NEW PHYSICS IN CP VIOLATION EXPERIMENTS

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ABSTRACT: CP violation plays a privileged role in our quest for new physics beyond the electroweak standard model (SM). In the SM the violation of CP in the weak interactions has a single source: the phase of the quark mixing matrix (the CKM matrix, for Cabibbo–Kobayashi–Maskawa). Most extensions of the SM exhibit new sources of CP violation. For instance, the truly minimal supersymmetric extension of the SM (CMSSM) has two new phases in addition to the CKM phase. Given that CP violation is so tiny in the kaon system, is still largely unexplored in B physics and is negligibly small in the electric dipole moments, it is clear that new physics may have a good chance to manifest some departure from the SM in this particularly challenging class of rare phenomena. On the other hand, it is also apparent that CP violation generally represents a major constraint on any attempt at model building beyond the SM. In this review we tackle these two sides of the relation between CP violation and new physics. Our
focus will be on the potentialities to use $CP$ violation as a probe on Supersymmetric (SUSY) extensions of the SM. We wish to clarify the extent to which such indirect signals for SUSY are linked to a fundamental theoretical issue: is there a relation between the mechanism that originates the whole flavor structure and the mechanism that is responsible for the breaking of supersymmetry? Different ways to answer this question lead to quite different expectations for $CP$ violation in B physics.

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1 INTRODUCTION

With the shutdown of the Large Electron–Positron Collider (LEP) at CERN, a crucial and glorious chapter of twentieth century physics has ended. Thanks to the efforts of experimentalists and theorists all over the world, we are now able to make a definitive statement concerning our knowledge of the physical world: the Standard Model (SM) represents the correct description of fundamental interactions up to energies of $O(100 \text{ GeV})$. The fact that we know the right theory of fundamental interaction working at distances as small as $10^{-18}$ m is a tremendous achievement.
Having emphasized this bright side of the last two decades, one has must also admit that the experimental and theoretical particle physics of these years has been rather unsuccessful in finding a road to follow beyond the SM. If we are all convinced that the SM cannot be the ultimate theory of “everything” (if only because it does not include gravity), we are in the dark about when, where and how new physics beyond the SM should manifest itself. Searches for new particles and interactions in leptonic and hadronic machines have failed to produce results. Theoretically, the most promising advances in recent years have led to theories (strings, branes, etc) which seem rather far from experimental tests.

Although the quest for new physics finds its most convincing answer in the traditional avenue of energies sufficient to produce and observe new particles, the lesson of recent decades has clearly indicated a second, indirect way of signalling the presence of new physics. We refer to the physics related to rare processes where Flavor Changing Neutral Currents (FCNC) and/or $CP$ violation occur. As is well known, the first indications of the existence of charm and of a heavy top quark came before their direct discovery from evidence that indicated their role as virtual particles in $K$ and $B$ mixings, respectively. Obviously rare processes are a privileged ground for indirect signals of new physics. One can hardly believe that new physics contributions can be observable when considering phenomena that arise at the tree level in the SM. On the contrary, FCNC and/or $CP$ violating processes offer the possibility for loops containing new particles to compete with loops containing SM particles, thus allowing a fair chance for new physics to emerge against the “SM background”.

We think that the indirect road to new physics will become particularly relevant in the next few years. As we said, high energy accelerators have established the
SM as the correct 100 GeV theory, but they have not yet succeeded at the equally important task of observing some signals of new physics. After the end of LEP, our attention in high energy accelerator physics turns to the upgraded Tevatron at Fermilab. The evidence from the late activity at LEP in favor of a light Higgs with a mass around 115 GeV feeds our hopes that a Higgs signal may be detected at the Tevatron. Even if this were to occur, we would have only an indirect indication that new physics must be far below some grand unification scale; the so-called “big desert” picture would be rejected (we remind the reader that if the SM is valid up to some very large scale close to the Planck scale the Higgs mass should exceed 130 GeV). The class of SM extensions that goes under the name of low energy supersymmetry (SUSY) would be favored, although they certainly do not exhaust the possibilities for new physics with a light Higgs. However, unless superpartners are light, it might be difficult to establish their existence at the Tevatron. If superpartners elude the Tevatron, we will have to wait for the Large Hadron Collider (LHC) at CERN to operate before deciding “directly” whether low energy SUSY is a reality or just an intriguing intellectual construction.

