The Continuum Spectrum of the 4N System. Results and Challenges

J. Carbonell

Institut des Sciences Nucléaires,
53 Av. des Martyrs, 38026 Grenoble, France

1. INTRODUCTION

The theoretical description of the A=4 scattering states constitutes a serious challenge for the existing NN interaction models. The reason for that is not purely technical, but lies rather in the richness of the continuum spectrum itself. Though far from the heavy nuclei imbroglio, it is the simplest system which presents the main characteristics – thresholds and resonances – of the nuclear complexity. Furthermore, they appear already in the low energy region which can be unlikely affected by the three nucleon forces, a keystone in the success encountered when describing the A=3 states and A=4 bound state [1–4].

Solutions of the 4N scattering states have been recently obtained by different groups [5–7], solving Schrödinger, Faddeev-Yakubovsky or AGS equations, with realistic NN potentials.

The aim of this contribution is to give an overview of the main theoretical results existing for the A=4 scattering, specially those obtained since the last Groningen Few-Body Conference. A more detailed review including references can be found in [8].

2. n-3H

The first topic concerns n+3H, the simplest A=4 system after the α particle. It is a pure T=1 isospin state free from the Coulomb problems. A simple look into its cross section and its comparison with the n-d case (Figures 1 and 2) illustrate well the qualitative difference with respect the A=3 case.

The n-3H scattering lengths with realistic potentials (AV14 [9], AV18 [10], Nijm II [11]) were presented in the last Few-Body Groningen Conference [12,13]. The singlet $a_0$ and triplet $a_1$ values (in fm) are summarized in the upper half part of Table 1 together with the deduced coherent scattering length $a_c = \frac{1}{4}a_0 + \frac{3}{4}a_1$ and the zero energy cross section $\sigma(0) = \pi(a_0^2 + 3a_1^2)$ in fm$^2$.

One can see on one hand similar (< 1%) values for different realistic potentials and on another hand a very good agreement using different methods. In [6,12,14] the Faddeev-Yakubovsky (FY) equations in configuration space were solved whereas authors of [13,5] used the Correlated Hyperspherical Method (CHH). The comparison with the experimental cross section – $\sigma(0) = 170 \pm 3$ mb from [15] – shows that NN realistic potentials fail in describing the zero energy cross section as they fail in reproducing the three- ($B_3$) and four-nucleon ($B_4$) binding energies. Unlike the n-d case, the 4N scattering states call
Table 1
n-t scattering length.

<table>
<thead>
<tr>
<th>NN</th>
<th>NNN</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_c$</th>
<th>$\sigma(0)$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AV14</td>
<td>—</td>
<td>4.31</td>
<td>3.79</td>
<td>3.92</td>
<td>194</td>
<td>[12,6]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.32</td>
<td>3.80</td>
<td>3.93</td>
<td>195</td>
<td>[13,5]</td>
</tr>
<tr>
<td>AV18</td>
<td>—</td>
<td>4.32</td>
<td>3.76</td>
<td>3.90</td>
<td>192</td>
<td>[13,5]</td>
</tr>
<tr>
<td>Nijm II</td>
<td>—</td>
<td>4.31</td>
<td>3.76</td>
<td>3.90</td>
<td>192</td>
<td>[12,6]</td>
</tr>
<tr>
<td>AV14</td>
<td>Hyperadial</td>
<td>4.00</td>
<td>3.53</td>
<td>3.65</td>
<td>168</td>
<td>[12,6]</td>
</tr>
<tr>
<td>AV14</td>
<td>Urbana VIII</td>
<td>4.08</td>
<td>3.59</td>
<td>3.71</td>
<td>174</td>
<td>[13,5]</td>
</tr>
<tr>
<td>AV18</td>
<td>Urbana IX</td>
<td>4.05</td>
<td>3.58</td>
<td>3.71</td>
<td>172</td>
<td>[13,5]</td>
</tr>
<tr>
<td>MT I-III</td>
<td>—</td>
<td>4.10</td>
<td>3.63</td>
<td>3.75</td>
<td>177</td>
<td>[14]</td>
</tr>
<tr>
<td>Exp</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>170±3</td>
<td>[15]</td>
</tr>
</tbody>
</table>

for three nucleon interaction (TNI) from the very beginning. Indeed in the n-$^3$H case the singlet and triplet contribution are of the same size (Figure 1) whereas the doublet n-d value - directly correlated to $B_3$ and so affected by TNI - turns to be one order of magnitude smaller than the quartet and has no visible effect in the total cross section (Figure 2). This smartness of nature made the inclusion of TNI unnecessary to reproduce the low energy n-d cross section, thought they play an important role at higher energies and could explain some anomalies in polarization observables ($A_y$) [3,16].

