COLLIDER VERIFICATION OF THE NEUTRINO
MASS MATRIX IN TWO SCENARIOS

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Abstract

If the origin of neutrino mass is at the TeV energy scale, collider experiments may
in fact map out all the elements of the $3 \times 3$ neutrino mass matrix, up to an overall
scale. Two examples [1, 2] are discussed and one is related [3] to the muon anomalous
magnetic moment.

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1 Introduction

In the minimal Standard Model with one Higgs doublet $\Phi = (\phi^+, \phi^0)$ and 3 lepton doublets $L = (\nu, l)_L$ and singlets $l_R$ only, neutrino mass must come from the effective dimension-5 operator [4, 5]

$$\frac{1}{\Lambda} LL\Phi\Phi = \frac{1}{\Lambda} (\nu\phi^0 - l\phi^+)^2,$$

which shows that the form of $m_\nu$ must necessarily be “seesaw”, i.e. $v^2/\Lambda$ where $v = \langle \phi^0 \rangle$, whatever the underlying mechanism for neutrino mass is.

The canonical seesaw mechanism [6] assumes 3 heavy right-handed singlet lepton fields $N_R$ with the Yukawa couplings $f LN\Phi$ and the Majorana mass $m_N$, hence Eq.(1) is realized with the famous expression

$$m_\nu = \frac{f^2 v^2}{m_N}.$$  (2)

Note that lepton number is violated by $m_N$ in the denominator and it should be large for a small neutrino mass, i.e.

$$m_\nu \sim \left(\frac{f}{10} \right)^2 \left(\frac{10^{13} \text{ GeV}}{m_N}\right) \text{ eV}.\quad (3)$$

2 Higgs Triplet Model

An equally satisfactory realization of Eq.(1) is to use a Higgs triplet [7, 8] $\xi = (\xi^{++}, \xi^+, \xi^0)$ with

$$L_{int} = f_{ij}[\xi^0\nu_i\nu_j + \xi^+(\nu_i l_j + l_i\nu_j)/\sqrt{2} + \xi^{++}l_i l_j]$$

$$+ \mu(\xi^0\phi^0\phi^0 - \sqrt{2}\xi^-\phi^+\phi^0 + \xi^{--}\phi^+\phi^+) + h.c.\quad (4)$$

Instead of having a negative $m_\xi^2$, make it positive and large, i.e. $m_\xi >> v$. We then find [8]

$$m_\nu = \frac{2f_{ij}\mu v^2}{m_\xi^2} = 2f_{ij}\langle \xi^0 \rangle.\quad (5)$$
Note that the effective operator of Eq.(1) is realized here with a simple rearrangement of the individual terms, i.e.

$$L_i L_j \Phi \Phi = \nu_i \nu_j \phi^0 \phi^0 - (\nu_i l_j + l_i \nu_j)\phi^+ \phi^0 + l_i l_j \phi^+ \phi^0.$$  \hspace{1cm} (6)

Note also that lepton number is violated in the numerator in this case. If $f_{ij} \sim 1$, then $\mu/m_\xi < 10^{-13}$ GeV$^{-1}$. Hence $m_\xi \sim 1$ TeV is possible, if $\mu < 100$ eV. To obtain such a small mass parameter, the “shining” mechanism of extra large dimensions [9] may be used. \[1\] Let $\chi$ be a singlet scalar in the bulk carrying lepton number $L = -2$, then

$$\langle \chi \rangle \sim \frac{\Gamma(\frac{n-2}{2})}{4\pi^2} M_s \left( \frac{M_s}{M_P} \right)^{2-n}.$$  \hspace{1cm} (7)

For $n = 3$, $M_s \sim 1$ TeV, $M_P = 2.4 \times 10^{18}$ GeV, $\langle \chi \rangle \sim 4.4$ eV. Therefore, if we replace $\mu$ by $h\chi$, the intriguing possibility of having $m_\xi \sim 1$ TeV is realized. In particular, the doubly charged $\xi^{\pm \pm}$ can be easily produced at colliders and $\xi^{++} \rightarrow l_i^+ l_j^+$ is a distinct and backgroundless decay which maps out $|f_{ij}|$, and thus determine directly the neutrino mass matrix up to an overall scale. \[1\]

