The nuclear equation of state probed by $K^+$ production in heavy ion collisions

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The dependence of $K^+$ production on the nuclear equation of state is investigated in heavy ion collisions. An increase of the excitation function of $K^+$ multiplicities obtained in heavy ($Au+Au$) over light ($C+C$) systems when going far below threshold which has been observed by the KaoS Collaboration strongly favours a soft equation of state. This observation holds despite of the influence of an in-medium kaon potential predicted by effective chiral models which is necessary to reproduce the experimental $K^+$ yields. Phase space effects are discussed with respect to the $K^+$ excitation function.

I. INTRODUCTION

From the very beginning kaons have been considered as one of the best probes to study dense and hot nuclear matter formed in relativistic heavy ion collisions [1]. In particular at incident energies below the corresponding production thresholds in free space $K^+$ mesons are created in the early and high density phase of such reactions and – due to strangeness conservation – are not reabsorbed by the nuclear environment. Furthermore, there exist strong evidences that kaons change their properties inside the nuclear medium as predicted by effective chiral models [2,3]. The investigation of a partial restoration of chiral symmetry in dense matter probed by $K$ mesons has strongly stimulated both experimental and theoretical efforts in the recent years [4–11].

The original motivation to study the kaon production in heavy ion reactions at inter-

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mediate energies, namely to extract information on the nuclear equation of state (EOS) at high densities is a matter of current debate. Already in the first theoretical investigations by transport models it was noticed that the $K^+$ yield reacts sensitive on the nuclear equation of state [1,12–14], i.e. it was found to be about a factor 2–3 larger when a soft EOS was applied compared to a hard EOS. At that time the available data [5] already favoured a soft equation of state. However, calculations as well as the experimental data were still burdened with large uncertainties. It was further noticed [12] that the influence of the repulsive momentum dependent part of the nuclear interaction leads to a strong suppression of the kaon abundances. An underprediction of $K^+$ yields using more realistic momentum dependent forces was due to the fact that the production mechanism is twofold: baryon induced processes $BB \rightarrow BY K^+$ where the kaon is created via binary baryon–baryon collisions ($B$ stands either for a nucleon or a $\Delta$–resonance and $Y$ for a $\Lambda$ or a $\Sigma$ hyperon, respectively) and processes $\pi B \rightarrow Y K^+$ induced by pion absorption. In the early studies on subthreshold $K^+$ production only the baryon induced channels have been considered [12,13]. As shown in [14–16] the pionic channel plays an important role, particular in heavy systems. Taking this fact into account the kaon yield could be explained adopting realistic momentum dependent nuclear interactions [9,11,15,16]. However, the dependence of the kaon production on the nuclear EOS turned now out to be too small for definite conclusions.

The a recent work [17] we studied the question if in the meantime decisive information on the nuclear EOS can be extracted from subthreshold kaon production in heavy ion collisions. There are several reasons why it appears worthwhile to do this: Firstly, there has been significant progress in the recent years towards a more precise determination of the elementary kaon production cross sections [18,19], based also on new data points form the COSY-11 for the reactions $pp \rightarrow pK^+X$ very close to threshold [20]. Secondly, the KaoS Collaboration has performed systematic measurements of the $K^+$ production far below threshold in heavy ($Au + Au$) and light ($C + C$) systems [21]. Looking at the ratios built from heavy and light systems possible uncertainties which might still exist in the theoretical calculations should cancel out to a large extent which allows to draw reliable conclusions. Furthermore, far be-
low threshold the kaon production is a highly collective process and a particular sensitivity to the compression of the participant matter is expected.