The failure in the zero energy region can be corrected by including TNI. In [12,6] an hyperadial TNI was added with parameters adjusted to ensure $B_3=8.48$ and $B_4=29.0$ MeV, a value which takes into account the Coulomb correction in $^4$He. In [5] the much more elaborate Urbana VIII and IX forces were included leading to $B_3 = 8.48$, $B_4 = 28.3$ MeV. The scattering length obtained in this way are displayed in lower part of Table 1.
The small differences come essentially from the slightly different $B_4$ values to which they were adjusted. The values for the MT I-III model potential [17] are also given and found to be very close to the realistic NN+NNN interactions. In view of that it seems – at least in what concerns bound and zero energy states – that the only role of TNI is to ensure the physical values for $B_3$ and $B_4$. If they lead to very close results despite their severe analytical differences one can hardly pretend to learn something about them in such kind of calculations alone. In practice they provide enough parameters to fit one number.

Figure 1 shows the $n-^3H$ cross section calculated including only the $J^π = 0^+$ and $1^+$ states. Experimental values are taken from [15,18]. Results obtained with the NN forces alone are in dot-dashed curve. Those including TNI are in solid line (separate contributions in long- and short-dashed lines) and provide an accurate cross section until $T_{lab} \approx 0.5$ MeV.

Table 2

<table>
<thead>
<tr>
<th>$a_c$</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.68 ± 0.05</td>
<td>3.91 ± 0.12</td>
<td>3.6 ± 0.1</td>
<td>19</td>
</tr>
<tr>
<td>3.82 ± 0.07</td>
<td>3.70 ± 0.62</td>
<td>3.70 ± 0.21</td>
<td>20</td>
</tr>
<tr>
<td>3.59 ± 0.02</td>
<td>4.98 ± 0.29</td>
<td>3.13 ± 0.11</td>
<td>21 I</td>
</tr>
<tr>
<td>3.59 ± 0.02</td>
<td>2.10 ± 0.31</td>
<td>4.05 ± 0.09</td>
<td>21 II</td>
</tr>
<tr>
<td>3.607 ± 0.017</td>
<td>4.53 ± 0.10</td>
<td>3.325 ± 0.011</td>
<td>22</td>
</tr>
</tbody>
</table>

If the very low energy cross section is accurately measured and reproduced, the situation with scattering lengths, summarized in Table 2, looks more precarious. These values are displayed in Figure 3 together with the theoretical ones previously discussed (horizontal lines).

Figure 3. Diamond from [19], triangles up from [20], squares from [21], stars from [22]

The best agreement is found with the results of [19]; in fact they contain a theoretical input, the ratio $a_1/a_0$, which turns to be very close to the one given by the realistic potentials from Table 1. The other compatible results are those of [20]. However, apart from the quite comfortable error bars in $a_0$, they have been obtained using a value of
$a_c = 3.82$ which is not compatible with the more recent and precise values of $[21,22]$. The values given in $[22]$ are quite close to the theoretical ones but they are extracted from a p-$^3$He R-matrix analysis in which the Coulomb interaction has been removed. Finally, as it was pointed out in $[5]$, the experimental values did not lie on the theoretical curves relying $a_i$ to $B_3$.

The usual way to get $a_i$ is by reversing the relations giving $\sigma(0)$ and $a_c$. This procedure is numerically quite unstable. Indeed by assuming an exact value $a_c = 3.60$, the small existing error in $\sigma(0)$ leads to a range of values $a_0 = 4.60 - 5.16$ and $a_1 = 3.08 - 3.27$. A more precise measurement of $\sigma(0)$ could be helpful to improve the present situation and in this respect the CERN TOF $[23]$ neutron facility could offer interesting possibilities.

The preceding results show that the resonance peak requires the inclusion of negative parity $J^\pi = 0^-, 1^-, 2^- n^3$H states, which become dominant already at $T_{cm} \approx 2.5$ MeV.

