We propose a framework where the string scale as well as all compact dimensions are at the electroweak scale $\sim \text{TeV}^{-1}$. The weakness of gravity is attributed to the small value of the string coupling $g_s \sim 10^{-16}$, presumably a remnant of the dilaton’s runaway behavior, suggesting the possibility of a common solution to the hierarchy and dilaton-runaway problems. In spite of the small $g_s$, in type II string theories with gauge interactions localized in the vicinity of NS5-branes, the standard model gauge couplings are of order one and are associated with the sizes of compact dimensions. At a TeV these theories exhibit higher dimensional and stringy behavior. The models are holographically dual to a higher dimensional non-critical string theory and this can be used to compute the experimentally accessible spectrum and self-couplings of the little strings. In spite of the stringy behavior, gravity remains weak and can be ignored at collider energies. The

---

1 On leave of absence from CPHT, Ecole Polytechnique, UMR du CNRS 7644.
Damour-Polyakov mechanism is an automatic consequence of our scenario and suggests the presence of a massless conformally-coupled scalar, leading to potentially observable deviations from Einstein’s theory, including violations of the equivalence principle.
1. Introduction

An obstacle to building a unified theory of all forces is the enormous disparity between
the gravitational and other forces, commonly referred to as the hierarchy problem. In the
standard framework of particle physics this is answered at the expense of postulating an
enormous energy desert separating the gravitational from the electroweak scale [1]. The
supersymmetric version of this picture [2], called the supersymmetric standard model, has
had a quantitative success: the unification prediction of the value of the weak mixing angle
[2], subsequently confirmed by the LEP and SLC experiments. This makes it tempting
to believe in the unification of the non-gravitational forces at a large energy scale \( \sim 10^{16} \)
GeV. Nevertheless, this picture leaves many fundamental questions unanswered. There are
125 parameters in the supersymmetric standard model that remain unexplained. These
include the masses of the three generations of particles and, above all, the incredible
smallness of the cosmological constant. This suggests that there are enormous gaps in our
understanding of Nature at low energies and that perhaps we will need a radical revision
of our fundamental view of the world at low energies, at least with respect to gravity.

On the other hand, string theory provides the only known framework for quantizing
gravity. The cost is to replace our fundamental concept of point particles by extended
objects whose quantum consistency requires the existence of extra dimensions. One of
the important consequences of the recent theoretical progress on the non-perturbative
dynamics of string theories is that the string and compactification scales are not necessarily
tied to the four-dimensional Planck mass [3,4,5]. This opens the exciting possibility that
string physics may become relevant at much lower energies with spectacular new effects in
future accelerators.

Such a possibility can also be used to explain the hierarchy problem, motivated by
the following string theoretic expression for the four-dimensional (4d) Planck mass [6]:

\[
M_P^2 = \frac{1}{g_s^2} M_s^8 V_6 ,
\]

where \( g_s \) is the string coupling, \( M_s \) the string scale and \( V_6 \) the volume of the six-dimensional
internal space. This relation shows that it is possible that there is only one fundamental
scale in the universe, the electroweak scale \( \sim \text{TeV} \), where all forces of nature, including
gravity, unify and therefore \( M_s \sim \text{TeV} \). Then the enormity of the Planck scale can be
accounted for in two distinct ways:
(1) A non-stringy way [6]:
This is realized if $V_6$ is enormously larger than the fundamental scale while keeping $g_s$ of order unity. In order to make such large dimensions consistent with observations, gauge interactions should be localized on branes transverse to them. A natural framework for realizing this scenario is type I string theory with the Standard Model (SM) confined on a collection of $D_p$-branes. Perturbative calculability requires the $p-3$ longitudinal dimensions to be compactified near the (TeV) string scale, while the $9-p$ transverse dimensions should be much larger in order to account for the observed weakness of gravitational interactions.

While this scenario can be naturally imbedded in type I string theories, it does not require string theory for its implementation at low energies, below the (TeV) scale of quantum gravity. The physical mechanism is the dilution of the strength of gravity by spreading it into extra dimensions, which could have been invented by Gauss two centuries ago. The hierarchy problem now turns into one of explaining dynamically the large magnitude of $V_6$.

(2) A stringy way:
This is realized by taking $V_6$ to be of the order of the fundamental scale $\sim \text{TeV}^{-6}$ and attributing the enormity of the Planck mass to a tiny $g_s \sim 10^{-16}$ [7] (see also [5]). The hierarchy problem is now equivalent to understanding the smallness of $g_s$, or equivalently the large value of the dilaton field in our universe.

Starting with [6], possibility (1) has been explored extensively in the last three years. Our objective in this paper is to study the second logical possibility (2) which is stringy in nature, at least in the sense that it involves $g_s$, and gives a new perspective to the hierarchy and other problems in physics. A fundamental question now becomes whether a string theory with such a small $g_s$ can contain the ordinary gauge interactions whose dimensionless couplings are of order unity. Fortunately the answer is yes in the context of special type II string theories whose gauge interactions are localized in the vicinity of NS5-branes, which we will utilize here. In these theories gauge couplings are given by the geometric sizes of new dimensions and are non-vanishing even if $g_s$ vanishes.

In the limit of vanishing $g_s$ [8] one obtains a theory without gravity, the so-called Little String Theory (LST); it was introduced in [9,8] (see also [10],[11], and for a review

\begin{footnote}{Actually, the value of $g_s$ is more likely $10^{-14}$ [7], corresponding in eq. (1.1) to a volume $V_6 \simeq (2\pi)^6$ of a toroidal compactification with all radii fixed at the string length.}

\end{footnote}
see [12] and references therein). LST is a partial string theory; although it does not include gravity, it has string excitations and therefore is not a normal local field theory. It is an intermediate logical possibility between full-fledged string theory and field theory. The main objective of our paper is to point out that this intermediate possibility can be realized at the experimentally accessible energy of \( \sim \) TeV and give us an alternate way to address the hierarchy problem which connects it with the dilaton-runaway problem. This therefore interpolates between the TeV-strings framework [6], which has full string theory at a TeV, and other field-theoretic possibilities for TeV physics such as supersymmetry, technicolor or warped compactifications [13].

