As seen recently in the Quark Matter 2001 conference [3], the first runs of the RHIC have provided many exciting results. With the anticipated much longer runtime, at a variety of colliding energies and nuclear species, the existence and properties of the high temperature phase of quantum chromodynamics (QCD) will be probed and tested further. This is an immediate reason to improve our theoretical knowledge of such matter.

Fully non-perturbative computations of quark number susceptibilities are important for three reasons. Firstly, there have been attempts to link them directly to experimental measurements of event-to-event fluctuations in particle production [4]. Secondly, experimental observations of a relative enhancement of strange quarks have been attributed to the formation of a QCD plasma [5]; a hypothesis which can be quantitatively tested against the computation of the strange quark susceptibility. Finally, earlier results [6–9] showed a strong jump in the susceptibility across the phase transition, but indicated a statistically significant 20% departure from weak-coupling physics. We report first results on the strange quark number susceptibility, \( \chi_s \), over a large range of temperatures, mainly in the plasma phase of QCD. \( \chi_s \) jumps across the phase transition temperature, \( T_c \), and grows rapidly with temperature above but close to \( T_c \). For all quark masses and susceptibilities in the entire temperature range studied, we found significant departures from ideal-gas values. We also observed a strong correlation between these quantities and the susceptibility in the scalar/pseudo-scalar channel, supporting ideas of “dynamical confinement” in the high temperature phase of the QCD plasma.
temperatures up to $1.5T_c$. However, the quark mass varied with temperature, since the quantity that was fixed was $m/T$.

In order to facilitate a comparison of our results with these, we report the same susceptibilities. In the remainder of this paper, $\chi_0$ and $\chi_3$ refer to these 2-flavour quantities. We also report the first determination of the strange quark susceptibility, $\chi_s$. This work improves on previous studies in four other ways—by covering a larger range of temperatures, by using a series of different quark masses at fixed $m/T_c$, by investigating finite spatial volume effects systematically, and by analysing the effect of increasing statistics in the stochastic determination of Fermion operators on the lattice. The study of volume dependence gives control over extrapolation to the thermodynamic limit. Our study of many different quark masses gives the strange quark susceptibility. The systematic study of the stochastic method yields a vastly improved determination of the flavour-singlet susceptibility, $\chi_0$.

These computations have been made on lattices with lattice spacing $a = 1/4T_c$ in the quenched approximation. Fermion loops are neglected in this approximation, making it substantially easier to handle numerically. Apart from an overall normalisation of the temperature scale, this approximation is known to reproduce all the qualitative features of the full QCD simulations. Furthermore, numerical agreement between the quenched and 2-flavour dynamical QCD results for $\chi_0,3$ are obtained by 5–10% correction of the former [7]. It has been shown recently that one can extract continuum results from the lattice spacing we employ [10]. Nevertheless, in future studies we will analyse the effects of relaxing these two approximations.

Successive configurations used in our computations are separated by 1000 sweeps of a Cabibbo-Marinari heat bath algorithm, so that the gauge fields are completely decorrelated by any measure one may choose to use. At $\beta = 5.8941$, corresponding to $T = 1.5T_c$, we have generated configurations on $4 \times 8^3$, $4 \times 12^3$ and $4 \times 16^3$ lattices. At other couplings, corresponding to $T = 0.75T_c$, $1.1T_c$, $1.25T_c$, $2T_c$ and $3T_c$, we have used only the $4 \times 12^3$ lattice. We have used quark masses $m/T_c = 3$, 1.5, 1, 0.75, 0.30 and 0.03. Using estimates of $T_c = 275$–290 for the quenched theory [10], we see that the strange quark mass lies between 0.3$T_c$ and 0.5$T_c$.

In this letter we use staggered Fermions exclusively. Since these are defined for four flavours, we have to normalise the susceptibilities appropriately [11]. We have

$$\chi_0 = \frac{1}{2}(O_1 + \frac{1}{2}O_2),$$
$$\chi_3 = \frac{1}{2}O_1,$$
$$\chi_s = \frac{1}{4}(O_1 + \frac{1}{4}O_2),$$

where the two operators involved are

$$O_1 = \frac{T}{V} \langle \text{Tr} \left( M''M'^{-1} - M'M^{-1}M'M^{-1} \right) \rangle,$$
$$O_2 = \frac{T}{V} \langle \left( \text{Tr} M'M^{-1} \right)^2 \rangle.$$

The traces here are sums over lattice points and colour indices, and angular brackets are averages over gauge field configurations. Primes denote derivatives of the Dirac matrix with respect to appropriate chemical potentials. The quark mass to be used in the Dirac operator for evaluating $\chi_3$ is, of course, different from that for $\chi_{0,3}$.