What can we do to search for new physics in the meantime? Here is where the FCNC and/or $CP$ violating processes come into full play in our challenging (sometimes desperate) effort to find hints of new physics. Two brand–new $B$ factories have just started operating, and there are plans for new experiments in rare kaon decays, in testing the electric dipole moments, in flavor violating leptonic processes, etc. The enormous and interesting project of probing the SM in rare processes will proceed at full speed in the pre–LHC epoch. It is important to critically discuss strategies to look for new physics in such processes. Our goal here is to report on some of the most relevant aspects of the search for
some signal of departure from the SM expectations in $CP$ violating phenomena. Admittedly, even if we find such signals it will be hard to establish what new physics is responsible for them. However, we indicate how, at least in the specific framework of low energy SUSY, the combination of several observations may shed important light on the new physics, hence directing direct searches towards specific targets.

We refer to “FCNC and/or $CP$ violation” in qualifying the interesting processes for indirect detection of new physics because we can consider three different classes of them: (a) phenomena that require FCNC but occur even without $CP$ violation as $K \rightarrow \bar{K}$ or $B \rightarrow \bar{B}$ mixing; (b) $CP$ violating processes without FCNC, namely Electric Dipole Moments (EDMs) of the neutron, electron, etc; and finally, (c) simultaneous FCNC and $CP$ violating phenomena, for instance the quantities $\varepsilon$ or $\varepsilon'/\varepsilon$ in $K$ decays into pions and the $CP$ asymmetries in $B$ decays.

In our view, among these three classes of rare processes, those related to the presence of $CP$ violation deserve a special attention for at least three reasons.

1. After nearly four decades of intensive experimental and theoretical work $CP$ violation still appears rather mysterious and, hence, a potentially good candidate to offer surprises in future tests (for general and recent reviews on $CP$ violation see (1)). We recently witnessed two major breakthroughs in our understanding of $CP$ violation. First, from the measurements (2) of the $\varepsilon'/\varepsilon$ parameters from both sides of the Atlantic, we got the information that $CP$ violation occurs not only in $K \rightarrow \bar{K}$ mixing ($\Delta S = 2$), but also in the direct $K$ decay amplitudes ($\Delta S = 1$). Hence we can definitely reject the idea that $CP$ violation arises only from some specific superweak interactions in processes that change strangeness by two units. The second
relevant piece of information, which we obtained last year, is that $CP$ violation is not present exclusively in the kaon system; there exists at least one other place where it shows up, namely in $B$ physics. This evidence emerged from the measurement of the $CP$ asymmetry in the decays of $B$ into $J/\psi K_s$ in three different experiments: $a_{J/\psi} = 0.34 \pm 0.20 \pm 0.05$ at BaBar (3), $a_{J/\psi} = 0.58^{+0.32}_{-0.34}^{+0.09}$ at BELLE (4) and $a_{J/\psi} = 0.79^{+0.41}_{-0.44}$ at CDF (5). Interestingly enough, although the measurement coming from CDF confirms the SM expectation for such asymmetry, the results from the two abovementioned $B$ factories indicate central values significantly lower than the SM expectations. The errors are still quite large and so the best advice is the usual “wait and see”, but this example illustrates how surprises in the $CP$ exploration can be lurking around the corner.

2. From the theoretical point of view, it is important to emphasize that new physics beyond the SM generically introduces new sources of $CP$ violation in addition to the usual CKM phase of the SM. Indeed, it is a common experience of model builders that if one tries to extend the SM with some low energy new physics one must somehow control the proliferation of new $CP$ violating contributions. Significant portions of the parameter spaces of new physics models can generally be ruled out by the severe constraints imposed by $CP$ violating phenomena (6). For instance, as we discuss below, even if one considers the minimal supersymmetric extension of the SM that passes all the FCNC tests unscathed, one still faces severe problems in matching the experimental results concerning processes with $CP$ violation, especially the constraints coming from the various bounds on electric dipole moments.
3. The third reason which makes us optimistic in having new physics playing a major role in $CP$ violation concerns the matter–antimatter asymmetry in the universe. Starting from a baryon–antibaryon symmetric universe, the SM is unable to account for the observed baryon asymmetry. The presence of new $CP$–violating contributions beyond the SM looks crucial to produce an efficient mechanism for the generation of a satisfactory $\Delta B$ asymmetry.