We propose three closely related frameworks for building realistic theories with little strings at a TeV. Their common feature is the existence of closed little strings with \( \sim \) TeV tension, whose self interaction and spectroscopy can be computed in some cases. In addition there can be string excitations of ordinary particles, with either the same or different tension, as well as KK and winding modes associated with \( \sim \) TeV\(^{-1}\) size dimensions.

An unexpected bonus of the framework is that the Damour-Polyakov mechanism, based on the universality of the dilaton coupling functions and normally considered improbable, becomes automatic. It may lead to small but potentially observable deviations from the equivalence principle [14].

In section 2, we discuss mass scales and couplings in type II string theories, and define our general framework. In section 3, we recall the possible descriptions of little string theories and in particular the double scaling limit which defines a sensible perturbation theory. In section 4, we touch on some basic phenomenological consequences of the framework. Section 5 addresses the hierarchy problem and suggests ways in which the dilaton field can have a naturally large value in our universe. In section 6, we remark on a possible implication of our framework for the cosmological constant problem and other topics.

2. Mass Scales and Couplings

In every perturbative string theory, gravity arises from closed strings that propagate in ten dimensions. As a result, the 4d Planck mass is given by eq. (1.1). Here, all internal dimensions are taken to be larger than the string length \( l_s \equiv M_s^{-1} \) by a suitable choice of T-dualities; thus, in this convention, all closed string winding modes are heavier than the string scale. The strength of gravity at energies above all compactification scales and
below the string scale, \( V_6^{-1/6} < E < M_s \), is determined by the ten-dimensional Planck mass
\[
M_{10}^8 = \frac{M_P^2}{V_6} = \frac{M_s^8}{g_s^2},
\]
which can be obtained by summing over all KK graviton excitations yielding a suppression proportional to \( (E/M_{10})^8 = (E/M_s)^8 g_s^2 \). It follows that at energies of order the string scale gravitational interactions are controlled by the string coupling \( g_s \).

On the other hand, in type II theories non-abelian gauge interactions arise non-perturbatively localized on (Neveu-Schwarz) NS5-branes, corresponding in the simplest case to D-branes stretched between the NS5-branes. In a T-dual picture, non-abelian fields in (supersymmetric) type IIA (IIB) theories emerge from D2 (D3) branes wrapping around collapsing 2-cycles of the compactification manifold [15]. Such 2-cycles are localized in a subspace of dimension 4 that defines (upon T-duality) the transverse position of the NS5-branes where gauge interactions are confined. Furthermore, the four-dimensional Yang-Mills (YM) coupling is determined by the geometry of the two-dimensional compact space along the NS5-branes, independently of the value of the string coupling \( g_s \).

For instance, let us consider a stack of NS5-branes extended in the directions \( X^{0,1,2,3,4,5} \), where \( X^{0,1,2,3} \) define our 1+3 dimensional spacetime. The extra two longitudinal directions \( X^{4,5} \) are compactified on a rectangular torus \( T^2 \) with radii \( R_{4,5} \), while the four transverse directions \( X^{6,7,8,9} \) are compactified on a manifold with size \( R_t \). The four-dimensional gauge coupling is then given by
\[
\text{type IIA : } \quad g_{YM}^2 = \frac{R_4}{R_5},
\]
\[
\text{type IIB : } \quad g_{YM}^2 = \frac{l_s^2}{R_4 R_5}.
\]

In summary, in type II theories gravitational interactions are controlled by the string coupling, while gauge interactions are governed by geometrical moduli along the 5-branes where they are confined. This is in contrast with type I theories, where the string coupling determines also the strength of gauge interactions confined on D-branes and is therefore fixed to be of order one \(^3\); thus, in type I theories gravitational interactions become strong at the string scale.

It follows that the type II string scale can be lowered at the TeV scale without introducing extra large transverse dimensions, but instead a tiny string coupling to account

\(^3\) Here we drop factors of \( \pi \) and for numerical estimates we use \( g_{YM} \simeq 0.1 \).
for the hierarchy $M_s/M_P$ [7]. In this case, the physics around the string scale is described approximately by a theory without gravity obtained in the weak coupling limit $g_s \rightarrow 0$ [8]. This theory, which is defined in the limit of coincident NS5-branes with vanishing string coupling, is called little string theory (LST) [8-12].

This theory lives in six dimensions and contains two sectors. The charged (non-abelian) sector confined on the NS5-branes and a neutral sector of closed fundamental strings trapped in the vicinity of the NS5-branes. In section 3, we review the main properties of these theories, while in section 4 we discuss their phenomenological consequences when the fundamental string scale is in the TeV region.

One may ask the question whether a tiny string coupling can be described alternatively, via some duality, in terms of large dimensions in the context of M-theory. Indeed, it was shown that the weakly coupled type II string compactified for instance on $K3 \times T^2$, with all compactified dimensions of string size, provides a dual description to the strongly coupled heterotic theory compactified on $T^4 \times T^2$ with the four dimensions of $T^4$ having the heterotic string size $l_H \sim M_P^{-1}$ while the two dimensions of $T^2$ being much larger, of the order of the type II string length [7]. If the type II string scale $M_s \sim \text{TeV}$, its string coupling $g_s \sim M_s/M_P \simeq 10^{-16}$, while the heterotic coupling is huge $g_H \sim l_s/l_H \sim 1/g_s$. Using heterotic – M-theory duality, one can find an alternative description in terms of M-theory compactified on the eleventh-dimensional interval $S^1/Z_2 \times T^4 \times T^2$. The M-theory length scale $l_M = g_H^{1/3} l_H$, so that $l_M^{-1} \sim 10^{14}$ GeV and the size of the eleventh dimension $R_{11} = g_H l_H \sim \text{TeV}^{-1}$. Thus, there are three large dimensions at the TeV ($R_{11}$ and $T^2$) and four small dimensions of Planck length (corresponding to $T^4$) which invalidate the effective field theory description and makes this M-theory interpretation of no practical use.