The traces are evaluated by the usual stochastic technique,

$$\text{Tr} A = \frac{1}{N} \sum_{i=1}^{N} R_i^t A R_i,$$

where $R_i$ are a set of $N$ uncorrelated vectors with components drawn independently from a Gaussian ensemble with unit variance. Each vector has three colour components at each site of the lattice. We improve on the definitions in eq. (5) by using half lattice versions of the Dirac operator for staggered Fermions. A detailed discussion of the stochastic evaluation of the squared trace in eq. (5) can be found in [6]. A systematic study of the optimum value of $N$ is shown in Figure 1. We find that $N \approx 80$ is needed in order to see that $\chi_0 = \chi_3$ within statistical errors [12]. In all our subsequent work we have used $N = 80$ [9]. Such a large value of $N$ also seems to decrease the variance in the average over gauge configurations.

In Figure 2 we exhibit the spatial volume dependence of our results at $T = 1.5T_c$. Note that the volume dependence is smaller than the statistical errors on $\chi$ at this temperature, and also much smaller than what could be expected for an ideal gas of Fermions. In view of this, we have chosen to perform the remaining computations with lattices of size $4 \times 12^3$. This volume allows us to measure thermodynamic quantities up to 3$T_c$. At larger $T$, spatial deconfinement sets in and distorts the
results unless larger lattices are used [13]. Measurements of finite volume effects on the QCD equation of state indicates that this lattice size can also be used down to $1.1 T_c$, below which the first order phase transition of the quenched theory causes strong finite volume effects. As a result, the quenched theory approximation to full QCD is expected to fail close to $T_c$. Another finite size effect appears in Fermion computations in the quenched theory. As the quark mass decreases at fixed temperature, the scalar/pseudo-scalar screening length increases. If this length becomes comparable to the spatial dimensions of the lattice, then finite volume effects cannot be neglected. Among all our computations, this happened only for $m/T_c = 0.03$ at $T = 1.1 T_c$.

\[ \chi = \chi_0 = 2 \chi_s \text{ within statistical errors} \] for $T < T_c$ all the susceptibilities are consistent with zero. Notice that the results lie significantly below the expectation from FFT even at temperatures as high as $3 T_c$. It is interesting to note that the departures from an ideal gas become stronger with increasing quark mass. We discuss later our checks that this is not a lattice artifact. The jump in $\chi_s$ across $T_c$, its rapid increase with temperature, and its becoming comparable with $\chi_{0.3}$ for $T \geq 2 T_c$, all have observable consequences in the pattern of relative strangeness enhancement, including strange baryon enhancement, in going from CERN to RHIC energies [15].

It is interesting to speculate upon the reasons for departure from the weak coupling theory. In view of the long-standing observation that screening masses in the scalar/pseudo-scalar channel (so-called pion screening masses) also depart strongly from the weak coupling theory, we would like to check whether these observations are related. Below $T_c$, at vanishing chemical potential, pair production of quarks can take place only by pair production of mesons. The lightest meson, the pion, will be most effective at producing quark pairs. Hence the pion susceptibility [16]

\[ \chi_\pi = G_\pi(k_0 = 0, k = 0) = \frac{1}{N_z} \sum_z C_\pi(z) \] (here $G_\pi$ is the pion propagator in momentum space and $C_\pi$ is the zero-momentum screening correlator) should determine $\chi_{0.3}$. For $T > T_c$, this logic does not necessarily follow, but in view of the strong correlation in the pion sector, it is interesting to test such a hypothesis.