II. THE MODEL

The present investigations are based on the Quantum Molecular Dynamics (QMD) transport model [22]. For the nuclear EOS we adopt soft and hard Skyrme forces corresponding to a compression modulus of $K=200$ MeV and $380$ MeV, respectively, and with a momentum dependence adjusted to the empirical optical nucleon-nucleus potential [22]. The saturation point of nuclear matter is thereby fixed at $E_B = -16$ MeV and $\rho_{\text{sat}} = 0.17$ fm$^{-3}$ [22]. The calculations include $\Delta$ and $N^*(1440)$ resonances with [23]. The QMD approach with Skyrme interactions is well tested, contains a controlled momentum dependence and provides a reliable description of the reaction dynamics in the SIS energy range, expressed e.g. by collective nucleon flow observables as well as particle production. Also the EOS predicted by microscopic approaches (G-matrix) [24] is similar to the soft version of the Skyrme interaction for densities up to $2\rho_{\text{sat}}$.

We further consider the influence of an in-medium kaon potential based on effective chiral models [2–4,9,26]. The $K^+$ mean field consists of a repulsive vector part $V_\mu = 3/8 f^*_\pi j_\mu$ and an attractive scalar part $\Sigma_S = m_K - m_K^* = m_K - \sqrt{m_K^2 - \Sigma_{\text{KN}}}/f_\pi \rho_s + V_\mu V^\mu$. Here $j_\mu$ is the baryon vector current and $\rho_s$ the scalar baryon density and $\Sigma_{\text{KN}} = 450$ MeV. Following [26] in the vector field the pion decay constant in the medium $f^*_\pi = 0.6 f_\pi^2$ is used. However, the enhancement of the scalar part using $f^*_\pi$ is compensated by higher order contributions in the chiral expansion [26], and therefore here the bare value is used, i.e. $\Sigma_{\text{KN}}/f_\pi$. Compared to other chiral approaches [3,4] the resulting kaon dispersion relation shows a relatively strong density dependence. The increase of the in-medium $K^+$ mass $\tilde{m}_K$, Eq. (2), with this parameterisation is still consistent with the empirical knowledge of kaon-nucleus scattering and allows to explore in-medium effects on the production mechanism arising from zero temperature kaon potentials. For the kaon production via pion absorption $\pi B \rightarrow Y K^+$
the elementary cross section of [25] are used. For the \( NN \rightarrow BYK^+ \) channels we apply the cross sections of Ref. [18] which give a good fit to the COSY-data close to threshold. For the case of \( N\Delta \rightarrow BYK^+ \) and \( \Delta\Delta \rightarrow BYK^+ \) reactions experimental are rare. Thus we rely on the model calculation of ref. [19]. In the case that a \( N^* \) resonance is involved in the reaction we used the same cross section as for nucleons. In the presence of scalar and vector fields the kaon optical potential in nuclear matter has the same structure as the corresponding Schroedinger equivalent optical potential for nucleons [24]

\[
U_{\text{opt}}(\rho, \mathbf{k}) = -\Sigma_S + \frac{1}{m_K} k_\mu V^\mu + \frac{\Sigma_S^2 - V_\mu^2}{2m_K} .
\]

and leads to a shift of the thresholds conditions inside the medium. To fulfil energy-momentum conservation the optical potential is absorbed into an newly defined effective mass

\[
\tilde{m}_K(\rho, \mathbf{k}) = \sqrt{m_K^2 + 2m_K U_{\text{opt}}(\rho, \mathbf{k})}
\]

which is a Lorentz scalar and sets the canonical momenta on the mass-shell \( 0 = k_\mu^2 - \tilde{m}_K^2 \).

Thus, e.g., the threshold condition for \( K^+ \) production in baryon induced reactions reads \( \sqrt{s} \geq \tilde{m}_B + \tilde{m}_Y + \tilde{m}_K \) with \( \sqrt{s} \) the centre-of-mass energy of the colliding baryons. For a consistent treatment of the thresholds the scalar and vector baryon mean fields entering into eq. (2) are determined from two versions of the non-linear Walecka model with \( K=200/380 \) MeV, respectively [13]. The hyperon field is thereby scaled by 2/3 which yields also a good description of the \( \Lambda \) flow [27]. Since the parameterisations chosen for the non-linear Walecka model yield the same EOS as the Skyrme ones, the overall energy is conserved. The kaon production is treated perturbatively and does generally not affect the reaction dynamics [6].