3. LST and D-branes in LST

In this section we recall some aspects of LST and its weakly coupled version (DSLST). Using the idea of holography we review a dual description of the theory as a “non-critical” string, which allows to compute the spectrum and couplings in some appropriate regime. Moreover, this description provides a geometric set up which has some analogy with the scenario of [13]. We shall emphasize the similarities in this section, and we will make some remarks at the end of the paper. Finally, we also consider a different family of theories, on D-branes in LST, which shares some of the properties of LST.
3.1. Little String Theory

For simplicity, we first consider the six-dimensional LST (for a review, see [12] and references therein). One way to define this theory is the following. We start with a stack of $k$ NS fivebranes in type II string theory with a string coupling $g_s$, and take the limit $g_s \to 0$. In this limit bulk degrees of freedom, including gravity, are decoupled, and one is left with a six-dimensional theory of strings without gravity. An alternative description is to consider the $g_s \to 0$ limit of type II on a singular K3 manifold. The two definitions are related by a T-duality.

This LST has the following properties [12] (in the conventions of the first definition):

(i) It has a unique scale $M_s$ – the string mass scale of the original type II string theory.
(ii) In type IIA, the low energy theory is an $N = (2, 0)$ six-dimensional SCFT, while in type IIB it is an $N = (1, 1)$, $SU(k)$ gauge theory with a gauge coupling $g_{YM} \sim 1/M_s$.
(iii) It has a Hagedorn density of states and the Hagedorn temperature is $T_H = M_s/(2\pi\sqrt{k})$.
(iv) It is argued [16] that this theory is “holographically” dual to a higher dimensional string theory (with gravity): the type II string on

$$\mathcal{M} = R^{5,1} \times R_\phi \times SU(2)_k.$$  \hspace{1cm} (3.1)

Here $R^{5,1}$ is the 5+1 dimensional Minkowski space in the directions of the worldvolume of the fivebranes. $R_\phi$ is the real line parameterized by a scalar $\phi$, with a linear dilaton

$$\Phi = -\sqrt{\frac{1}{2k}} \phi.$$  \hspace{1cm} (3.2)

$\phi$ is related to the radial direction $r$ of the $R^4$ space transverse to the fivebranes: $\phi \sim \log r$. The $SU(2)_k$ is a level $k$ WZW SCFT on the $SU(2) \simeq S^3$ background, where this three sphere is related to the angular coordinates of the transverse $R^4$. This background is obtained in the near horizon limit of the fivebranes, and describes the SCFT on the infinite “throat” (see figure 1(a)). The fivebranes might be thought of as sitting deep down the throat, namely, at $\phi \to -\infty$ $(r \to 0)$ where the theory is strongly coupled ($\exp(2\Phi)$ is large (3.2)). On the other hand, as $\phi \to \infty$ $(r \to \infty$, towards the decoupled asymptotically flat space far from the fivebranes) the theory is weakly coupled.

(v) Off-shell observables in LST correspond to on-shell observables in string theory on $\mathcal{M}$ (3.1). Observables in the theory correspond to non-normalizable vertex operators,
namely, those whose wave function is exponentially supported at the weak coupling regime \( \phi \to \infty \). There are also \( \delta \) function normalizable operators whose role in the theory is less clear. The latter form a continuum of states, whose contribution to the density of states is a small fraction.

The holographic description above is useful to identify observables and their properties under the symmetries of the theory. However, correlation functions cannot be computed in perturbation theory because they are sensitive to the strong coupling regime down the throat. To resolve this strong coupling problem we shall “chop” the strong coupling regime of the throat (see figure 1).

![Diagram ofthroat and branes](image)

**Fig. 1:** (a) The infinite throat background dual to strongly coupled LST; (b) The strong coupling region is chopped into a cigar-like geometry, whose tip is associated with a SM brane while the asymptotic region is associated with a Planck brane.
It is convenient to decompose the $SU(2)_k$ SCFT on $S^1_Y \times SU(2)_k/U(1)$, where $S^1_Y$ is the Cartan sub-algebra of $SU(2)$, parameterized by a scalar $Y$, and the $SU(2)_k/U(1)$ quotient SCFT is equivalent to a level $k$, $N=2$ minimal model. Then the throat SCFT becomes the product of an infinite cylinder $R_\phi \times S^3_Y$ (with a linear dilaton) times an $N=2$ minimal model:

$$R_\phi \times S^3 \simeq R_\phi \times S^1_Y \times SU(2)_k/U(1).$$

(3.3)

One way \cite{17} to chop the strong coupling regime of the throat is to replace the infinite cylinder $R_\phi \times S^1_Y$ with the semi-infinite cigar \cite{18} SCFT $SL(2)_k/U(1)$ (see figure 1(b)):

$$\mathcal{M} = R^{5,1} \times R_\phi \times S^1_Y \times \frac{SU(2)_k}{U(1)} \to R^{5,1} \times \frac{SL(2)_k}{U(1)} \times \frac{SU(2)_k}{U(1)}.$$  

(3.4)