The aim of this article is to discuss the above points with the goal of exploring the potentialities of $CP$ violation in our quest for new physics. Because of the vastness of the subject, we focus on a promising class of new physics that goes under the generic name of low energy SUSY (7). We emphasize that low energy SUSY does not denote a well defined model; rather, it includes a variety of SM extensions (with a variety of phenomenological implications). We characterize this huge class of models according to their main features in relation to $CP$ violation. We want to avoid losing our readers in endless “botanic” classifications of SUSY models, but we also want to avoid the idea that one can discuss $CP$ violation in “the” SUSY model as if one were discussing a specific construction such as the SM.

As appealing as low energy SUSY may appear, at least to some physicists, one should not forget that after more than twenty years of searches we do not have any experimental hint of SUSY particles. Furthermore, one could argue theoretically that the idea that superpartners appear on the TeV scale is motivated only by the hope that SUSY resolves the gauge hierarchy problem. The unification of the gauge couplings when low energy SUSY particles are included in their running can be considered only circumstantial evidence in favor of SUSY. In spite of all this, our choice to stick to low energy SUSY for most of this article in exploring the
impact of $C P$ violation on new physics is justified by a simple and uncontroversial fact: we do not know of any other “complete” model of new physics that tackles the gauge hierarchy problem and successfully passes the impressive list of direct and indirect experimental tests of new physics. In any case, in the final part of this article we comment on $C P$ violation in other alternative possibilities for new physics.

Our discussion follows the general lines of distinction between $C P$ violating processes with and without FCNC that we underlined above. In the next Section we focus on $C P$ violation in the electric dipole moments (hence without FCNC) and we introduce the first aspect of what is called the “SUSY $C P$ problem”. Then in Section 3 we move to flavor changing $C P$ violation in SUSY and we deal with the second aspect of the SUSY $C P$ problem. Section 4 is devoted to an overview of $C P$ violation in other extensions of the SM. Finally in Section 5 we present our conclusions and outlook.

2 SUSY $C P$ VIOLATION AND ELECTRIC DIPOLE MOMENTS

Before entering the more specific discussion of the next sections, we offer a quick overview on $C P$ violation in SUSY at a more introductory level.

When one decides to supersymmetrize the SM one has many options (7). First, the fields of the SM are embedded into superfields containing also the SUSY partners of each known particle; but how many superfields should one introduce? Here we stick to the minimal option: we introduce the minimal amount of superfields that are strictly demanded to obtain a viable supersymmetrization of the SM. This means that each particle will be accompanied by a superpartner, except in the Higgs sector where we have to introduce a second Higgs doublet in addition
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Some models introduce new singlet superfields or even more complicate structures, and such models may have new sources of CP violation. A second important limitation in the class of SUSY models that we consider here concerns the imposition of R parity. This discrete symmetry is usually added to the gauge and super–symmetries in order to prevent excessive baryon and lepton number violations. Although some symmetry must indeed be imposed to inhibit proton decay, R parity is not the only way to achieve this. A vast class of alternative models imposes some discrete symmetry other than R, allowing for either baryon or lepton number violations. In such models, many new Yukawa couplings exist and then the number of CP violating phases of the theory is sizeably increased.

Even in minimal Supersymmetric versions of the SM (MSSM) where the minimal number of superfields is introduced and R parity is imposed, one is still left with more than 100 free parameters, almost half of them given by CP violating phases (8). Fortunately most of this huge parameter space is already phenomenologically ruled out. Indeed, FCNC and CP violating processes play a major role in drastically reducing the parameter space. Obviously it is difficult to make phenomenological predictions with so many free parameters, and so through the years many theoretical further restrictions have been envisaged for the MSSM class. The most drastic reduction on the SUSY parameter space leads to what is called the constrained MSSM (CMSSM) or minimal supergravity (7). In the absence of phases, this model is characterized by only four parameters plus the sign of a fifth parameter.

