Evidence for neutrino oscillations is pointing to the existence of tiny but finite neutrino masses. Such masses may be naturally generated via radiative corrections in models such as the Zee model where a singlet Zee-scalar plays a key role. We minimally extend the Zee model by including a right-handed singlet neutrino $\nu_R$. The radiative Zee-mechanism can be protected by a simple $U(1)_X$ symmetry involving only the $\nu_R$ and a Zee-scalar. We further construct a class of models with a single horizontal $U(1)_{FN}$ (à la Froggatt-Nielsen) such that the mass patterns of the neutrinos and leptons are naturally explained. We then analyze the muon anomalous magnetic moment $(g_\mu - 2)$ and the flavor changing $\mu \rightarrow e\gamma$ decay. The $\nu_R$ interaction in our minimal extension is found to induce the BNL $g_\mu - 2$ anomaly, with a light charged Zee-scalar of mass $100 - 300\,\text{GeV}$.

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The minimal Zee model [3] contains the three active left-handed neutrinos of the SM and a bilepton singlet Zee-scalar which plays a key role for radiative generation of their Majorana masses. There is no underlying reason that forbids the existence of light right-handed singlet neutrinos ($\nu_R$), as $\nu_R$ can also be naturally contained in various extensions of the SM (such as the models with left-right symmetry or $SO(10)$). The introduction of $\nu_R$ thus provides the simplest possible extension of Zee model. However, a simple embedding of $\nu_R$ results in the loss of the prediction of neutrino masses as they are rendered arbitrary by the tree-level Dirac mass terms [7,8,5,6]. In this work, we first build a class of minimally extended Zee models including $\nu_R$ (called Type-I) and invoke a simple $U(1)_X$ symmetry (or its discrete subgroup $Z_2$) to effectively protect the radiative Zee-mechanism by forbidding the mixings between the $\nu_R$ and the active neutrinos. Such an extension is nontrivial since the successful embedding of a singlet $\nu_R$ requires the addition of a second singlet Zee-scalar in the minimal extension. Next we note that the original Zee model neither predicts the size of the Zee-scalar Yukawa couplings nor provides any insight on generating the lepton masses and their hierarchy. Though our simplest $U(1)_X$ in Type-I models protects the radiative Zee-mechanism, we can use this same $U(1)_X$ group as a horizontal symmetry involving both neutrinos and leptons (called Type-II), and thus explain the mass patterns of the neutrinos and leptons in a natural way, à la Froggatt-Nielsen [9]. In both Type-I and -II models, the $\nu_R$ can interact with the right-handed muon and Zee-scalar with a natural $O(1)$ Yukawa coupling, which has striking phenomenological consequences.

Finally, we apply the Type-I and -II models to analyze the Zee-scalar-induced contributions to the muon anomalous magnetic moment $g_\mu - 2$ and the lepton-flavor-violation decay $\mu \rightarrow e\gamma$. We find that the recent BNL $g_\mu - 2$ anomaly [10] can be explained with a light charged Zee-scalar of mass around $100 - 300\,\text{GeV}$. Our models also have the $\mu \rightarrow e\gamma$ decay branching ratio around or below the current experimental limit.

Minimal Extension of the Zee Model with a Right-handed Singlet Neutrino

The minimal Zee model [3] introduces one extra singlet charged scalar ($S_1^\pm$) together with the usual two-Higgs-doublet sector. By assuming no right-handed $\nu_R$, as in the SM, this scalar only interacts with left-handed neutrinos and leptons. Thus, the Zee model contains the following additional Lagrangian,

$$\Delta L = \sum_{j,j'} \frac{f_{jj'}}{2} \bar{\nu}_{\alpha j} T^a_{\alpha j'} L_{b j'} S_1^+ + m_{ab} \bar{H}_1^a H_2^b S_1^+ + \text{h.c.}$$

$$= [f_{12} (\bar{\nu}_{\alpha L}^c \mu_L - \bar{\nu}_{\alpha L}^c e_L) + f_{13} (\bar{\nu}_{\alpha L}^c \tau_L - \bar{\nu}_{\alpha L}^c e_L) + \cdots]$$