A first attempt was done in $[12]$ using FY equations in configuration space. The iteration was limited to $V^{<1}_{NN} + 3^{3}PF_2$ and the partial wave expansion of FY amplitudes to $l_{y,z} \leq 2$. Results obtained with AV14 interaction are shown in Figure 4. It was found that the peak region was poorly described by the NN forces alone and that the inclusion of hyperadial TNI still lowered the cross section. The calculations show a high sensitivity to the inclusion of P-waves in $V_{NN}$, a fact also pointed out by Fonseca in dd-dd and dd-p$^3$H polarization observables $[24,25]$. Their effect is shown in Figure 5 with a zoom at $T_{cm}=3.5$ MeV: including $V_{P_1,3}P_0,3P_1$ still reduces the cross section and the $V_{3PF_2}$ rises substantially the value to compensate this reduction but not enough to fit the experimental points. This failure was attributed either to a lack of convergence in the partial wave expansion of FY amplitudes or to a failure in NN current models $[6]$.

![Figure 4. S+P wave n-$^3$H cross section](image1.png)

![Figure 5. NN P-waves at $T_{cm}=3.5$ MeV](image2.png)

A more recent calculation was done by Fonseca using AGS equations in momentum space $[7]$. This calculation is restricted to 1-rank separable expansion in the $T^{NN}$-matrix but the partial wave expansion of AGS amplitudes was pushed until $l_{y,z} \leq 3$, what represent a sizeable increase in the number of FY amplitudes, specially those describing the
internal structure of triton. Using AV14 and Bonn-B $V_{NN}$ models, this author found a reasonable description of the total and differential cross section data. The $V_{NN}$ P-waves – essentially $^{3}\text{PF}_2$ – make all the difference.

This interesting result deserves some comments. First to remark the unusually high sensitivity to NN P-waves in the low energy cross section. Whereas it has almost no effect in the Nd scattering and triton binding energy, the only contribution of the $V_{NN}^{3P_2}$ accounts at $T_{cm}=3.5$ MeV for half of the n-$^3\text{H}$ P-waves cross section, which in its turn represents half of the total cross section. Second to notice that the analyzing power $A_y$ shows the same kind of discrepancy than for n-d but not solved by small changes in the $^3P_j$ NN phaseshifts, as done in [26]. Similar disagreements are also found in the dd-p$^{3}\text{H}$ reaction.

It is worth noticing that – despite its a priori crude approximation – the 1-rank T-matrix expansion provides very close results to those obtained by solving FY equations for the same number of amplitudes [27]. If the result concerning the n-$^3\text{H}$ resonant cross section is confirmed by independent methods or by increasing the number of terms in the T-matrix, it would speak very much in favour of low rank separable expansions.

![Figure 6. n-$^3\text{H}$ differential cross section with MT I-III model](image)

Figure 6. n-$^3\text{H}$ differential cross section with MT I-III model

It could have some interest to notice the ability of a trivial NN model like MT I-III in describing such a non trivial thing. This potential acts only in $V_{NN}^{L=0}$ waves, has no tensor, nor spin-orbit forces, even not pion tail and triton wavefunction contains only S-wave Faddeev components. It provides however a very good agreement with experimental results, specially in the resonance peak [14] and even for differential cross sections, as can be seen in Figure 6. Only the zero energy is slightly overestimated due to small differences in binding energies: $B_3 = 8.53$ MeV instead of 8.48 and $B_4 = 30.3$ MeV instead of 29.0, once removed Coulomb corrections. In this model the n-$^3\text{H}$ resonant cross section has nothing to do with NN P-waves: it is created by the exchange mechanism between the incoming and target nucleons, what results into an effective 1+3 potential generated only by S-wave NN interactions. This shows that nothing is trivial beyond $A=2$ and the difficulty to disentangle the NN from the N-A interaction.
Table 3
Experimental and theoretical values for p-\(^{3}\)He scattering length.