This corresponds to separating the $k$ NS fivebranes on a transverse circle of radius $L$ in the double scaling limit $g_s, L \to 0$ such that $g_s/L$ is held fixed \cite{19,17}. The string coupling takes its maximal value at the tip of the cigar where

$$g_s(\text{tip}) \equiv g_{\text{lst}} \sim \frac{g_s}{LM_s},$$

(3.5)

while it approaches 0 as one goes away from the tip ($\phi \to \infty$) along the radial direction $\phi$ of the cigar. The scalar $Y$ parameterizes the angular direction of the cigar whose radius is $R_{\text{cigar}} \sim \sqrt{k}/M_s$ asymptotically. The separation of the fivebranes introduces another scale in the theory. In type IIB it is the mass of a gauge boson corresponding to a D-string stretched between two NS5-branes, giving rise to a charged particle in the low energy $SU(k)$ gauge theory with mass

$$M_{\text{IIB}}^{W} \sim T_{D1}L = \frac{M_s^2 L}{g_s} = \frac{M_s}{g_{\text{lst}}},$$

(3.6)

where $T_{D1} = M_s^2/g_s$ is the D-string tension. One may regard the above as chopping the infinite throat by SM branes (separated on a circle) near the tip of the cigar (see figure 1(b)).

So far we have considered the theory decoupled from gravity. The decoupling limit corresponds, in particular, to the limit $M_P \sim M_s/g_s \to \infty$. To keep gravity at the finite (although large) scale $M_P$ observed in nature, we should relax the limit $g_s \to 0$ although the string coupling is still very small, as discussed in section 2. One may regard this as chopping the weak coupling regime of the semi-infinite cigar – the other side of the
original throat – by a Planck brane \(^4\) (see figure 1(b)). We shall work in a scenario where \(M_s, M_W^{IIB} \ll M_P\) and, therefore, the effects of gravity for \(E \sim M_s\) are negligible.

This Double Scaled LST (DSLST) [17] has a weak coupling expansion parameter \(g_{lst} = M_s/M_W^{IIB}\) when \(M_W^{IIB} > M_s\) (we may however keep \(M_W^{IIB} \ll M_P\)). This allows one, in principle, to compute correlation functions perturbatively for processes at energies even larger than \(M_s\) (as long as they are sufficiently lower than \(M_W^{IIB}\)). On-shell correlators in the string theory (3.4) correspond, via holography, to off-shell Green’s functions in the six-dimensional spacetime theory. From the analytic structure of the two point functions one can read the physical spectrum while the three point functions give rise to the couplings of physical states, via the LSZ reduction. The two and three point functions were computed in [17,21], with the following results:

1. **2-p-f:** The two point functions have a series of single poles, from which one can read the mass spectrum, followed by a branch cut (the poles correspond to the principal discrete series in the unitarity range of the \(SL(2)/U(1)\) SCFT while the branch cut is due to the principal continuous series). The massless states correspond to photon multiplets in the low energy theory. They are followed by a discrete spectrum organized into Regge trajectories due to string excitations. The interpretation of the continuum is less clear, and is probably associated with “long strings” (see [22,23] and references therein). When \(g_s\) is finite the continuum in the spectrum is discretized.

2. **3-p-f:** The three point couplings allow, in particular, the decay of a massive discrete state into two massless states. Hence, one expects the stringy states to affect the form factor of the “photon” at energies of the order \(M_s\).

Four-dimensional theories (at low energy) can be constructed in various ways, for instance:

(i) By compactifying two directions longitudinal to the fivebranes on a two torus, as described in section 2. The theory at energies below \(M_s\) and the compactification scale is an \(N = 4, SU(k)\), four-dimensional gauge theory.

(ii) Four-dimensional LST whose low energy limit is an \(N = 2\) SCFT in the moduli space of pure \(N = 2, SU(n)\) gauge theory can be studied by considering the near horizon of a fivebrane wrapping a (singular) Riemann surface. Its holographic dual is [24] a type II string on

\[
\mathcal{M} = R^{3,1} \times \frac{SL(2)_k}{U(1)} \times \frac{SU(2)_n}{U(1)},
\]

\(^4\) More precisely, it is done by keeping the asymptotically flat regime “glued” to the throat, with the four-dimensional space transverse to the NS5-branes being compactified on \(T^4/Z_2\) (similar to [20]) and with the appropriate number of NS5-branes as required by global issues.
with $k = \frac{2n}{n+2}$.

(iii) Richer four-dimensional LSTs can be obtained by replacing the level $n$, $N = 2$ minimal model in (3.7) with a richer $N = 2$ SCFT. For instance, replacing $\frac{SU(2)_n}{U(1)}$ by $[\frac{SU(2)_n}{U(1)} \times \frac{SU(2)_n}{U(1)}]/Z_n$, with $k = n/2$, leads [25] to an $N = 2$ SCFT with quark flavors.

(iv) Theories with $N = 1$ supersymmetry can be obtained by variations of the theories above, for instance, by orbifolding and/or by considering the decoupled theory on fivebranes in the heterotic string [26].