(1)
where $\ell_j, \ell'_j \in (\mu, \tau)$ and $L_j = (\nu_j, \ell_j)^T$ is the left-handed doublet of the $j$th family. The $(H_1, H_2)$ are the usual two-Higgs-doublets with hypercharge $(1/2, -1/2)$, where $H_1 = (-H_1^+, H_1^-)^T$, $H_2 = (H_2^0, H_2^-)^T$, and $H_1 = i\tau_2 H_1^+$. The Yukawa sector can conserve total lepton number by assigning to $S_2^\pm$ the lepton numbers $\mp 2$. Thus, the total lepton number is only softly violated by the dimension-3 trilinear Higgs operator in Eq. (1). As such, the small Majorana neutrino masses are radiatively generated at one-loop and are automatically finite.

We minimally extend the Zee model by including a single right-handed Dirac neutrino $\nu_R$ with following Yukawa interactions,

$$\Delta L_2 = \left[ f_1 \overline{\nu_R} e_R + f_2 \overline{\nu_R} \mu_R + f_3 \overline{\nu_R} \tau_R \right] S^+_2 + \text{h.c.} \quad (2)$$

where $S_2^\pm$ is a second singlet Zee-scalar. The nontrivial issue with embedding $\nu_R$ is to avoid arbitrary tree-level Dirac mass terms generated by the Yukawa interactions $L_j H_2 \nu_R$ and $T_j H_1 \nu_R$ (which mix $\nu_L$ and $\nu_R$), so that the predictive power of the radiative Zee-mechanism can be effectively protected. We achieve this goal by noting that the Yukawa sector (2) of $\nu_R$ possesses a global $U(1)_X$ symmetry, which can properly forbid the neutrino Dirac mass terms once a Zee-scalar $S_2^\pm$ is included together with $\nu_R$. It can be shown, by assigning the most general $U(1)_X$ quantum numbers for the Zee-model with $\nu_R$, that the unwanted tree-level neutrino Dirac masses cannot be removed without $S_2^\pm$. We define our simplest Type-I models with $U(1)_X$ in Table 1, where only $\nu_R$ and $S_2^\pm$ carry $U(1)_X$ charges while all other fields are singlets of $U(1)_X$. Hence, the Type-I extension gives a truely minimal embedding of $\nu_R$ into the Zee model.

**Table 1.** Quantum number assignments for Type-I and -II models. The hypercharge is defined as $Y = Q - I_3$.

<table>
<thead>
<tr>
<th>Operators</th>
<th>$U(1)_Y$</th>
<th>$U(1)_X$</th>
<th>$U(1)^{\mu}_F$</th>
<th>$U(1)^{\nu}_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_j \ell_j R$</td>
<td>$\nu_R$</td>
<td>$H_1$</td>
<td>$H_2$</td>
<td>$S^+_2$</td>
</tr>
<tr>
<td>$U(1)_Y$</td>
<td>$-1/2$</td>
<td>$-1$</td>
<td>$0$</td>
<td>$1/2$</td>
</tr>
<tr>
<td>$U(1)_X$</td>
<td>$0$</td>
<td>$0$</td>
<td>$x$</td>
<td>$0$</td>
</tr>
<tr>
<td>$U(1)^{\mu}_F$</td>
<td>$0$</td>
<td>$y_j$</td>
<td>$x'$</td>
<td>$0$</td>
</tr>
<tr>
<td>$U(1)^{\nu}_F$</td>
<td>$u_j$</td>
<td>$y_j$</td>
<td>$x'$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Table 2 classifies all (dis-)allowed operators of Type-I up to dimension-4. It shows that, as long as $x \neq 0$, the radiative Zee-mechanism is protected and the $\nu_R$ remains massless. Such a massless $\nu_R$ does not contribute to the invisible $Z$-width as it carries no weak charge. A special case of our Type-I is to consider its discrete subgroup $Z_2$ under which $\nu_R$ and $S_2^\pm$ transform as, $\nu_R \rightarrow i\nu_R$, $S_2^\pm \rightarrow \mp i S_2^\pm$, while all other fields remain invariant. Other non-minimal variations of our Type-I can be easily constructed.

