Constraints on Neutrino Oscillations Using 128 Days of Super-Kamiokande Solar Neutrino Data
TABLE I: Flux, uncertainty and definition of zenith angle and energy bins. The systematic uncertainty in the last two columns is split into energy-uncorrelated and energy-correlated uncertainty. The systematic uncertainty is assumed to be fully correlated in zenith angle.

<table>
<thead>
<tr>
<th>cosθz-Range</th>
<th>Flux±statistical uncertainty in units of SSM</th>
<th>Energy-correlated syst. uncert. in %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Day/ Mantle 1/ Mantle 2/ Mantle 3/ Mantle 4/ Mantle 5/ Core</td>
<td></td>
</tr>
<tr>
<td>5.0–5.5 MeV</td>
<td>0.436±0.046</td>
<td>0.436±0.046</td>
</tr>
<tr>
<td>5.5–6.5 MeV</td>
<td>0.431±0.022</td>
<td>0.442±0.048</td>
</tr>
<tr>
<td>6.5–8.0 MeV</td>
<td>0.461±0.013</td>
<td>0.524±0.036</td>
</tr>
<tr>
<td>8.0–9.5 MeV</td>
<td>0.437±0.014</td>
<td>0.449±0.038</td>
</tr>
<tr>
<td>9.5–11.5 MeV</td>
<td>0.434±0.015</td>
<td>0.432±0.042</td>
</tr>
<tr>
<td>11.5–13.5 MeV</td>
<td>0.456±0.026</td>
<td>0.466±0.071</td>
</tr>
<tr>
<td>13.5–16.0 MeV</td>
<td>0.482±0.056</td>
<td>0.532±0.155</td>
</tr>
<tr>
<td>16.0–20.0 MeV</td>
<td>0.476±0.149</td>
<td>0.476±0.149</td>
</tr>
</tbody>
</table>

The sample is divided into seven zenith angle bins (one day bin and six bins in cos θz for the night); within each zenith angle bin, the data are divided into eight recoil electron bins. We will refer to this binning of the data as the “zenith angle spectrum” (see Fig. 4). We define the zenith angle θz of an event as the angle between the vertical direction and the solar direction at the time of the event. Day events have cos θz ≤ 0 and night events cos θz > 0. The size of the sample (already divided into seven zenith angles) does not allow a subdivision into 19 energy bins shown in [3]. Due to this statistical limitation the lowest (5.0-5.5 MeV) and the highest (16.0-20.0 MeV) energy bin combine the flux of all zenith angles. Table 4 shows the flux, statistical and systematic uncertainty for all zenith angle and energy bins. The expected SSM flux of a particular energy bin is calculated from the total 8B and hep flux of BP2000 [3] and the neutrino spectrum from Ortiz et al. [2]. This neutrino spectrum is based on an improved measurement of the β-delayed a spectrum of the 8B decay with a small and well-controlled systematic uncertainty. Earlier reports used the neutrino spectrum by Bahcall et al. [1].

This zenith angle spectrum is analyzed in a two-neutrino oscillation scenario, which can be described with a mixing angle θ and a mass difference Δm². We consider two cases: (i) νs ↔ νµ,τ and (ii) νe ↔ νs(Katrin). For each set of neutrino oscillation parameters (sin² θ and Δm²) the expected number of solar neutrinos and their zenith angle spectrum are calculated. First, the probability P1 (P2) of a solar neutrino to be in the mass eigenstate ν1 (ν2) on the surface of the sun is obtained from a numerical calculation which propagates a neutrino wave function from the production point in the core to the surface. This calculation uses models for the distribution of the neutrino production point in the sun [1], the electron density in the sun [1], and the neutrino spectrum [2]. Above Δm² = 1.8 × 10⁻³ eV² the propagation of the two mass eigenstates from the sun to the earth and inside the earth can be assumed to be incoherent. The survival probability at the detector is given by

\[ P(\nu_e \rightarrow \nu_e)_{SK} = P_1 P_1 + P_2 P_2, \]
The survival probability is then

\[
P_{\text{surv}} = (1 - 2\sin^2\Theta) \left(1 + \frac{1}{3} \sin^2\Theta \right).
\]

where \(\Theta\) is the zenith angle.

Each energy bin \(i\) has a separate \(7 \times 7\) error matrix \(V_i\) describing the energy-uncorrelated uncertainty. \(V_i\) is the sum of the statistical error matrix and the energy-uncorrelated systematic error matrix (see Table I), the latter of which is constructed assuming full correlation in zenith angle. The flux normalization factor \(a\) is unconstrained to make the \(\chi^2\) independent of the total solar neutrino flux. The correlation parameter \(\delta_{\text{corr}}\) is constrained within \(\sigma_{\text{corr}}\). The size and shape of the correlated error are calculated as in [10]. The \(hep\) contribution to the neutrino flux is not constrained.