In any MSSM, at least two new “genuine” SUSY CP–violating phases are present. They originate from the SUSY parameters $\mu$, $M$, $A$ and $B$. The first of
these parameters is the dimensionful coefficient of the $H_u H_d$ term of the super-potential. The remaining three parameters are present in the sector that softly breaks the $N = 1$ global SUSY. $M$ denotes the common value of the gaugino masses, $A$ is the trilinear scalar coupling, and $B$ denotes the bilinear scalar coupling. In our notation, all these three parameters are dimensionful. The simplest way to see which combinations of the phases of these four parameters are physical (9) is to notice that for vanishing values of $\mu$, $M$, $A$ and $B$ the theory possesses two additional symmetries (10). Indeed, if $B$ and $\mu$ are set to zero, a $U(1)$ Peccei–Quinn symmetry emerges, which in particular rotates $H_u$ and $H_d$. If $M$, $A$ and $B$ are set to zero, the Lagrangian acquires a continuous $U(1)_R$ symmetry. Then we can consider $\mu$, $M$, $A$ and $B$ as spurions that break the $U(1)_{PQ}$ and $U(1)_R$ symmetries. In this way, the question concerning the number and nature of the meaningful phases translates into the problem of finding the independent combinations of the four parameters that are invariant under $U(1)_{PQ}$ and $U(1)_R$ and determining their independent phases. There are three such independent combinations, but only two of their phases are independent. We use here the commonly adopted choice:

$$\phi_A = \arg (A^* M), \quad \phi_\mu = \arg (\mu^* M).$$  \hfill (1)

where also $\arg (B\mu) = 0$, i.e. $\phi_B = -\phi_\mu$.

The main constraints on the SUSY phases come from their contribution to the electric dipole moments of the neutron and of the electron. For instance, the effect of $\phi_A$ and $\phi_\mu$ on the electric and chromoelectric dipole moments of the light quarks ($u$, $d$, $s$) leads to a contribution to $d_n$ of order (11)
\[ d_n \sim 2 \left( \frac{100 \text{ GeV}}{\tilde{m}} \right)^2 \sin \phi_{A,\mu} \times 10^{-23} \text{e cm}, \]  

(2)

where \( \tilde{m} \) here denotes a common mass for sleptons and gauginos. The present experimental bound, \( d_n < 1.1^{-25} \text{ e cm} \), implies that \( \phi_{A,\mu} \) should be < \( 10^{-2} \), unless one pushes SUSY masses up to \( \mathcal{O}(1 \text{ TeV}) \). A possible caveat to such an argument calling for a fine–tuning of \( \phi_{A,\mu} \) is that uncertainties in the estimate of the hadronic matrix elements could relax the severe bound in Equation 2 (12).

These considerations lead most authors dealing with the MSSM to simply put \( \phi_A \) and \( \phi_\mu \) equal to zero. Actually, one may argue in favor of this choice by considering the soft breaking sector of the MSSM as resulting from SUSY breaking mechanisms that force \( \phi_A \) and \( \phi_\mu \) to vanish. For instance, it is conceivable that both \( A \) and \( M \) originate from one source of \( U(1)_R \) breaking. Since \( \phi_A \) measures the relative phase of \( A \) and \( M \), in this case it would “naturally” vanish. In some specific models, it has been shown (13) that through an analogous mechanism \( \phi_\mu \) may also vanish.

In recent years, the attitude towards the EDM problem in SUSY and the consequent suppression of the SUSY phases has significantly changed. Indeed, options have been envisaged allowing for a conveniently suppressed SUSY contribution to the EDM even in the presence of large (sometimes maximal) SUSY phases. Methods of suppressing the EDMs consist of cancellation of various SUSY contributions among themselves (14), non universality of the soft breaking parameters at the unification scale (15) and approximately degenerate heavy sfermions for the first two generations (16). In the presence of one of these mechanisms, large supersymmetric phases are expected yet EDMs should be generally close to the experimental bounds. In the following we discuss the implications of these new
$CP$ violating phases that may be present in SUSY, keeping in mind that some of the above mechanisms may be required to satisfy the constraints coming from the EDM’s.

3 FLAVOR CHANGING $CP$ VIOLATION

Despite of the large sensitivity of EDMs to the presence of new phases, so far only neutral meson systems, $K^0$–$\bar{K}^0$ or $B^0$–$\bar{B}^0$, show measurable effects of $CP$ violation. This fact is, at first sight, surprising because in the neutral mesons $CP$ violation is associated with a change in flavor and hence is CKM suppressed, whereas EDMs are completely independent of flavor mixing. The reason for this is that, in the SM, $CP$ violation is intimately related to flavor, to the extent that observable $CP$ violation requires, not only a phase in the CKM mixing matrix, but also three non–degenerate families of quarks (17). As shown in the previous section, the supersymmetrized SM contains new sources of $CP$, both flavor independent or flavor dependent. Although the new phases are, in principle, strongly constrained by the EDM experimental limits, we have seen that several mechanisms allow us to satisfy these constrains with large supersymmetric phases.