**Table 2.** Summary of $U(1)$ charges carried by the effective operators in Type-I and -II models.

<table>
<thead>
<tr>
<th>Operators</th>
<th>$U(1)_X$</th>
<th>$U(1)^{\nu}_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_j H_1 \ell_j R$</td>
<td>$0$</td>
<td>$y_j$</td>
</tr>
<tr>
<td>$T_j H_2 \ell_j R$</td>
<td>$0$</td>
<td>$y_j - z$</td>
</tr>
<tr>
<td>$T_j H_3 \nu_R$</td>
<td>$x$</td>
<td>$x'$</td>
</tr>
<tr>
<td>$\nu_R \ell_j L S^+_2$</td>
<td>$-x$</td>
<td>$-x - y$</td>
</tr>
<tr>
<td>$\nu_R \ell_j R S^+_2$</td>
<td>$-x'$</td>
<td>$-x' - y$</td>
</tr>
<tr>
<td>$\tilde{H}_1 H_2 S^+_2$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\tilde{H}_1 H_2 S^-_2$</td>
<td>$-x$</td>
<td>$z - x - y$</td>
</tr>
<tr>
<td>$\nu_R \overline{\nu_R}$</td>
<td>$2 x$</td>
<td>$2 x'$</td>
</tr>
<tr>
<td>$S_1^+ S_2^-$</td>
<td>$x + y - z$</td>
<td>$x + y - z$</td>
</tr>
<tr>
<td>$\epsilon_{ab} H^b_1 H^a_2$</td>
<td>$0$</td>
<td>$z$</td>
</tr>
</tbody>
</table>

**Neutrino Oscillations, Lepton Masses and Horizontal $U(1)_F$ Symmetry**

While the above Type-I models give the most economic embedding of $\nu_R$ with all the good features of the original Zee-model retained, they do not provide any insight on two important issues: (i) There is no theory prediction on the size of the Zee-scalar Yukawa couplings $f_{ij}$ in Eq. (1), but the neutrino oscillation data requires the following hierarchy $[4]$:

$$\frac{f_{12}}{f_{13}} \simeq \frac{m_2^2}{m_1^2} \simeq 3 \times 10^2, \quad \frac{f_{13}}{f_{23}} \simeq \frac{\sqrt{2} \delta m^2_{\text{atm}}}{\delta m^2_{\odot}} \approx 10^2 \text{ or } 10^3,$$

(3)

where $f_{13}/f_{23} \simeq 10^2$ (or $10^3$) corresponds to the MSW large angle solution (vacuum oscillation solution). (ii) The small lepton masses and their large hierarchy are not understood. Our goal is to construct this same $U(1)_F$ group as a horizontal symmetry involving all the leptons so that these two issues can be naturally explained à la Froggatt-Nielsen (FN) $[9]$. [This $U(1)_F$ will be called $U(1)_{FN}$,] The basic idea is to consider a horizontal $U(1)_{FN}$ spontaneously broken by the vacuum expectation value $\langle S^0 \rangle$ of a singlet scalar $S^0$. We can assign $U(1)_{FN}$ charges for relevant fields such that different mass terms are suppressed by different powers of $\epsilon \equiv \langle S^0 \rangle / \Lambda$ where $\Lambda$ is the scale at which the $U(1)_{FN}$ breaking is mediated to the light fermions. For instance, a low energy effective operator carrying a net $U(1)_{FN}$ charge $q$ (either $\geq 0$ or $< 0$) will
The tau Yukawa coupling itself can be estimated as 