<table>
<thead>
<tr>
<th>NN Method</th>
<th>a(_0)</th>
<th>a(_1)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>MT I-III</td>
<td>CHH</td>
<td>10.0</td>
<td>[28]</td>
</tr>
<tr>
<td>CR</td>
<td></td>
<td>8.2</td>
<td>7.7</td>
</tr>
<tr>
<td>AV18</td>
<td></td>
<td>12.9</td>
<td>10.0</td>
</tr>
<tr>
<td>AV14 Urbana VII VMC</td>
<td>10.1±0.5</td>
<td>30]</td>
<td></td>
</tr>
<tr>
<td>AV14 Urbana VIII</td>
<td>CHH</td>
<td>10.3</td>
<td>9.13</td>
</tr>
<tr>
<td>AV18 Urbana IX</td>
<td>CHH</td>
<td>11.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Exp.</td>
<td>10.8±2.6</td>
<td>8.1±0.5</td>
<td>[32]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.2±1.5</td>
<td>[31]</td>
</tr>
</tbody>
</table>

From the strong interaction point of view, the natural partner of n-\(^{3}\)H is p-\(^{3}\)He which differs only by Coulomb force. The p-\(^{3}\)He reactions are however much more accessible experimentally though the resonant behaviour is somehow hidden due to the absence of total cross section. The situation concerning the low energy parameters is summarized in Table 3. One can remark a much bigger theoretical "dispersion" than for the n-\(^{3}\)H case and a need for precise experimental values of a\(_0\) and a\(_1\). Triplet scattering length is a relevant quantity in calculating the weak proton capture p+\(^{3}\)He→\(^{4}\)He+e\(^+\)+ν cross section. A firmly established a\(_1\) value is needed but here – as in many other radiative processes – the main uncertainties come from the transition operators. The MEC contribution can modify the result by a factor 5 [30]. There exists also some preliminary calculations at non-zero energy [33]. The differential cross sections at \(E_{cm} = 3\) MeV shows some lack at backward scattering angles and a rather large underprediction in the analyzing power \(A_y\).

3. The \(^4\)He continuum

Next in complexity is the continuum spectrum of the \(^4\)He represented in Figure 7. Calculations of p-\(^{3}\)H scattering are complicated by the existence of the first \(J^\pi = 0^+\) excitation of \(^4\)He in its threshold vicinity. This resonance located at \(E_R=0.40\) MeV above p-\(^{3}\)H covers with its \(\Gamma=0.5\) MeV width the scattering region below n-\(^{3}\)He.

\[
\begin{array}{c}
1^- \\
2^- \\
0^- \\
0^+ \\
\hline
\end{array}
\]

\(d + d\)

\(n+^{3}\)He

\(p+^{3}\)H

\(^4\)He

Figure 7. \(^4\)He continuum spectrum

Figure 8. p-\(^{3}\)H differential cross section at \(\theta_{cm} = 120^\circ\) versus center of mass energy
It turns out that most of the calculations performed until now find this state below the $p^{-3}H$ threshold, that is as a second $^4He$ bound state, probably because they did not include Coulomb corrections. Due to that, the sign of the strong $p^{-3}H$ scattering length is wrong and the interference with the Coulomb amplitude leads to senseless results in the interthreshold region.

Table 4
Low energy N+3N parameters (fm)

<table>
<thead>
<tr>
<th>S</th>
<th>T</th>
<th>$a$</th>
<th>$r_0$</th>
<th>$v_0$</th>
<th>$q_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>14.75</td>
<td>6.75</td>
<td>0.308</td>
<td>0.505</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>4.13</td>
<td>2.01</td>
<td>0.462</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>3.25</td>
<td>1.82</td>
<td>$\simeq$0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3.73</td>
<td>1.87</td>
<td>0.231</td>
<td>-</td>
</tr>
</tbody>
</table>

Among the few data existing in this energy range, we found the differential cross section at $\theta_{cm} = 120^\circ$ as a function of the energy in [34]. An attempt to describe this cross section can be done using the low energy scattering parameters given in [14] and summarized in Table 4. One infers from them, in the isospin approximation, the strong $p^{-3}H$ scattering lengths $a^{S=0} = 9.44$ and $a^{S=1} = 3.49$ fm. The $p^{-3}H$ amplitude reads $f(\theta) = f_c(\theta) + f_{sc}(\theta)$ where $f_c$ is the pure Coulomb term and $f_{sc}$ the strong amplitude in a Coulomb field [35]. Limiting $f_{sc}$ to S-wave and using the Coulomb corrected effective range approximation, one can estimate the low energy $p^{-3}H$ differential cross section. The results obtained with the values of Table 4 and an effective range $r_0 = 2$ fm for both spin states are displayed in Figure 8 (dashed line) and fail to reproduce the observed structure. A good fit (solid lines) is obtained with large and negative values $a^{S=0} \approx -20$ fm, indicating the above threshold position of the first $^4He$ excitation. In terms of isospin components, this corresponds to a value $a_0^{S=0} \approx -40$. The precise location of this state is a very strong requirement for the NN models. Without it, there is no hope to have a good description of the low energy $p^{-3}H$ and also $n^{-3}He$ reaction. We remark that a direct CRM calculation was presented in [29]. These authors found $a_{0}^{S=0} = -22.6$ fm, $a_{1}^{S=1} = 4.6$ fm and extracted from the $p^{-3}H$ phaseshifts the position $E_R = 0.15$ MeV and width $\Gamma = 0.23$ MeV of the resonance.