III. EOS DEPENDENCE OF \( K^+ \) PRODUCTION

In Fig. 1 the \( K^+ \) excitation function for \( Au + Au \) and \( C + C \) reactions starting from 0.8 A·GeV which is far below threshold (\( E_{\text{thr}} = 1.58 \) GeV) are shown. The calculations are
performed for a soft/hard EOS including the in-medium kaon potential. For both systems the agreement with the KaoS data [8,21] is very good when a soft EOS is used. In the large system there is a visible EOS effect which is absent in the light system. To estimate the influence of the in-medium kaon potential for $C + C$ also calculations without potential are shown. Already in the light system the $K^+$ yield is reduced by about 50% by the influence of the potential which is essential to reproduce the data [8].

![Excitation function of the $K^+$ production cross section in $Au + Au$ (scaled by $10^{-1}$) and $C + C$ reactions. The calculations are performed with in-medium kaon potential and using a hard/soft nuclear EOS and are compared to data from the KaoS Collaboration [8,21]. For $C + C$ also calculations without kaon potential are shown.](image)

To extract more clear information on the nuclear EOS next we consider the ratio $R$ of the kaon multiplicities obtained in $Au + Au$ over $C + C$ reactions, normalised to the corresponding mass numbers. In Fig. 2 central ($b=0$ fm) collisions are analysed. Going far below threshold $R$ strongly increases when a soft EOS is applied whereas the increase of $R$ is much less pronounced using the stiff EOS and $R$ even slightly drops at the lowest energy.
FIG. 2. Excitation function of the ratio of $K^+$ multiplicities obtained in central (b=0 fm) $Au+Au$ over $C+C$ reactions. The calculations are performed with/without in-medium kaon potential and using a hard/soft nuclear EOS.

Hence, this ratio reflects the higher compression achieved in the heavy system. The $C+C$ system, on the other hand, is too small to develop a significantly larger compression in the case of a soft EOS compared to the hard EOS. Moreover, in the latter case the slightly more energetic binary collisions lead even to a higher $K^+$ yield in the case of a hard EOS at 0.8 A·GeV. Remarkably, this behaviour is seen despite of the presence of an in-medium kaon potential which acts opposite to the EOS effect: a higher compression increases the kaon yield but also the value of the in-medium kaon mass which, on the other hand, tends to lower the yield again. However, the increase of the in-medium mass goes linear with density whereas the collision rate per volume increases approximately with $\rho^2$. E.g. in central $Au+Au$ reactions at 0.8 A·GeV the average density $<\rho>$ at kaon production is enhanced from 1.47 to 1.57 $\rho_{\text{sat}}$ switching from the hard to the soft EOS. This leads to an average shift of the in-medium mass (2) compared to the vacuum value of 55/61 MeV using the hard/soft EOS, i.e. a relative shift of 6 MeV between soft and hard. However, collective effects like the accumulation of energy by multiple scattering show a higher sensitivity on the compression resulting in an enhancement of the available energy $<\sqrt{s}> = 90$ MeV applying the soft
EOS. For $C + C$ this effect is reverse ($<\sqrt{s}> = -45$ MeV) since the system is too small to develop a significant difference in compression and more repulsive collisions enhance the $K^+$ yield at low energies. This effect disappears above 1.0 A-GeV. There exists thus a visible EOS dependence of the kaon multiplicities.

FIG. 3. Excitation function of the ratio $R$ of $K^+$ multiplicities obtained in inclusive $Au + Au$ over $C + C$ reactions. The calculations are performed with in-medium kaon potential and using a hard/soft nuclear EOS and compared to the experimental range of $R$ (shaded area) given by the data from the KaoS Collaboration [21].