$$m_\tau : m_\mu : m_\tau \simeq \epsilon^4 : \epsilon^1 : \epsilon^0,$$  

which require,  

$$(y_1 - u_1) - (y_3 - u_3) = \pm 4, \quad (y_2 - u_2) - (y_3 - u_3) = \pm 1.$$  

The tau Yukawa coupling itself can be estimated as $\gamma_\tau \simeq (m_\tau/m_\mu) \tan \beta \simeq 10^{-2} \tan \beta \sim \epsilon^1$ (with $\tan \beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle$), in the typical range of $\tan \beta \simeq 10 - 40$, and this restricts the $U(1)_{FN}$ charges of $\tau$ as $y_3 - u_3 = \pm 1$.

Table 3 summarizes three explicit realizations of Type-II models. From Table 3 and Eq. (2), the Yukawa couplings of $\nu_R$ are predicted as, 

Type Ia : $(f_1, f_2, f_3) \sim (\epsilon^3, 1, 1)$;  
Type IIb1 : $(f_1, f_2, f_3) \sim (\epsilon^5, 1, 1)$;  
Type IIb2 : $(f_1, f_2, f_3) \sim (\epsilon^{10}, 1, 1)$.  

From Table 3 and Eq. (1), we further predict the left-handed Yukawa couplings $f_{jj'}$, 

Type Ia : $(f_{12}, f_{13}, f_{23}) \sim \epsilon z$;  
Type IIb1 : $(f_{12}, f_{13}, f_{23}) \sim (\epsilon^4 z, \epsilon^6 z, \epsilon^8 z)$;  
Type IIb2 : $(f_{12}, f_{13}, f_{23}) \sim (\epsilon^{3} z, \epsilon^{5} z, \epsilon^{12} z)$;

where the allowed values of $z$ are defined in Table 3. Thus, Type-IIa suppresses $f_{jj'}$ couplings to $O(10^{-2} - 10^{-4})$. The models in Type-IIb1 (IIb2), however, nicely accommodate the hierarchy (3) for the MSW large angle solution (vacuum oscillation solution), while the predicted size of $f_{12} \sim 10^{-3} - 10^{-6}$ is also of the right order [4]. Finally, it is trivial to extend these models with more than one singlet $\nu_R$ (i.e., $\nu_{Rj}$ with $j = 1, \cdots, N_{\nu_R}$ and $N_{\nu_R} = 3$ for instance), by simply defining them to share the same $U(1)$ charges as in Tables 1 and 3.

Table 3. $U(1)_{FN}$ quantum number assignments for Type-IIa, -IIb1 and -IIb2 models. [The defined range of $z$ is $|z| \simeq 3$ for Type-IIa and $0 \leq z \leq 3$ for Type-IIb1 and -IIb2.]

<table>
<thead>
<tr>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$L_3$</th>
<th>$e_R$</th>
<th>$\mu_R$</th>
<th>$\tau_R$</th>
<th>$\nu_R$</th>
<th>$H_1$</th>
<th>$H_2$</th>
<th>$S_1^+$</th>
<th>$S_2^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IIa</td>
<td>0</td>
<td>0</td>
<td>-5</td>
<td>-2</td>
<td>-1</td>
<td>$x+1$</td>
<td>0</td>
<td>$z-z$</td>
<td>1-x</td>
<td></td>
</tr>
<tr>
<td>IIb1</td>
<td>-1</td>
<td>-3</td>
<td>-5</td>
<td>4</td>
<td>-1</td>
<td>-4</td>
<td>$x$</td>
<td>0</td>
<td>$z-z$</td>
<td>1-x</td>
</tr>
<tr>
<td>IIb2</td>
<td>2</td>
<td>-5</td>
<td>-7</td>
<td>3</td>
<td>-6</td>
<td>$x$</td>
<td>0</td>
<td>$z-z$</td>
<td>3-x</td>
<td></td>
</tr>
</tbody>
</table>