There exists some calculation for the $n^{-3}He$ scattering length. Using MT I-III and CRM, [29] found $a_0 = 7.5 + 4.2i$ and $a_1 = 3.0 + 0.0i$. Using AV14 and Urbana VII TNI, [36] found $a_1 = 3.5 \pm 0.25$, in agreement with the experimental value although the coupling to $p^{-3}H$ was neglected. The zero energy wavefunction was used to obtain the $n^{-3}He \rightarrow ^4He + \gamma$ cross section which was found overpredicted by a factor 2 and 1.5 in [37] where the $\Delta$ degree of freedom was included. Results for the $n^{-3}He$ elastic cross section at higher energy have been presented in this conference [38]. None of these calculations have been performed taking into account the full complexity required.

Several d-d calculations have been performed at different levels of approximation. Using VMC authors of [39] calculate the d+d\rightarrow ^4He+\gamma cross section with a dd wavefunction decoupled from n^{-3}He and p^{-3}H channels and conclude that the optimal variational wavefunction does not explain the data. FY equations in momentum space and separable potential were used in [40–42] to calculate total and differential $d + d \rightarrow p + ^3H$ cross sections at 20-120 keV and found very good agreement with data. Also at threshold energies, FY equation in configuration space have been solved using simple MT I-III model and
isospin approximation for the N+NNN thresholds. A strong J=0+ T=0 dd scattering length \( a_0 = 4.91 - 0.011i \) fm was found. Using CRM with the same interaction and Coulomb taken into account authors of [29] found \( a_0 = 10.2 - 0.2i \) and \( a_2 = 7.5 \) fm. At higher dd energies, there exists AGS calculations of polarization observables indicating large disagreement with data [7]. Finally we would like to mention the extensive RGM calculations of \(^4\)He bound and scattering states done by [43] in which the phase shifts of different two-fragment channels were obtained.

4. SUMMARY

The continuum of \( A=4 \) is an open door to a higher degree in the nuclear complexity and offers an enormous field of work for a generation of motivated researchers.

The n-\(^3\)H resonant peak acts as a zoom for the internal structure of triton as well as for the NN P-waves. A first task is to clarify with independent calculations the ability of NN models in describing the elastic total and differential cross section in the resonance region. The position and width of the underlying resonances could be then calculated. The results obtained using the complex rotation method in \( A=3 \) system seems very promising [44].

A major point is the description of the p-\(^3\)H and n-\(^3\)He thresholds, dominated by the first excitation of \(^4\)He. This calculations imply coupled channel four-body equations with Coulomb interaction taken into account. This structure completed with the d-d channel constitutes the formal skeleton to be acquainted with in order to access the \( A=4 \) continuum.

The calculation of the numerous weak and electromagnetic capture processes is a redoubtable challenge. The access to a high quality of nuclear wavefunction would allow to test and fix the many ambiguities in the transition operators.

The last remark concerns relativity. The increasing complexity of the three nucleon forces requires a numerical investment which becomes comparable to a relativistic description. An effort, conceptual and numerical, in this direction should be of highest interest. The pioneering result using Gross equation [45] shows the possibility to reach a consistent description of the three nucleons system using only NN forces. The description of a nuclear system will probably remain always phenomenological – nucleons are already very complicated objects – but the relativistic approach could provide a simpler framework.

Acknowledgments: The author is deeply grateful to A. Fonseca, H. Hofmman, A. Kievsky, S. Oryu, E. Uzu, B. Pfützinger and M. Viviani for helpful discussions and remarks during the preparation of this contribution and to M. Mangin-Brinet for a careful reading of the manuscript.

REFERENCES

34. N. Jarmie, M.G. Silbert, D.B. Smith, J.S. Loos, Phys. Rev. 130 (1963) 1987
38. E. Uzu, contribution to this conference
44. E.A. Kolganova, A.K. Motovilov, Y.K. Ho, contribution to this conference