Next, we analyze this possibility and the effects in flavor changing $CP$ violation observables. We first concentrate on a MSSM with a flavor blind SUSY breaking, and then we study a general MSSM in which the soft breaking terms include all kinds of new flavor structures.

3.1 Flavor Blind SUSY Breaking and $CP$ Violation

The first step in our review of Supersymmetric $CP$ violation is the analysis of a MSSM with flavor blind SUSY breaking. Flavor–blind refers to a softly broken
Supersymmetric SM in which the soft breaking terms do not introduce any new flavor structure beyond the Yukawa matrices whose presence in the superpotential is required to reproduce correctly the fermion masses and mixing angles. Supersymmetry is broken at a large scale, that we identify with $M_{GUT}$, and from here, the parameters evolve with the standard MSSM renormalization group equations (RGE) (18, 19) down to the electro–weak scale. In these conditions the most general allowed structure of the soft–breaking terms at $M_{GUT}$ is,

\[
(m^2_Q)_{ij} = m^2_Q \delta_{ij} \quad (m^2_U)_{ij} = m^2_U \delta_{ij} \quad (m^2_D)_{ij} = m^2_D \delta_{ij}
\]

\[
(m^2_L)_{ij} = m^2_L \delta_{ij} \quad (m^2_E)_{ij} = m^2_E \delta_{ij} \quad (m^2_{H_1}) = m^2_{H_1} \quad (m^2_{H_2}) = m^2_{H_2}
\]

\[
(A_U)_{ij} = A_U e^{i\phi_{A_U}} (Y_U)_{ij} \quad (A_D)_{ij} = A_D e^{i\phi_{A_D}} (Y_D)_{ij}
\]

\[
(A_E)_{ij} = A_E e^{i\phi_{A_E}} (Y_E)_{ij}.
\]

where all the allowed phases are explicitly written except possible phases in the Yukawa matrices that give rise to an observable phase in the the CKM matrix, $\delta_{CKM}$. It is important to emphasize that, in this flavor blind MSSM, $\delta_{CKM}$ is the only physical phase in the Yukawa matrices and all other phases in $Y_U$ and $Y_D$ can be rephased away in the same way as in the SM. The absence of flavor structure in the scalar sector means that quarks and squarks can be rotated parallel already at the GUT scale and hence only $\delta_{CKM}$ survives. This is not true in the presence of new flavor structures, where additional Yukawa phases cannot be rephased away from quark–squark couplings (20). Furthermore, we also assume unification of gaugino masses at $M_{GUT}$ and the universal gaugino mass can always be taken as real.

The soft breaking terms structure in Equation 3 includes, as the simplest example, the CMSSM where all scalar masses and $A$–terms are universal and the
number of parameters is reduced to 6 real parameters once we require radiative symmetry breaking, \((m_{1/2}, m_0^2, A_0, \tan \beta, \phi_\mu, \phi_A)\) (19, 21, 22). More general soft–breaking terms in the absence of new flavor structures can arise in GUT models (23). For instance, in a \(SU(5)\) model, we expect common masses for the particles in the 5 and in the 10 multiplets, and, in general, different masses for the two Higgses. The new parameters in the soft–breaking sector would then be \((m_{1/2}, m_5^2, m_{10}^2, m_{H_1}^2, m_{H_2}^2, A_5, A_{10}, \tan \beta, \phi_\mu, \phi_{A_5}, \phi_{A_{10}})\) (24). We take this structure as a representative example of Equation 3, since it already shares all the relevant features. In any case, although the number of parameters is significantly increased with respect to the CMSSM, it can still be managed and a full RGE evolution and analysis of the low–energy spectrum is possible.

In this framework, we consider SUSY effects on flavor changing \(CP\) violation and, in particular, the \(CP\) asymmetry in the \(b \to s\gamma\) decay, \(\varepsilon_K\) and \(B^0\) \(CP\) asymmetries. However, we include also two \(CP\) conserving observables that are relevant in the fit of the unitarity triangle, namely \(\Delta M_{B_d}\) and \(\Delta M_{B_s}\). All these processes receive two qualitatively different Supersymmetric contributions.