### 3 Leptonic Higgs Doublet Model

Another simple and interesting way to have the origin of neutrino mass at the TeV scale has just been proposed. \[2\] As in the canonical seesaw model, we have again $3 N_R$‘s but they are now assigned $L = 0$ instead of the customary $L = 1$. Hence the Majorana mass terms are allowed but the usual $LN\Phi$ terms are forbidden by lepton-number conservation. The $LL\Phi\Phi$ operator of Eq.(1) is not possible and $m_\nu = 0$ at this point.

We now add a new scalar doublet $\eta = (\eta^+, \eta^0)$ with $L = -1$, then $fLN\eta$ is allowed, and the operator $LL\eta\eta$ will generate a nonzero neutrino mass if $\langle \eta^0 \rangle \neq 0$. The trick now is to show how $f\langle \eta^0 \rangle < 1$ MeV can be obtained naturally, so that $m_N \sim 1$ TeV becomes
possible and amenable to experimental verification, in contrast to the very heavy $N_R$'s of the canonical seesaw mechanism.

Consider the following Higgs potential:

\[ V = m_1^2 \Phi^\dagger \Phi + m_2^2 \eta^\dagger \eta + \frac{1}{2} \lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_2 (\eta^\dagger \eta)^2 \]

\[ + \lambda_3 (\Phi^\dagger \Phi) (\eta^\dagger \eta) + \lambda_4 (\Phi^\dagger \eta) (\eta^\dagger \Phi) + \mu_{12} (\Phi^\dagger \eta + \eta^\dagger \Phi), \]

(8)

where the $\mu_{12}^2$ term breaks lepton number softly and is the only possible such term. Let $\langle \phi^0 \rangle = v$, $\langle \eta^0 \rangle = u$, then the equations of constraint for the minimum of $V$ are given by

\[ v [m_1^2 + \lambda_1 v^2 + (\lambda_3 + \lambda_4) u^2] + \mu_{12}^2 u = 0, \]

(9)

\[ u [m_2^2 + \lambda_2 u^2 + (\lambda_3 + \lambda_4) v^2] + \mu_{12}^2 v = 0. \]

(10)

Consider the case $m_1^2 < 0$, $m_2^2 > 0$, and $|\mu_{12}^2| \ll m_2^2$, then

\[ v^2 \simeq - \frac{m_1^2}{\lambda_1}, \quad u \simeq - \frac{\mu_{12}^2 v}{m_2^2 + (\lambda_3 + \lambda_4) v^2}. \]

(11)

Hence $u$ may be very small compared to $v (= 174$ GeV). For example, if $m_2 \sim 1$ TeV, $|\mu_{12}^2| \sim 10$ GeV$^2$, then $u \sim 1$ MeV and

\[ m_\nu \sim \left( \frac{f}{1.0} \right)^2 \left( \frac{1 \text{ TeV}}{m_N} \right) \text{ eV}. \]

(12)

Since both $m_N$ and $m_2$ are now of order 1 TeV, they may be produced at future colliders and be detected. (I) If $m_2 > m_N$, then the physical charged Higgs boson $h^+$, which is mostly $\eta^+$, will decay into $N$, which then decays into a charged lepton and a $W$ boson via $\nu - N$ mixing:

\[ h^+ \rightarrow l_i^+ N_j, \quad N_j \rightarrow l_k^\pm W^\mp. \]

(13)

(II) If $m_N > m_2$, then

\[ N_i \rightarrow l_j^\pm h^\mp, \quad h^+ \rightarrow t\bar{b}, \]

(14)
the latter coming from $\Phi - \eta$ mixing. In either case, $m_2$ and $m_N$ can be determined kinematically, and $|f_{ij}|$ measured up to an overall scale.