3.2. Theories on D-branes in LST

In this subsection we discuss the 3+1 dimensional theory on D4-branes stretched between NS5-branes when the string scale $M_s$ is set, say, around 1 TeV. In particular, in such theories we will be able to discuss the spectrum and couplings of charged particles in theories with TeV strings without gravity.
Consider the brane configuration in figure 2 (for a review, see [27]). An NS fivebrane is separated a distance \( \ell \) from a (possibly differently oriented) NS’ fivebrane, and \( N_c \) D4-branes are stretched between them. The low energy theory on the \( R^{3,1} \) directions common to all the branes is an \( SU(N_c) \) gauge theory. The amount of supersymmetry of the theory depends on the relative orientation of the fivebranes. For instance, if the fivebranes are parallel the four-dimensional theory is \( N = 2 \) supersymmetric; a certain relative rotation of the fivebranes breaks it to \( N = 1 \) [28]. The (classical) YM coupling is

\[
g_{YM}^2 = \frac{g_s l_s}{\ell} = g_{lst}(\ell). \tag{3.8}
\]

Following the discussion of the previous subsection, as \( g_s \to 0 \) (as well as \( \ell/l_s \to 0 \) such that \( g_{YM}^2 = g_{lst} \) is held fixed), the decoupled theory is dual to a theory of D-branes near the tip of a cigar geometry (see figure 2) with \( g_s(\text{tip}) \equiv g_{lst}(\ell) \) given in (3.8). At energies of the order \( M_s \) the spectrum becomes similar to that of LST: in the \( N = 2 \) case it is like a six-dimensional LST on \( R^{5,1} \times SL(2)_2/U(1) \), while in the \( N = 1 \) case it is similar to a four-dimensional LST on \( R^{3,1} \times SL(2)_1/U(1) \).

We can add matter to the theory in several ways. One way is to add D6-branes to the configuration and another is to add D4-branes on the other sides of the NS5-branes. In the second case the group that lives on the D4-branes in the center is called the “color group,” whereas the one on the D4-branes sticking to the right (and/or to the left) is called the “flavor group.” If the number \( N_c \) of colors equals (up to a model dependent numerical factor) the number of flavors \( N_f \), the four-dimensional theory is conformal (in which case (3.8) is the exact gauge coupling). Moreover, we compactify the space transverse to \( R^{3,1} \) on a six-dimensional space with volume \( V_6 \). \(^5\) The four-dimensional Newton’s constant is

\[
\frac{1}{G_4} \sim M_P^2 \sim \frac{V_6}{g_s^2 l_s^8}. \tag{3.9}
\]

Consider turning one of the NS fivebranes in the configuration above into a stack of fivebranes. Separating these fivebranes a distance \( L \) (say, on a circle transverse to the D4-branes) corresponds in the low energy four-dimensional SYM theory to changing certain

\(^5\) Once this is done one should take care of global issues by allowing the appropriate total number of NS5-branes, introduce orientifolds, anti-D-branes, etc. (see comments below); we assume that this is done.
parameters in the superpotential \([28,29]\). Such configurations allow to consider the physics of color-flavor open strings which are bound to the stack of NS5-branes perturbatively in \(g_{st}(L) = g_s l_s / L \) (3.5). Such open strings in the background of NS5-branes were studied in [30]. In particular, observables corresponding to “quarks” \((N_c, N_f)\) multiplets and their excitations were identified and their two point functions were computed using the idea of holography, following [17,21]. The spectrum of charged particles in such four-dimensional theories is thus very similar to the spectrum discussed in the previous subsection: the massless particles correspond to quarks, followed by a discrete Hagedorn spectrum with masses of the order \(M_s\), and a continuum. Following the disk computations in [30] for the three point functions one expects, in particular, that a massive color-color string excitation can decay into a pair of quark-anti-quark.

Comments:

1. The configurations discussed above are only part of a globally consistent brane configuration which includes more fivebranes, orientifolds, D-branes and anti-D-branes. All these extra objects can be located a distance of the order \(l_s\) away from the configurations above, hence physics due to the additional structure will show up at energies above the string scale.

2. Some systems of D4 and anti-D4-branes stretched between non-parallel NS5-branes are expected to be stable non-BPS brane configurations [31]. One might expect the supersymmetry breaking scale \(M_{SUSY}\) in such theories to be of the order of the string scale: \(M_{SUSY} \sim M_s\).

3. The system discussed in this subsection is related to systems discussed in the previous subsection: there is a U-duality relating D4-branes intersecting NS5-branes to NS5-branes wrapped on a Riemann surface, which is holographically dual to the string theory on (generalizations of the) backgrounds of the form (3.7).

4. Phenomenology of TeV LST

Here, we discuss the main phenomenological implications of the above theories when the string tension and compactification scales are in the TeV region. Because of the tiny value of the string coupling \(g_s \sim M_s / M_P \simeq 10^{-16}\), for all low energy consequences we can take the limit \(g_s \rightarrow 0\), in which case gravity decouples and one is left over with LST having two sectors. The non-abelian (Standard Model) particles confined on NS5-branes (described by D-branes stretched between the NS5-branes) or on D-branes stretched
between NS5-branes (described by open strings ending on the D-branes), and a sector of gauge singlet closed little strings trapped in the vicinity of the NS5-branes. Thus, there are three types of possible excitations revealing new physics:

1. KK and winding modes of ordinary particles, signaling new dimensions at a TeV.
2. String oscillations of the quarks, leptons and familiar gauge bosons.
3. Vibrational excitations of the little string at the TeV scale. These are unique to this framework.

There are three types of theories, each with different signatures: type IIB, type IIA and theories with D-branes stretched between NS5-branes. We will consider them in turn.

Type IIB models:

In these the Standard Model gauge interactions are described by a six-dimensional theory of 0-branes obtained as endpoints of D-strings on the NS5-branes. Their tension $T_0$ is determined by eq. (3.6) and can be identified with the mass of the W-boson, $T_0 \equiv T_{D1} \simeq M_W$. The four-dimensional gauge coupling (2.2) is determined by the area of the two-dimensional compact space along the NS5-branes, which implies that the compactification scale is an order of magnitude lower than the string scale:

$$M_c \equiv \frac{1}{R_5} \sim \frac{1}{R_4} = g_Y M_s.$$ (4.1)

It follows that the first effects of charged particles beyond the Standard Model that would be encountered in particle accelerators are due to the production of KK excitations in the two extra dimensions [32]. Neutral states will also appear at the TeV range; we shall discuss them below.