The comparison to the KaoS data [21] is finally made in Fig. 3. Here only calculations including the kaon potential are shown since it is already clear from Fig. 1 that without the potential one is not able to reproduce the experimental yields. The calculations are performed under minimal bias conditions with $b_{\text{max}} = 11$ fm for $Au + Au$ and $b_{\text{max}} = 5$ fm for $C + C$ and normalised to the experimental reaction cross sections [8,21]. Both calculations show an increase of $R$ with decreasing incident energy down to 1.0 A-GeV. As already seen for the central collisions this increase is much less pronounced using the stiff EOS. In the latter case $R$ even drops for 0.8 A-GeV whereas the soft EOS leads to an unrelieved increase of $R$. At 1.5 A-GeV which is already very close to threshold the differences between the
two models tend to disappear. The overall behaviour of $R$ is found to be quite independent of the various production channels with initial states $i = NN, πN, NΔ, πΔ, ΔΔ$. Ratios $R_i$ built separately for the individual channels show in both cases (soft or hard) a similar energy dependence as the total $R$ (except of $R_{πΔ}$ which tends to remain large also at high energies). The transport calculations further demonstrate that the increase of $R$ is not is not caused by a trivial, i.e. EOS independent limitation of phase space at low energy in the small system. This is supported by the fact that the number of collisions which the involved particles encountered prior to the production of a kaon and which is a measure of the collectivity provided by the system does not reach a sharp limit for $C + C$ at low energies. The strong increase of $R$ can be directly related to higher compressible nuclear matter. The comparison to the experimental data from KaoS [21] where the increase of $R$ is even more pronounced strongly favours a soft equation of state.

IV. PHASE SPACE FOR $K^+$ PRODUCTION

To obtain a quantitative picture of the explored density effects in Fig. 4 the baryon densities are shown at which the kaons are created. The energy is chosen most below threshold, i.e. at 0.8 A·GeV and only central collisions are considered where the effects are maximal. $dM_{K^+}/dρ$ is defined as

$$dM_{K^+}/dρ = \sum_{i}^{N_{K^+}} \frac{dP_i}{dρ_B(x_i, t_i)}$$

(3)

where $ρ_B$ is the baryon density at which the kaon $i$ was created and $P_i$ is the corresponding production probability. For the comparison of the two systems the curves are normalised to the corresponding mass numbers.
FIG. 4. Kaon multiplicities (normalised to the mass numbers of the colliding nuclei) as a function of the baryon density at the space-time coordinates where the $K^+$ mesons have been created. Central ($b=0$ fm) $Au+Au$ and $C+C$ reactions at 0.8 A-GeV are considered. The calculations are performed with in-medium kaon potential and using a hard/soft nuclear EOS.

Fig.4 illustrates several features: Only in the case of a soft EOS the mean densities at which kaons are created differ significantly for the two different reaction systems, i.e. $<\rho/\rho_{\text{sat}}>=1.46/1.40$ ($C+C$) and $1.47/1.57$ ($Au+Au$) using the hard/soft EOS. Generally, in $C+C$ densities above $2\rho_{\text{sat}}$ are rarely reached whereas in $Au+Au$ the kaons are created at densities up to three times saturation density. Furthermore, for $C+C$ the density distributions are weakly dependent on the nuclear EOS. The situation changes completely in $Au+Au$. Here the densities profile shows a pronounced EOS dependence [13]. Moreover, the excess of kaons obtained with the soft EOS originates almost exclusively from high density matter which demonstrates that compression effects are probed.
FIG. 5. As a measure for the available phase space for $K^+$ production the mean number of collisions $<N_C>$ per particle which the hadrons ($N, \Delta, \pi$) did undergo before they produce a $K^+$ meson is considered. The upper panel shows $<N_C>$ in central $Au + Au$ and $C + C$ collisions. The lower panel shows the ratio of this quantity in $Au + Au$ over the same in $C + C$ reactions. The calculations are performed with/without in-medium kaon potential and using a hard/soft nuclear EOS.

