Is It Possible To Clone Using An Arbitrary Blank State?

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Abstract

We show that using a fixed cloning machine, whether deterministic inexact or probabilistic exact, one can take an arbitrary blank state.

Quantum information cannot be cloned. There cannot exist a machine which can produce two (or more) exact copies of an arbitrary state in a deterministic manner [1]. Replication is not allowed even when the input state is taken at random from a given set of two non-orthogonal states [2]. It has been further shown that probabilistic cloning is not possible if the input state is from a given linearly dependent set [3]. Henceforth we deal explicitly with qubits although all the results can be extended to higher dimensions.

Although exact cloning is not possible, one can approximately clone an arbitrary input state [4, 5, 6]. In this scheme a fixed state is taken as the blank state, depending on which and the initial machine state, the cloning operation is constructed. We extend this operation such that any arbitrary (pure or mixed) state taken as the blank copy can do the job.

The optimal universal 1 → 2 inexact qubit cloner of Bruß et al.[5] takes an arbitrary qubit $|\psi\rangle \langle \psi| = \frac{1}{2}(I + \sigma_Z \sigma_Z)$ along with a fixed blank qubit $|b\rangle$ and a machine state $|M\rangle$ as input. An entangled state of the three qubits is produced as the output such that the reduced density matrices of the first two qubits are two similar approximate copies $\rho = \frac{1}{2}(I + \eta \sigma_Z \sigma_Z)$ of $|\psi\rangle \langle \psi|$ with $\eta = \frac{2}{3}$. The unitary operator realizing this process is defined on the combined Hilbert space of the input qubit, the blank qubit and machine by

$$U' |0\rangle |b\rangle |M\rangle = \frac{\sqrt{2}}{3} |00\rangle |m\rangle + \frac{\sqrt{1}}{6} (|01\rangle + |10\rangle) |m\perp\rangle$$

$$U' |1\rangle |b\rangle |M\rangle = \frac{\sqrt{2}}{3} |11\rangle |m\rangle + \frac{\sqrt{1}}{6} (|01\rangle + |10\rangle) |m\perp\rangle$$

where $|b\rangle$ is a fixed blank state (in a two-dimensional Hilbert space), $|M\rangle$ is a fixed machine state, $|m\rangle$ and $|m\perp\rangle$ being two mutually orthonormal states of the machine Hilbert space. The two clones are to surface at the first and second qubits. Note that the machine has turned out to be a qubit.

As it stands, the unitary operator $U'$ depends on the blank state $|b\rangle$ and the machine state $|M\rangle$. And the quality of the clones could be badly affected if $|b\rangle$ gets changed to an unknown state, say by some environment-induced decoherence. We show that by suitably constraining the unitary

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operator it is possible to keep the clones intact, even in this changed scenario. We carry over this considerations to the case of probabilistic exact cloning.

Let us suppose that the machine state \(|M\rangle\) belongs to a four-dimensional Hilbert space. And let \(U\) be defined on the combined Hilbert space of the input qubit, blank qubit and the four-dimensional Hilbert space of the machine by

\[
U |0\rangle |b\rangle |M\rangle = \sqrt{\frac{2}{3}} |00\rangle |M_0\rangle + \sqrt{\frac{1}{6}} (|01\rangle + |10\rangle) |M_1\rangle
\]

\[
U |1\rangle |b\rangle |M\rangle = \sqrt{\frac{2}{3}} |11\rangle |M_1\rangle + \sqrt{\frac{1}{6}} (|01\rangle + |10\rangle) |M_0\rangle
\]

\[
U |0\rangle |b_\perp\rangle |M\rangle = \sqrt{\frac{2}{3}} |00\rangle |M_2\rangle + \sqrt{\frac{1}{6}} (|01\rangle + |10\rangle) |M_3\rangle
\]

\[
U |1\rangle |b_\perp\rangle |M\rangle = \sqrt{\frac{2}{3}} |11\rangle |M_3\rangle + \sqrt{\frac{1}{6}} (|01\rangle + |10\rangle) |M_2\rangle
\]

where \(|M_i\rangle |M_j\rangle = \delta_{ij} (i, j = 0, 1, 2, 3)\) and \(|b\rangle |b_\perp\rangle = 0\).