Type IIA models:

In these the gauge degrees of freedom are described by strings, obtained as endlines of D2-branes on the NS5-branes. Their tension $T_1 \equiv T_{D2}L$ can be obtained by T-duality

---

6 In theories with large supersymmetry there are also magnetically charged particles, however, those can be pushed above the $M_s \sim$ TeV scale. More realistic models where supersymmetry is broken down to $N = 1$ or $N = 0$ can be obtained, say, by appropriate orbifoldings; in such theories magnetically charged particles are projected out.
from type IIB along, say, the direction $X^4$, so that the gauge coupling (2.2) is given by the ratio of the two radii. As a result, $T_1 = T_0/R_4 \equiv M_W/R_4$ leading to

$$T_1 = \frac{M_W}{R_4} = \frac{M_WM_c}{g_{YM}^2}. \quad (4.2)$$

On the other hand, $T_1 = T_{D2L} = M_s^3L/g_s$, which combined with eq. (3.5) yields:

$$T_1 = \frac{M_s^3}{T_1}. \quad (4.3)$$

From eq. (4.2), $M_W$ is identified with the dual compactification scale along the $X^4$ direction with respect to the charged string tension $T_1$. If the fundamental string tension $M_s^2$ is lighter than $T_1$, then $M_s^2R_4 < T_1R_4 = M_W$ and closed little strings have windings at energies lower than $M_W$, which is excluded experimentally. Thus, the tension $T_1$ of the charged gauge states is less than that of the little strings, $\sqrt{T_1} < M_s$, and in the energy interval between the two tensions we will have an effective superconformal theory of tensionless strings (see also [33]). However, from eq. (4.3), $g_{lst} > 1$ and we cannot reliably compute in little string perturbation theory. It follows that in this case the first effects of charged particles that would be encountered in particle accelerators are KK modes of one dimension along the $X^5$ direction and/or charged string excitations with tension $T_1$, depending whether $M_c \equiv R_5^{-1}$ is less or bigger than $M_W / g_{YM}^2$. In both the type IIA and IIB frameworks the coupling of the little strings to the standard model matter is unknown.

D-branes in LST:
In type II models where gauge interactions emerge from D-branes stretched between NS5-branes, the low energy physics is described by the theories on D-branes in LST, discussed in section 3.2. Note that the mass of W bosons corresponds now to the separation of the D-branes and is independent of separations of NS5-branes.

In this case, Standard Model particles have charged excitations due to windings of open strings in the directions transverse to the D4-branes but along the two extra compact dimensions of the NS5-branes. The energy of these excitations is $M_s^2R_c$, where $R_c$ is the compactification scale. If $R_c < l_s$ we can T-dualize $R_c \rightarrow \tilde{R}_c = l_s^2/R_c > l_s$. In this case the D4-branes turn into D5-branes wrapped on the compact direction $\tilde{R}_c$, and $g_{lst}(\ell)$ in eq. (3.8) turns into $\tilde{g}_{lst}(\ell) = g_{lst}(\ell)l_s/R_c$. Charged excitations in this direction correspond now to KK modes of open strings which are somewhat lighter than the string scale. In fact, the weak coupling condition $\tilde{g}_{lst} < 1$ gives $l_s > R_c > g_{lst}l_s = g_{YM}^2l_s$. Thus, both
the compactification and the string scales are in the TeV region and in all these cases the energy of such charged particles is around the TeV scale while little strings are weakly coupled.

There are also KK modes of open strings in the direction along which the D4-branes are stretched as well as windings along the directions transverse to both the D4 and the NS5-branes; those are very weakly coupled (and decouple in the $g_s \to 0$ limit). In addition, there are of course fundamental open string oscillator modes that are also charged under SM gauge interactions and have TeV masses.

The common thread of all three cases is the existence of a neutral sector described by closed fundamental little strings that survive in the limit $g_s \to 0$. They have non-trivial interactions among themselves with a coupling $g_{lst}$ given in eq. (3.5). Perturbative computations can therefore be trusted when $g_{lst} < 1$. Using eqs. (4.3), (4.2) and (3.6), one obtains:

$$g_{lst} = \frac{M_s^2}{T_1} = g_{YM}^2 \frac{M_s^2}{M_W M_c} ,$$

(type IIA)

$$g_{lst} = \frac{M_s}{M_W} ,$$

(type IIB)

D–branes in LST : $g_{lst} = g_{YM}^2 = \frac{g_s}{\ell M_s}$.

Recall that for theories of D-branes in the presence of NS5-branes, $g_{lst}$ is independent of the W boson mass which is determined by the separation of the D-branes and not of the NS5-branes.

It follows that the discussion of the perturbative spectrum of section 3 is strictly speaking valid for the theories of D-branes in LST if the separation of NS5-branes is larger than $g_s l_s \sim 1/M_P$. The little string excitations can be produced in particle accelerators if they dispose sufficient energy, or they can lead to indirect effects in various processes, as the effects of TeV string models based on type I theory [34].