In Figure 3 we collect together all our data. At all these points, $\chi_0 = \chi_3 = 2 \chi_s$ within statistical errors [14]. For $T < T_c$ all the susceptibilities are consistent with zero. Notice that the results lie significantly below the expectation from FFT even at temperatures as high as $3 T_c$. It is interesting to note that the departures from an ideal gas become stronger with increasing quark mass. We discuss later our checks that this is not a lattice artifact. The jump in $\chi_s$ across $T_c$, its rapid increase with temperature, and its becoming comparable with $\chi_{0.3}$ for $T \geq 2 T_c$, all have observable consequences in the pattern of relative strangeness enhancement, including strange baryon enhancement, in going from CERN to RHIC energies [15].

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\[ \chi_i = a_i^0(T) + a_i^1(T)m + a_i^2(T)m \log m + O(m^2), \] (8)

where $i = 0, 3, s$ or $\pi$ [18]. Eliminating $m$ between $\chi_\pi$ and $\chi_3$, we can always expand one in terms of the other, albeit with temperature dependent coefficients. The surprise, shown in Figure 4, is that the curves for different $T$ can be scaled to lie on top of each other. The scaling function which achieves this goes to 1 as $T \to T_c$. Thus, for $T \leq 2 T_c$ and $m/T \geq 1.5$, the co-variance of $\chi_\pi$ and $\chi_3$ at $T_c$ determines that at higher temperatures. Since the scalar/pseudoscalar screening mass is smallest at finite temperature, and very much smaller than other screening masses, therefore the observed scaling may be interpreted as a demonstration of dynamical confinement in much the same way as at a second order phase transition.

Why is this not a trivial observation? After all, as $m \to 0$, $\chi_\pi/\chi_3$ is a $T$-dependent number, which defines the scaling function. The point is that the scaling is
observed not only for \( m = 0 \), but for a whole range of masses. This defines an universal curve at \( T_c \), not just a single value. As a result, this scaling also gives us a partial ability to test for lattice artifacts. The very fact that scaling is seen for all \( m/T_c \geq 1.5 \), when results for the lightest mass are consistent with previous computations in two flavour dynamical QCD, implies that there are neither strong quenching artifacts, nor other strong lattice artifacts in our measurements.

In summary, we have presented new and precise results on quark number susceptibilities over a wide range of temperatures and quark masses in the high temperature phase of QCD. The susceptibilities differ significantly from the ideal gas expectations (Figure 3). These deviations increase with mass and decrease at higher \( T \). As a result, we expect the relative strangeness enhancement seen in heavy-ion collisions to increase with temperature as quantitatively determined here. The linear relation between \( \chi_s \) and \( \chi_3 \) (Figure 4) is the clearest evidence to date for the hypothesis of “dynamical confinement” in the high temperature phase of the plasma [17]. However, it also shows that such a picture becomes less effective with increasing temperature. It is perhaps not a coincidence that the temperature at which the quenched plasma becomes free of this phenomenon is also the temperature at which dimensional reduction becomes quantitatively correct [19]. For \( T \geq 2T_c \), the light quark susceptibility is known accurately enough to test dimensional reduction and various other ideas which have emerged in trying to explain lattice results on the QCD equation of state.

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[2] Permanent address: Dept. of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India. Electronic mail: sgupta@tifr.res.in
[11] In the QCD computation the number of flavours is fixed by the valence quarks. The number staggering schemes give 4 flavours by default, which is accounted for by raising the \( detM \) in eq. (1) to the power \( N_f/4 \) for obtaining \( N_f \) flavours. We have \( N_f = 2 \) of light quarks and one of strange.
[12] It is interesting to note that \( C_2 \), which gives the difference between \( \chi_3 \) and \( \chi_0 \), is zero in free field theory. It is also subleading at large \( N_c \).
[14] At comparable quark masses and temperatures, there is better agreement between the quenched and 2-flavour dynamical QCD results of [6] than in the earlier quenched computation [7]. We believe that the source of this difference lies in spatial volume effects close to \( T_c \) which affect the quenched and dynamical simulations differently.
[18] Such an expansion is only valid for \( \chi_s \) in the high temperature phase, as follows from the results of [16]. In the low temperature phase, a similar expansion can only be made for \( 1/\chi_s \) with \( a_\pi = 0 \). The importance of the chiral logs is clearest in the expansion for the chiral condensate.

FIG. 4. Scaled values of \( \chi_s \) vary with \( \chi_3 \) independently of the values of \( T \leq 2T_c \) and \( m/T_c \leq 1.5 \). The meanings of the symbols are the same as in Figure 3.