The \(\chi^2\) values are calculated in the parameter space, \((10^{-4} \leq \sin^2\theta \leq 1, 10^{-11} \leq \Delta m^2 \leq 10^{-3} \text{ eV}^2)\). In the case of active neutrinos, the minimum \(\chi^2\) value is 36.1 with 40 degrees of freedom at \((\sin^2\theta = 1, \Delta m^2 = 6.53 \times 10^{-11} \text{ eV}^2)\). The best-fit flux normalization is \(a = 0.788\), the correlation parameter is \(\delta_{\text{corr}} = -0.06\sigma_{\text{corr}}\) and the \(hep\) flux is 0. The shaded areas in Fig. 2 are excluded at 95% C.L. from this flux independent analysis. Most of the SMA and just-so solutions are disfavored with this C.L. In the case of sterile neutrinos, the minimum \(\chi^2\)
is 0. The shaded areas in Fig. 3 show the excluded regions (95% C.L.). The best-fit flux normalization is $\alpha = 0.917$, the correlation parameter is $\delta_{\text{corr}} = 0.06\sigma_{\text{corr}}$ and the hep flux is 0. The shaded areas are allowed at 95% in a combined fit to the fluxes measured at GALLEX[4], SAGE[3], Homestake[1] and Super-Kamiokande[5].

Using the theoretical uncertainty of the $^{8}$B flux $\sigma_{\text{max}} = 2.3 \pm 1.6^{\text{SSM}}$, an analysis combining flux and zenith angle spectrum has also been performed. In the active neutrino case, the minimum $\chi^2$ value is 37.8 with 41 d.o.f. at the same position as the unconstrained case. The flux normalization changes to $\alpha = 0.789$ and the correlation parameter to $\delta_{\text{corr}} = -0.02\sigma_{\text{corr}}$. The minimum $\chi^2$ point is within the just-so solution, but some LMA $\chi^2$ are similar to the minimum. For example, $\chi^2 = 39.1$ at $(\sin^2 2\theta = 0.87, \Delta m^2 = 7 \cdot 10^{-5} \text{ eV}^2)$ with a hep flux of 2.9×BP2000. The dotted lines in Fig. 3 show the contours of the 95% C.L. allowed regions. In the sterile neutrino case, the minimum $\chi^2$ value is 35.9 with 41 d.o.f at the same position as the unconstrained case. Flux normalization, correlation parameter and hep flux are unchanged. The inside of dotted lines in Fig. 3 is allowed at 95% C.L.. Since the allowed area from the combined flux analysis does not overlap these regions, oscillations into only sterile neutrinos are disfavored at this confidence level.

Figures 2 and 3 are based on the $\chi^2$ analysis of the zenith angle spectrum. We have also performed an oscillation search using the “day/night spectrum”, which, in contrast to the zenith angle spectrum, divides the data into two zenith angle bins (day and night bin). Each of these bins is then divided into 19 energy bins [5]. The $\chi^2$ is defined as follows:

$$\chi^2 = \sum_{E_i,N} \sum_{i=1}^{19} \left( \frac{\phi_{i,N}^{\text{osc}}}{\phi_{i,N}^{\text{SSM}}} - \alpha \times f(E_i, \delta_{\text{corr}}) \times \frac{\phi_{i,N}^{\text{osc}}}{\phi_{i,N}^{\text{SSM}}} \right) / \sigma_i^2$$
The notation is analogous to that used in the \( \chi^2 \) definition of the zenith angle spectrum analysis. \( \sigma_i \) is the sum of statistical and uncorrelated errors added in quadrature.

The minimum \( \chi^2 \) value is 28.2 with 34 degrees of freedom at \( \sin^2 2\theta = 0.4 \) and \( \Delta m^2 = 1.38 \times 10^{-10} \text{ eV}^2 \). The best-fit flux normalization is \( \alpha = 0.488 \) and the correlation parameter is \( \delta_{\text{corr}} = -0.2\sigma_{\text{corr}} \). Figure 4 shows the 95% excluded regions using the shape of this day/night spectrum. The excluded area is similar to that obtained in the zenith angle spectrum, but more restrictive in the SMA region. The differences at the LMA and near the LOW solution are due to the zenith angle variations within the night bin. The lower left corner of the SMA predicts a slight depression of the core flux resulting in a day flux prediction that is larger than the night flux. SK measures a 1.3\( \sigma \) excess of the night flux over the day flux, but the flux in the core bin is below the day flux. This leads to a slightly better fit of these parameters to the zenith angle spectrum than to the day/night spectrum. The lower left corner of the SMA 95% C.L. region is excluded at 93% C.L. by the zenith angle spectrum and at 97% C.L. by the day/night spectrum analysis. Other differences are due to the use of different binnings.

In summary, Super-Kamiokande precisely measured the energy dependence and zenith angle dependence of the solar \(^8\)B neutrino flux. The data do not show a significant distortion of the spectrum or zenith angle variation. This places strong constraints on neutrino oscillation solutions to the solar neutrino problem independently of the flux expectation. If oscillations into active neutrinos are assumed, just-so and the SMA solutions are disfavored at 93% (zenith angle spectrum) to 97% C.L. (day/night spectrum) and the LMA solutions are preferred. In conjunction with the SK \(^8\)B flux measurement, two allowed areas at large mixing remain. All possible oscillation solutions into only sterile neutrinos are disfavored at 95% confidence level.

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**REFERENCES**

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[3] Present address: Department of Physics, University of Utah, Salt Lake City, UT 84112, USA
[9] S. Fukuda et al., to be submitted.
[20] In the case of vacuum oscillations $P_1 = \cos^2 \theta$, $P_2 = \sin^2 \theta$, and this reduces to $1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$.
[21] This solution only appears at 99% C.L.; however it is usually discussed as a possible solution.