The perturbative spectrum occurs at [17,21]

$$M_{n,m}^2 = \frac{2M_s^2}{k}(n-1)(2m-n) , \quad 2m + 1 > 2n > 2m + 1 - k , \quad n \in Z .$$

(4.5)

The pole at $n = 1$ corresponds to the light SM particle. The other poles at $M_{n,m}^2 \sim M_s^2/k$ are KK-type excitations, due to the asymptotic radius of the cigar. Each set of poles on the up-side-down parabola (4.5) is followed by a branch cut starting at the maximum of the parabola, that we discussed in the previous section.
Similarly, observables corresponding to string excitations $N$ create from the vacuum particles with masses

$$M_{n,m;N}^2 = M_{n,m}^2 + NM_s^2. \quad (4.6)$$

Hence, each of the particles in (4.5) is followed by a Regge trajectory of string excitations.$^7$

It is interesting to consider the thermodynamics of LST at a TeV; this can be done using its holographic description $^{35,36,37,38,39}$. Strongly coupled LST has a Hagedorn density of states and the Hagedorn temperature is $T_H = M_s/(2\pi \sqrt{k})$ $^{35}$. At high energy the entropy is

$$S = \beta_H E + \alpha \log E + O(1/E), \quad (4.7)$$

leading to the temperature-energy relation

$$\beta = \frac{\partial S}{\partial E} = \beta_H + \alpha/E + O(1/E^2). \quad (4.8)$$

The sign of $\alpha$ indicates if $T_H = 1/\beta_H$ is a limiting temperature (where the energy density diverges as $T$ approaches $T_H$ from below) or a temperature where a phase transition might occur. Recently, $\alpha$ was computed and was shown to be negative $^{39}$. This suggests that a phase transition is expected at $T \sim M_s$, similar to QCD. The nature of the high temperature behavior of the theory might have some interesting consequences in the physics of the early universe.

Weakly coupled LST ($g_{\text{lst}} < 1$) has of course the same high energy thermodynamics (when $E \gg M_s/g_{\text{lst}}$). However, at intermediate energies $M_s < E < M_s/g_{\text{lst}}$ the weakly coupled little string excitations possess a Hagedorn density of states with $T_H = M_s/(2\pi)$ $^{17}$.

Finally, we remark that gauge coupling Unification, the one concrete quantitative success of the supersymmetric standard model $^2$, can be accommodated in a way parallel to $^{40}$. There it was shown that under general conditions, placing the color and weak interactions on two different sets of branes (extended in different directions in the internal compact space) implies one relation among the three gauge couplings which naturally leads to the correct value of the weak mixing angle, provided that we choose the fundamental scale to be at a few TeV.

$^7$ For charged open little strings (open strings in the background of NS5-branes), some factors of 2 should be added in eqs. (4.5), (4.6) relative to the neutral closed little strings sector; see for instance eqs. (B.14), (B.15) in $^{30}$. 

17
5. The Hierarchy Problem

In the framework we described here, the hierarchy between the Planck and the string scales is attributed to the smallness of the string coupling. A small string coupling is rather natural when supersymmetry is broken due to the runaway potential generated for the dilaton [41]. In a usual scenario where the YM couplings are of the order $g_s$, such a runaway behavior is a serious problem. On the other hand, in the LST scenario considered in this note, $g_{YM}$ is determined by geometrical data, while $g_s$ is an independent parameter. Yet, although $g_s \ll 1$ is required to set the observed Newton’s constant, it should not runaway all the way to 0. In this section, we describe a mechanism determining the expectation value (VEV) of the dilaton and the conditions for generating the desired hierarchy.

Let us first remark that if the value of the string coupling is chosen to be very small by hand, the resulting hierarchy is obviously stable under radiative corrections even around a non-supersymmetric string vacuum. This should be contrasted with the large dimension framework [6] where stability of hierarchy requires that massless bulk fields propagate in more than one large compact dimensions [42]. Moreover, the vacuum energy in a non-supersymmetric vacuum of the theories we described here behaves at most as $M_s^4 \sim (\text{TeV})^4$ (see also [43] and the discussion below). This should be again contrasted with the framework of large dimensions which suffers in general from the usual quadratic divergences $\sim M_s^2 M_P^2$, unless the bulk is supersymmetric [6,44].

Dynamically determining the dilaton by minimizing an effective potential faces the following problem. Since the dilaton plays the role of string loop expansion parameter, a generic non-trivial potential would mix several orders of perturbation theory (as well as eventually non-perturbative effects) and, in general, the minimum would be at a point where different powers of $g_s$ compete and, as a result, perturbation theory is unreliable. Moreover, the value of the coupling is in general expected to be of order unity. A possible exception using non-perturbative contributions such as several condensates [45] appears very unnatural in our case, since non-perturbative factors are extremely suppressed in the desired very weak coupling limit.

One way to evade this problem is through the appearance of logarithms. These can arise from loops of particles having gauge interactions with masses depending on the string coupling. The first difficulty is that gauge theories on NS5-branes are independent of the string coupling. One should therefore introduce a new gauge (hidden) sector living on
D-branes and thus having a gauge coupling given by $g_s^{1/2}$. The second difficulty is that massive particles on D-branes have in general masses set by their separation, their motion or the string scale itself (for string excitations) all of which are independent of $g_s$. In these cases, loop effects cannot produce logarithms of $g_s$. However when masses are induced radiatively, they depend on the string coupling and can give rise to logs. One such example arises when there is an anomalous $U(1)$. The anomaly is cancelled by an appropriate shift of an axion from the Ramond-Ramond sector and the abelian gauge field acquires a mass $m_A = g_s^{1/2} M_s$. Integrating out this field, one obtains a potential term proportional to $m_A^4 \ln m_A$, or equivalently (in the string frame):

$$V_{\text{eff}} = g_s^2 (v_1 \ln g_s + v_2) M_s^4 + c M_s^4,$$

(5.1)

with $v_{1,2}$ and $c$ numerical constants. The first two terms proportional to $v_{1,2}$ correspond to two string loops contributions (genus 2), while $c$ arises at the one loop (genus 1).

The effective potential (5.1) has an extremum at

$$\langle g_s \rangle = e^{-1/2 - v_2/v_1},$$

(5.2)

which is a minimum when $v_1$ is positive. This minimum can be exponentially small when $v_2$ is just one or two orders of magnitude bigger than $v_1$, which is not unreasonable since $v_1$ is determined entirely from the loop of the anomalous $U(1)$ while $v_2$ receives contributions from all string modes.