Zee Scalars, Muon $g-2$ and $\mu \rightarrow e\gamma$

The above minimally extended Zee-type models economically incorporate the $\nu_R$ and naturally explain the mass patterns of the neutrinos and leptons. The Zee-scalar Yukawa couplings with the neutrinos/leptons also exhibit an interesting spectrum. Now we are ready to analyze their phenomenological impact. The Brookhaven E821 collaboration has announced a 2.6 standard deviation in the muon anomalous magnetic moment $a_\mu = (g_\mu - 2)/2$, i.e., $\Delta a_\mu = a_{\mu,\exp} - a_{\mu,SM} = (42.6 \pm 16.5) \times 10^{-10}$ [10], which gives a 90% C.L. range for new physics,

$$15.5 \times 10^{-10} \leq \Delta a_\mu \leq 69.7 \times 10^{-10}.$$  

Different authors [11] have interpreted this anomaly in terms of supersymmetry, muon compositeness, extra $Z'$, leptoquarks and extended neutrino models. We attempt to explain it from the contribution of the Zee-scalars and the singlet $\nu_R$ in our minimal Type-(I-II) models.

The Zee-scalars $S_1^\pm$ and $S_2^\pm$ in Type-I/II contribute to $g_\mu - 2$ via the Yukawa couplings $f_{12,23}$ with $(\mu_L, \nu_{e,\tau})$ and $f_2$ with $(\mu_R, \nu_R)$, respectively. Thus, we have,

$$\Delta a_\mu = \frac{m_\mu^2}{96\pi^2} \left( \frac{|f_{12}|^2 + |f_{23}|^2}{M_{S_1^\pm}^2} + \frac{|f_2|^2}{M_{S_2^\pm}^2} \right) \simeq 11.8 \times 10^{-10} \times |f_2|^2 \left( \frac{100 \text{ GeV}}{M_{S_2^\pm}} \right)^2,$$  

with $M_1^2 = (\cos^2\theta/M_{S_1^\pm}^2 + \sin^2\theta/M_{S_2^\pm}^2)^{-2}$. Here $(M_{S_1^\pm}, M_{H^\pm})$ are the mass-eigenvalues of the two charged scalars in Eq. (1) and $\phi$ is their mixing angle. Our models forbid or highly suppress the mixing between $S_1^\pm$.
Note that the $f_{12,23}$ terms in (10) are negligible compared to the $f_2$ term for Type-I and Type-II (with $|z| \geq 1$), cf. Eqs. (7)-(8). The precision bound from $\mu \rightarrow \nu e e$ decay gives [4], $f_{12}/M^2_1 < 0.18/\text{TeV}$, which, combined with Eq. (3), also renders the $f_{12,23}$ terms irrelevant for the $g_\mu - 2$ anomaly. Hence, the original Zee-model cannot resolve the $g_\mu - 2$ anomaly. From (9) and (10), we deduce,

$$41.1 \text{ GeV} \leq \frac{M_{S_2}}{|f_2|} \leq 87.2 \text{ GeV}. \quad (11)$$

Since the LEP2 direct search for charged particles requires $M_{S_2} \gtrsim 100 \text{ GeV}$, $|f_2|$ is constrained as,

$$1.1 \lesssim |f_2| \lesssim O(1), \quad (12)$$

where the upper bound is imposed by perturbativity. Eq. (12) is in the predicted range of $f_2$ for our Type-II models [cf. Eq. (7)]. Thus, combining (11) and (12), we conclude that, to fully accommodate the BNL $g_\mu - 2$ data, the Zee-scalar $S_2^\pm$ in our Type-I and -II models has to be generically light, around $100 - 300 \text{ GeV}$. This leads to the possibility of discovering the light charged Zee-scalar at the Tevatron Run-2, the LHC or a future high energy Linear Collider. Since $f_2 \gg f_{1,3}$ for Type-II, the $S_2^\pm \rightarrow \mu R^\pm \nu_R$ decay has a large branching ratio, and dominates over the $e R^\pm \nu_R$ and $\tau R^\pm \nu_R$ modes. Though Type-I models do not predict $f_{1,3}$, they allow $f_2 = O(1)$ while $f_{1,3} \ll f_2$ is forced by the $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ bounds below. We have the partial decay width,

