead of the symmetry of the vacuum the idea of vortex enters in the algebra of the operators $W(C)$, the creator of a Wilson loop, and $B(C)$, the creator of a dual loop:

$$B(C)W(C') = W(C')B(C)e^{in_{CC'}2\pi}$$

Eq.(1) implies that whenever $W(C)$ obeys the area law at large extensions, $B(C)$ obeys the perimeter law and viceversa.

Monopoles are identified by a procedure known as abelian projection[16], which in the simple case of $SU(2)$ works as follows. Let $\Phi = \Phi/|\Phi|$ be the orientation of $\Phi$ in color space: $\Phi$ is defined everywhere except at zeros of $\Phi$. A field strength can be defined

$$F_{\mu\nu} = \Phi G_{\mu\nu} - \frac{1}{g} \Phi(D_{\mu} \Phi \wedge D_{\nu} \Phi)$$

with $G_{\mu\nu}$ the usual gauge field strength and $D_{\mu}$ the covariant derivative. Both terms in
eq.(2) are gauge invariant, and they are chosen in such a way that bilinear terms in $A_\mu A_\nu$ cancel. By simple algebra

$$F_{\mu\nu} = \hat{\Phi}(\partial_\mu A_\nu - \partial_\nu A_\mu) - \frac{1}{g} \hat{\Phi}(\partial_\mu \hat{\Phi} \wedge \partial_\nu \hat{\Phi}) \quad (3)$$

In the gauge in which $\hat{\Phi} = \text{const.}$ the second term drops and $F_{\mu\nu}$ is an abelian field. By defining $j^M_\mu = \partial_\nu F^{\star}_{\mu\nu}$, with $F^{\star}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F_{\rho\sigma}$, $\partial_\mu j^M_\mu$ is identically zero, and a conserved magnetic charge exists. This happens for any choice of $\hat{\Phi}$. There exist a huge infinity of conserved magnetic charges: it is not clear if they are independent from each other. One can investigate if they condense in the vacuum by constructing an operator $\mu$ which has nonzero magnetic charge, and by measuring its vev $\langle \mu \rangle$. If $\langle \mu \rangle \neq 0$ in the confined phase and tends to zero at the deconfining transition, dual superconductivity is at work and $\langle \mu \rangle$ is a disorder parameter for confinement. In formulae this means $\langle \mu \rangle \neq 0 \, T < T_c$ and

$$\langle \mu(T) \rangle \simeq (1 - \frac{T}{T_c})^\beta \quad \beta = \frac{2N}{g^2} \quad (4)$$

In a lattice $\left(1 - \frac{T}{T_c}\right) \sim (\beta_c - \beta)$. It proves more convenient to measure $\rho = \frac{d}{dT} \ln \langle \mu \rangle$.

A behaviour like (4) corresponds to a negative peak in $\rho$ at $\beta_c$, which becomes sharper and sharper as the spatial volume increases. A finite size scaling analysis allows to reach the infinite volume limit, and determines the value of $\beta_c$ and of the critical exponents of the correlation length and of $\langle \mu \rangle$ itself, $\delta$. A typical behaviour is shown in Fig.1.

The net result of the investigation is that there is condensation of monopoles in all the abelian projections. Dual superconductivity is at work and confinement does indeed stem from a symmetry[12]. The mechanism is the same in presence of fermions, in agreement with the ideas of $N_c \to \infty$. The dual excitations which should describe the disordered phase, must have nonzero magnetic charge in all the abelian projections.

Vortices can also give hints to identify the dual fields. A consequence of the algebra (1) is that the dual Polyakov line $\langle B(C) \rangle$ corresponding to a path $C$ which is a stright line in space going from $-\infty$ to $+\infty$ with periodic b.c., should play the dual role of the analogous Wilson loop (the Polyakov line) going from $-\infty$ to $+\infty$ along the time axis.

This is indeed what happens. What is very suggestive for further investigations is that the behaviour of $\langle B(C) \rangle$ across the phase transition is identical to that of $\langle \mu \rangle$, the disorder parameter of monopole condensation (Fig.2)[18].

4. Conclusions

Confinement is indeed related to symmetry of the vacuum: the same symmetry independent of the presence of dynamical quarks. Magnetic symmetry is broken in all abelian projections. Dual excitations must have non zero magnetic charge in all of them. The dual Polyakov line is also a good disorder parameter, and pratically equal to the one defined in terms of monopoles.

It is not yet clear what exactly the dual excitations are.
Figure 1. Monopoles: $\rho$ versus $\beta$ for different spatial sizes.

Figure 2. $\rho$ versus $\beta$ for monopoles and vortices.

REFERENCES