Note that in general, in the presence of D-branes, one may expect an additional contribution to $V_{\text{eff}}$ proportional to $g_s$, arising from genus 3/2. Such a term would destabilize the minimum (5.2) and is assumed to vanish. In fact this condition is related to the problem of fine tuning the cosmological constant. In the above example (5.1), we should therefore impose

$$c = -\langle g_s \rangle^2 (v_1 \ln \langle g_s \rangle + v_2) + O(\langle g_s \rangle^4).$$

(5.3)

Another example of logarithmic corrections to the potential may be provided in models of the Coleman-Weinberg type, where a classically massless scalar field with a tree-level potential acquires a non-trivial VEV driven by a negative squared mass generated radiatively. In this case, the scalar potential takes the form $V_{\text{eff}} \sim \Phi^4/g_s - \mu^2 \Phi^2$, up to $\ln \Phi$ corrections in both terms. Then, its minimization fixes $\langle \Phi \rangle \propto g_s^{1/2}$ and leads to $V_{\text{eff}} \sim g_s \ln g_s$ which is similar to the expression (5.1) that we studied above.
An alternative possibility of fixing the dilaton without generating a potential would be during the cosmological evolution of the universe, following the suggestion of Damour and Polyakov [14]. The basic requirement is that all couplings and masses of the effective theory should depend on the dilaton through the same function. If in addition this function has an extremum, the cosmological evolution will “push” the dilaton towards this extremum. This happens during matter dominated era, crossing mass thresholds in radiation dominated, as well as during any period of inflation. As a result, the dilaton couples quadratically to matter and its mass can vanish without causing any dangerous long range force.

The main requirement of a universal functional dependence seems however very unlikely to be satisfied in heterotic and type I string vacua. On the contrary, theories on NS5-branes seem to provide a natural framework for realizing such a requirement, since in the string frame the matter action is independent of the dilaton while graviton kinetic terms may acquire a non-trivial dilaton dependence:

\[
\mathcal{L} = F(g_s)R + \mathcal{L}_{\text{matter}}, \tag{5.4}
\]

where \(F(g_s) = 1/g_s^2 + \) higher order and non-perturbative corrections. It follows that upon rescaling the metric into the Einstein frame, all mass parameters of the matter Lagrangian will depend on the single “universal” function \(F\). It is not however clear under what conditions \(F\) would have an extremum at a tiny value of \(g_s\).

The dilaton may approach but not precisely reach the extremum of \(F\) in cosmological time [46]. This results in a small universal linear coupling of the dilaton to matter – but not to radiation – proportional to the fractional deviation \(\alpha\) of the dilaton’s present position away from its minimum. Such a scalar admixture to gravity has several possible observational consequences, including the bending of light and the Shapiro time delay of signals [46,47]. The most stringent bounds come from primordial nucleosynthesis, and they constrain the present value of \(\alpha\) to be less than a few percent [47]. A precision test, possibly improving the present limit of \(\alpha\) by over an order of magnitude, will take place in the relativistic gyroscope (or Gravity Probe B) experiment that will be launched in 2002.

In general, small flavor-dependent effects are expected to spoil the exact universality of the coupling of the dilaton through the function \(F\). In the string frame these are expected to show up as small \(g_s\)-dependent corrections to the various gauge-invariant terms in the matter Lagrangian of eq. (5.4). This results in a linear coupling of the dilaton to matter which is flavor-dependent and, therefore, leads to violations of the principle of equivalence.
estimated to be proportional to the product of $\alpha \times g_s$. These are potentially observable in the upcoming satellite experiment STEP, which will test the principle of equivalence to one part in $10^{18}$ [14]. Since $g_s$ grows with $M_s$ and $V_6$, the predicted violations are larger for string scale above a TeV or bulk volume above a TeV$^{-6}$ (see footnote 2).

6. Remarks

We end with some comments, first on the cosmological constant. In models with infinitesimal string coupling, it seems that the vacuum energy may be consistent with the present experimental bound if the perturbative contributions are arranged to vanish in one and two loops, while non-perturbative corrections appear to be extremely suppressed. Indeed, the three loop contribution is of order $g_s^4 M_s^4 \sim M_s^8 / M_P^2$ which is just of the order suggested by present observations for $M_s \sim 1$ TeV. This may provide a new framework for explaining the smallness of the cosmological constant which deserves further investigation.

The theory of NS fivebranes with the string scale set equal to the electroweak scale and with a very small asymptotic string coupling realizes several recent ideas in explicit string theory backgrounds.

For example, the tip of the cigar is a concrete realization in string theory of what one would call in [13] “the $d$-dimensional negative tension brane” (see figure 1). Unlike possible realizations of warped compactification scenaria in string theory, here the theory at high energies is not a CFT; it is a string theory with a scale $M_s$ coupled (weakly) to gravity. Nevertheless, there is an analogy between the dilaton and the $y$ coordinate responsible for the exponential hierarchy in warped compactification scenaria.

Acknowledgements: We would like to thank Nima Arkani-Hamed, Micha Berkooz, Thibault Damour, Nemanja Kaloper, David Kutasov, Peter Mayr, Sasha Polyakov, Massimo Porrati, Eva Silverstein, Herman Verlinde, Robert Wagoner and Edward Witten for valuable discussions. AG thanks the Theory Division at CERN, the Ecole Polytechnique and the ITP at Stanford University for their warm hospitality. This work is supported in part by the EEC TMR contract ERBFMRX-CT96-0090 and RTN contract HPRN-CT-2000-00122, and in part by the INTAS project 991590. The work of SD is supported by NSF grant PHY-9870115-003. The work of AG is supported in part by the BSF – American-Israel Bi-National Science Foundation, by the GIF – German-Israel Foundation for Scientific Research, and by the Israel Academy of Sciences and Humanities – Centers of Excellence Program.
References