$$\Gamma[S_2^\pm \rightarrow \mu R^\pm \nu_R] \simeq \frac{f_2}{16\pi} M_{S_2} \simeq O(1) \text{ GeV}, \quad (13)$$

for $M_{S_2} \approx 100 - 300 \text{ GeV}$. Hence, $S_2^\pm$ is a very narrow spin-0 resonance. The predicted branching ratio $\text{Br}[S_2^\pm \rightarrow \mu R^\pm \nu_R] \approx 1$ in Type-(I, II) suggests that $S_2^\pm$ can be best detected via muon plus missing energy.

Our models have further implications for the flavor-violating rare decay $\mu \rightarrow e\gamma$, whose branching ratio is bounded by, $\text{Br}[\mu \rightarrow e\gamma] < 1.2 \times 10^{-11}$, at the 90% C.L. The Type-(I, II) models give the following contributions,

$$\text{Br}[\mu \rightarrow e\gamma] = \frac{\alpha_{\text{em}} v^4}{384 \pi} \left( \frac{|f_1 f_2|^2}{M_{S_2}^4} + \frac{|f_{13} f_{23}|^2}{M_{S_2}^2} \right) \simeq \frac{\alpha_{\text{em}}}{384\pi} |f_1 f_2|^2 \left( v/M_{S_2} \right)^4, \quad (14)$$

where $v = (\sqrt{2}G_F)^{-1/2} \simeq 246 \text{ GeV}$. Thus, we derive,

$$|f_1 f_2| < 2.3 \times 10^{-4} \left( M_{S_2}/100 \text{ GeV} \right)^2. \quad (15)$$

Combining this with the $g_\mu - 2$ bound in (11), we find,

$$|f_1|/f_2 < 1.8 \times 10^{-4}, \quad \Rightarrow \quad f_1 \lesssim (2 - 6) \times 10^{-4}. \quad (16)$$

Comparing this with Eq. (7), we see that Type-IIb1 has $f_1$ just below the current bound while Type-IIb2 is well below it. On the other hand, the $f_1$ coupling in Type-IIa lies slightly above the limit by a factor of $2 - 3$; given the uncertainty of the parameters, it can be easily adjusted to stay within the bound. Also, a much weaker bound on $f_3$ can be derived from $\tau \rightarrow \mu\gamma$ decay, i.e., $|f_3| \lesssim 0.06 - 0.16 \sim O(0.1)$ at 90% C.L., for $1.1 \lesssim |f_2| \lesssim 3$, which is consistent with the Type-II predictions in (7). Finally, if we include $N_{\nu_R}(\geq 2)$ singlet $\nu_{Rj}$ with the same Yukawa coupling $f_2$, the upper [lower] bound in Eqs. (11) and (16) [Eq. (12)] will be relaxed by a factor of $\sqrt{N_{\nu_R}}$.

In summary, the Zee model naturally generates small neutrino Majorana masses by radiative corrections, but it neither predicts the Zee-scalar Yukawa couplings nor provides any insight on the lepton mass hierarchy. We have constructed a class of minimally extended Zee-models with the right-handed neutrino $\nu_R$ embedded, where a $U(1)$ symmetry protects the radiative neutrino masses while generating the lepton mass hierarchy, the hierarchy of the Zee-scalar Yukawa couplings required by the neutrino oscillation data, the hierarchy of Zee-scalar Yukawa couplings necessary for consistency with the $\mu \rightarrow e\gamma$ bound, and the size of the Zee-scalar Yukawa coupling needed for the BNL $g_\mu - 2$ anomaly. Furthermore, a light Zee scalar $S_2^\pm$ is predicted in our models, with a mass around $100 - 300 \text{ GeV}$.

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