As shown in the previous section, Supersymmetry introduces new \(CP\) violation phases that can strongly modify these observables through their effects in SUSY loops. On the other hand, even with vanishing SUSY phases, the presence of the CKM phase in loops containing SUSY particles induces new contributions that modify the SM predictions for these observables.

Concerning the first possibility, we consider the following extreme situation: we analyze the effects of both \(\phi_\mu\) and a flavor independent \(\phi_A\) in flavor changing \(CP\) violation experiments, ignoring completely (as a first step) EDM bounds. The result looks rather surprising at first sight: in the absence of the CKM phase,
a general MSSM with all possible phases in the soft-breaking terms, but no new flavor structure beyond the usual Yukawa matrices, can never give a sizeable contribution to $\varepsilon_K$, $\varepsilon'/\varepsilon$ or hadronic $B^0$ CP asymmetries (25). It is possible to understand the main reasons for this behavior (see (22,25) for a complete discussion). In a flavor blind MSSM, all scalar masses are flavor universal at $M_{GUT}$. After RGE evolution any off-diagonal entry in these matrices is necessarily proportional to a product of at least two Yukawa matrix elements, which finally can be translated into a quark mass squared and two CKM matrix elements, plus a loop factor. Thus, these $LL$ or $RR$ sfermion matrices remain diagonal and real in very good approximation. A similar situation takes place in the trilinear matrices: at $M_W$, they continue to remain proportional to the corresponding Yukawa matrix up to small RGE corrections of the same kind, so they are nearly diagonalized in the basis of diagonal quark Yukawas. In summary, the situation in the sfermion mass matrices is exactly analogous to the Constrained MSSM, and even though the $LR$ matrices are complex, flavor off-diagonal elements are negligible. Hence, gluino and neutralino contributions are too small once we take into account the lower bounds on squark and gaugino masses. The remaining possibility is chargino loops where the necessary flavor change is provided by the usual CKM mixing matrix. In this case, we have both chirality conserving and chirality changing transitions. It is easy to prove (22, 26) that in the limit of flavor diagonal sfermion masses and a real CKM matrix, the chirality conserving chargino contributions are exactly real, irrespective of the phases in the squark and chargino mixings. Therefore, only chirality changing chargino loops could give rise to a Supersymmetric contribution proportional to the new SUSY phases. Still, in the case of kaon physics, these contributions are suppressed by the square
of the $s$ quark Yukawa coupling and then completely negligible with respect to the experimentally measured value. In the $B$ system we could argue that the $b$ Yukawa coupling can be large (26). However, these transitions are closely related to the decay $b \to s \gamma$; constraints from this decay render the chargino chirality changing transitions negligible (22). In other words, the effects of SUSY phases in a flavor blind MSSM are restricted in practice to $LR$ transitions, such as the EDM or the $CP$ asymmetry in the $b \to s \gamma$ decay, and the effects in observables with dominant chirality conserving contributions are negligible even with maximal SUSY phases.

Accordingly, the most interesting $CP$ violation observable in these conditions is probably the $CP$ asymmetry in the $b \to s \gamma$ decay. However, we must take into account that the branching ratio itself is a very strong constraint in any SUSY model (27). The $CP$ asymmetry is defined as,

$$A_{CP}^{b \to s \gamma} = \frac{BR(\bar{B} \to X_s \gamma) - BR(B \to X_s \gamma)}{BR(\bar{B} \to X_s \gamma) + BR(B \to X_s \gamma)} \approx \frac{1}{|C_7|^2} \left( 0.012 \text{Im}\{C_2 C_7^*\} - 0.093 \text{Im}\{C_8 C_7^*\} \right) + 0.001 \text{Im}\{C_2 C_8^*\},$$

where the different $C_i$ are the Wilson coefficients of the current–current, $Q_2 = (\bar{s}_L \gamma^{\mu} c_{L_a}) (\bar{c}_L \gamma^{\mu} b_{L_a})$, magnetic, $Q_7 = \frac{e m_b}{16 \pi^2} \bar{s}_L \sigma^{\mu \nu} b_{R_a} F_{\mu \nu}$, and chromomagnetic, $Q_8 = \frac{g_s m_b}{16 \pi^2} \bar{s}_L T^a_{\alpha \beta} \sigma^{\mu \nu} b_{R\beta} G_{\alpha \mu \nu}$, dipole operators, evaluated at the scale $\mu = m_b$ (28). This asymmetry is predicted to be below 1% in the SM (28, 29).