Let \(|B\rangle = c |b\rangle + d |b_\perp\rangle\) be an arbitrary state of the Hilbert space of the blank qubit. Then

\[
U |0\rangle |B\rangle |M\rangle = \sqrt{\frac{2}{3}} |00\rangle |X\rangle + \sqrt{\frac{1}{6}} (|01\rangle + |10\rangle) |X'\rangle
\]

\[
U |1\rangle |B\rangle |M\rangle = \sqrt{\frac{2}{3}} |11\rangle |X'\rangle + \sqrt{\frac{1}{6}} (|01\rangle + |10\rangle) |X\rangle
\]

(2)

where

\(|X\rangle = c |M_0\rangle + d |M_2\rangle\)

\(|X'\rangle = c |M_1\rangle + d |M_3\rangle\)

are orthogonal. This form is exactly the same as in equation (1). Thus an arbitrary input state \(|\psi\rangle\) would be just as well cloned by equation (2) as it would be through equation (1). It is obvious that an arbitrary mixed state on the Hilbert space of the blank qubit can also be used in place of \(|B\rangle\) keeping the same \(U\).

Here we parenthetically note that the blank state can be made arbitrary even for a nonoptimal cloner [5]. And this is also true for an \(m \to n (m < n)\) cloner [6]. One only requires to take a higher dimensional machine.

Similar considerations carry over to the case of probabilistic exact cloning. Instead of the \(U_1\) defined by\[3\]

\[
U_1 |\psi_0\rangle |b\rangle |M\rangle = \sqrt{\gamma} |\psi_0\rangle |\psi_0\rangle |m\rangle + \sqrt{1 - \gamma} |\Phi\rangle
\]

\[
U_1 |\psi_1\rangle |b\rangle |M\rangle = \sqrt{\gamma} |\psi_1\rangle |\psi_1\rangle |m\rangle + \sqrt{1 - \gamma} |\Phi\rangle
\]

with \(\gamma = 1/(1 + |\langle \psi_0 | \psi_0 \rangle|)\) \(|m\rangle\) and \(|\Phi\rangle\) are orthogonal, \(|\psi_0\rangle\), \(|\psi_1\rangle\) being two non-orthogonal states which are to be probabilistically cloned, we define \(U_1\) as

\[
U_1 |\psi_0\rangle |b\rangle |M\rangle = \sqrt{\gamma} |\psi_0\rangle |\psi_0\rangle |M_0\rangle + \sqrt{1 - \gamma} |\Phi\rangle
\]

\[
U_1 |\psi_1\rangle |b\rangle |M\rangle = \sqrt{\gamma} |\psi_1\rangle |\psi_1\rangle |M_0\rangle + \sqrt{1 - \gamma} |\Phi\rangle
\]

\[
U_1 |\psi_0\rangle |b_\perp\rangle |M\rangle = \sqrt{\gamma} |\psi_0\rangle |\psi_0\rangle |M_1\rangle + \sqrt{1 - \gamma} |\Phi'\rangle
\]

\[
U_1 |\psi_1\rangle |b_\perp\rangle |M\rangle = \sqrt{\gamma} |\psi_1\rangle |\psi_1\rangle |M_1\rangle + \sqrt{1 - \gamma} |\Phi'\rangle
\]

where \(|M_0\rangle\), \(|M_1\rangle\), \(|\Phi\rangle\), \(|\Phi'\rangle\) are mutually orthogonal. Then

\[
U_1 |\psi_0\rangle |B\rangle |M\rangle = \sqrt{\gamma} |\psi_0\rangle |\psi_0\rangle |m'\rangle + \sqrt{1 - \gamma} |\Phi''\rangle
\]
\[ U_1 |\psi_1\rangle |B\rangle |M\rangle = \sqrt{\gamma} |\psi_1\rangle |\psi_1\rangle |m'\rangle + \sqrt{1-\gamma} |\Phi''\rangle \]

for an arbitrary blank qubit \(|B\rangle = c |b\rangle + d |b\perp\rangle \) so that \(|m'\rangle = c |M_0\rangle + d |M_1\rangle \) and \(|\Phi''\rangle = c |\Phi\rangle + d |\Phi'\rangle \) are orthogonal states. Consequently, the probabilistic cloning goes through with the same optimal efficiency even if we use an arbitrary blank qubit.

To summarize, we have shown that an unknown blank state is equally good for cloning, whether it is deterministic inexact or probabilistic exact.

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References