Similar to the density shown before a quantitative impression of the importance of collective and phase space effects can be obtained. Thus in Fig. 5 the average number of collisions for those hadrons ($N, \Delta, \pi$) which were involved in the $K^+$ production are displayed. Again only central collisions are considered where the effects are maximal. $<N_C>$ is defined as

$$<N_C> = \sum_{i}^{N_{K^+}} \frac{1}{2}(N^i_{C_1} + N^i_{C_2})P_i / \sum_{i}^{N_{K^+}} P_i$$

with $N^i_C$ being the number of collisions which particles (1, 2) experienced before they produced kaon $i$, and $P_i$ is the corresponding production probability. It is seen that in average the particles undergo about twice as much relevant collisions in the heavy compared to the
light system. Furthermore, the collectivity, i.e. the accumulation of energy by multiple scattering, increases with decreasing incident energy. This feature is found to be independent of the system size. Thus one can conclude that the increase of $R$ is not due to a trivial phase space effect, namely the fact that far below threshold the $C + C$ system is simply too small to provide enough collectivity for kaon production. If such a scenario - which could model independently as well explain the rise of $R$ seen in the KaoS data - would be true, $< N_C >$ would have to saturate for $C + C$ at low energies. This is obviously not the case. Moreover, building also here the ratio (lower panel of Fig. 5) it seems that the relative enhancement of available phase space for $K^+$ production in the large system is decreasing at low energies. In the last figure we show $< N_C >$ (b=0 fm, SMD, with kaon pot.) for an even smaller system $Li^6$. Also here the number of NN-collisions involved in the production of a $K^+$ increases going far below threshold. This demonstrates that $K^+$ production far below threshold always requires a certain amount of collectivity which can be provided also in a very small colliding system, though such processes are extremely rare. There is, however, no sharp limit were such collision histories become impossible. Thus trivial phase space effects can be excluded for an explanation of the increase of $R$.

![Graph showing mean number of collisions](image)

**FIG. 6.** Mean number of collisions $< N_C >$ (see Eq.(4) ) for central $Au + Au$, $C + C$ and $Li + Li$ collisions.

In [21] a similar argument was based on the measurement of high energy pions which
can test the phase space available for particle production. Astonishingly, the corresponding ratio built from high energy pions does not show any energy dependence but the excitation function remains almost constant. This feature contradicts a thermal interpretation where one expects particle abundances to be exclusively governed by phase space, independent from the fact if the necessary energy goes into mass or momentum. Above threshold $K^+$ production shows $m_t$ scaling according to available phase space, even if the particles are not produced from an equilibrated source [28]. However, even if one assumes a thermal picture (which is, we believe that is does not hold for kaons at SIS energies since it is in contradiction to the knowledge from transport calculations) kaons and high $p_t$ pions should behave different below and around threshold. As shown in [29] a thermal description at SIS energies must account for strangeness conservation in the $K^+$ production mechanism which, via the associated hyperons, introduces a dependence on the baryon chemical potential which is absent for the thermal pion yield. Thus $K^+$ production is coupled to the baryon density whereas high energy pions depend only the temperature. This simple consideration illustrates that it can not a priori be expected that high energy pions behave like kaons. However, the constant pion ratio supports our phase space arguments given above and the fact that non-trivial, i.e. EOS effects, are responsible for the strong increase of $R$ seen for the kaons.

V. SUMMARY

To summarise, we find that at incident energies far below the free threshold $K^+$ production is a suitable tool to study the dependence on the nuclear equation of state. Using a light system as reference frame there is a visible sensitivity on the EOS when ratios of heavy ($Au + Au$) over light ($C + C$) systems are considered. Transport calculations indicate that the $K^+$ production gets hardly affected by compressional effects in $C + C$ but is highly sensitive to the high density matter ($1 \leq \rho/\rho_{sat} \leq 3$) created in $Au + Au$ reactions. Results for the $K^+$ excitation function in $Au + Au$ over $C + C$ reactions as measured by the KaoS
Collaboration, strongly support the scenario with a soft EOS. This statement is also valid when an enhancement of the in-medium kaon mass as predicted by chiral models is taken into account.

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