In summary, the particle spectrum of the leptonic Higgs doublet model consists of the usual Standard-Model particles, including the one physical Higgs boson $h^0_1$, 3 heavy $N_R$'s at the TeV scale, and a heavy scalar doublet $(h^\pm, h^0_2, A)$ of individual masses $\sim m_2$. The charged Higgs boson $h^\pm$ can be pair-produced at hadron colliders, whereas $N_R (h^\pm)$ can be produced at lepton colliders via the exchange of $h^\pm (N_R)$.

4 The Size of Lepton Number Violation

It has been shown in the above that whereas Majorana neutrino masses have to be tiny, the actual magnitude of lepton number violation may come in all sizes.

(1) Large: $m_N \sim 10^{13}$ GeV in the canonical seesaw mechanism.

(2) Medium: $|\mu^2| \sim 10 \text{ GeV}^2$ in the leptonic Higgs doublet model with $m_N \sim 1 \text{ TeV}$.

(3) Small: $\lambda_0 \langle \chi \rangle \sim 10 \text{ eV}$ in the Higgs triplet model ($m_\xi \sim 1 \text{ TeV}$) with a singlet bulk scalar in extra large dimensions.

In (2) and (3), direct experimental determination of the relative magnitudes of the elements of $M_\nu$ is possible at future colliders.

5 Muon Anomalous Magnetic Moment

The recent measurement [10] of the muon anomalous magnetic moment appears to disagree with the Standard-Model prediction [11] by 2.6$\sigma$, i.e.

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} > 215 \times 10^{-11}$$

(15)
at 90% confidence level. The origin of this discrepancy may be directly related to the TeV physics responsible for neutrino mass. In the leptonic Higgs doublet model, this has the following consequences. [3]

Assume all $m_N$’s are equal with its Yukawa coupling matrix given by

$$h_{ij} = \begin{bmatrix} 2ch_1 & -\sqrt{2}sh_1 & \sqrt{2}sh_1 \\ 2sh_2 & \sqrt{2}ch_2 & -\sqrt{2}ch_2 \\ 0 & \sqrt{2}h_3 & \sqrt{2}h_3 \end{bmatrix}, \quad (16)$$

with $h_1 \leq h_2 \leq h_3$ and $s = \sin \theta, \ c = \cos \theta$, we then find

$$m_\eta < 371\sqrt{a_h} \text{ GeV}, \quad (17)$$

where $h_1 \simeq h_2 \simeq h_3$ has been assumed. In other words, the neutrino mass matrix has nearly degenerate mass eigenvalues. If not, then satisfying Eq. (15) would be in conflict with the experimental upper limit on the $\tau \to \mu \gamma$ branching fraction. As it is, we also have the interesting prediction of

$$\frac{\Gamma(\mu \to e\gamma)}{m_{\mu}^5} : \frac{\Gamma(\tau \to e\gamma)}{m_{\tau}^5} : \frac{\Gamma(\tau \to \mu\gamma)}{m_{\tau}^5} = 2s^2c^2(\Delta m^2)^{sol} : 2s^2c^2(\Delta m^2)^{sol} : (\Delta m^2)^{atm}_{\text{atm}} \quad (18)$$

In Fig. (1), the branching fractions of $\tau \to \mu\gamma$ and $\mu \to e\gamma$, and the $\mu - e$ conversion ratio in $^{13}$Al are plotted using the lower bound of Eq. (15), as a function of the common neutrino mass $m_\nu$. Using Eq. (16), the values

$$\quad (\Delta m^2)_{\text{atm}} = 3 \times 10^{-3} \text{ eV}^2, \quad (\Delta m^2)_{\text{sol}} = 3 \times 10^{-5} \text{ eV}^2, \quad (19)$$

have been chosen according to present data from neutrino-oscillation experiments. At $m_\nu \simeq 0.2 \text{ eV}$, which is in the range of present upper limits on $m_\nu$ from neutrinoless double beta decay, $B(\mu \to e\gamma)$ and $R_{\mu e}$ are both at their present experimental upper limits. Hence Eq. (18) will be tested in new experiments planned for the near future which will lower these upper limits.
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References

Figure 1: Lower bounds on \( B(\tau \to \mu \gamma) \), \( B(\mu \to e \gamma) \), and \( R_{\mu e} \).