On the other hand, we have seen that the new SUSY phases can modify the $b \to s \gamma$ transition significantly. In fact, several studies showed that the MSSM in the presence of large $\phi_\mu$ and $\phi_A$ can enhance the $CP$ asymmetry up to 15% (24, 30–33) which could be easily accessible at $B$ factories. In any case, it is im-
important to remember that this scenario is viable only if some mechanism reduces the SUSY contributions to the EDM. In the case of the CMSSM or a flavor blind MSSM with possible EDM cancellations this analysis was repeated in (24,32,33) and the asymmetry can reach at most a few per cent. Figure 1 shows that without EDM constraints (open grey circles) the asymmetry can be above 5% at any value of the branching ratio and can reach even 13% for low branching ratios. In Figure 1 the points of the parameter space that fulfill EDM constraints are represented by black dots. The effect on the $CP$ asymmetry can be sizeable at low values of the branching ratio (24), but for larger values of the branching ratio the asymmetry is again around 1%. In the plot, the points of the parameter space fulfilling EDM constraints are represented by black dots.

Still, even with $\phi_{A_i} = \phi_\mu = 0$, SUSY loops can modify the amplitudes proportional to the CKM phase. Indeed, in a flavor blind MSSM, gluino and neutralino exchange contributions to FCNC are subleading with respect to chargino ($\chi^{\pm}$) and charged Higgs ($H^{\pm}$) exchanges. This simply reflects the absence of flavor mixing at tree level in gluino and neutralino contributions, whereas CKM mixing is present in the chargino and charged Higgs contributions. Hence, when dealing with $CP$ violating FCNC processes in these models, we can confine the analysis to $\chi^{\pm}$ and $H^{\pm}$ loops. This scenario was analyzed (34–36) soon after the heavy top quark discovery opening the possibility of relatively light stops, and more recently in Reference (37). Specifically, these works analyze the effects of a Minimal Flavor Violation MSSM at the electroweak scale. That is, they consider an MSSM with the CKM matrix as the only source of flavor mixing even at the scale $M_W$. However, although they apply all the relevant low energy constraints, they do not consider the RGE effects in the evolution of the soft–breaking terms and
hence they take sparticle masses as completely unrelated.

Our point of view here is more restricted and we follow the analysis of Reference (24), where the flavor blind conditions are specified at a large scale $M_{\text{GUT}}$ and the standard MSSM RGEs (18,38) are used to evolve the initial conditions down to the electroweak scale. We consider two representative examples of flavor blind MSSM, the CMSSM as the simplest model and the $SU(5)$ inspired model defined above. Hence, we are mainly interested in the following part of the low energy spectrum: $\chi^+, H^+$ and $\tilde{t}$. Their masses are evolved to $M_W$ and then, all the relevant experimental constraints are imposed:

- Absence of charge and color breaking minima and directions unbounded from below (39).
- Lower bounds on masses from direct searches (40), in particular $m_{\chi^+_i} > 90$ GeV, $m_{\tilde{t}_i} > 90$ GeV, $m_{\chi^0} > 33$ GeV and $m_{\tilde{\nu}} > 33$ GeV.
- Branching ratio of the $b \rightarrow s \gamma$ decay (41).
- Neutralino as the Lightest Supersymmetric Particle.

In this way, the complete supersymmetric spectrum at the electroweak scale is obtained in terms of 6 or 11 parameters in the CMSSM or $SU(5)$ inspired model respectively. Within a MSSM scenario, this kind of analysis was first made in the work of Bertolini et al. (19) and has been updated several times since then (42). We follow Bartl et al. (24), who developed a specialized study of the spectrum relevant for FCNC and $CP$ violation experiments. Indeed, the most interesting point of this work is the strong correlation among different SUSY masses that have a strong impact on low energy FCNC and $CP$ violation studies. Figure 2 shows scatter plots of the mass of the lightest chargino versus the lightest
stop mass. In these plots we vary the scalar and gaugino masses at $M_{GUT}$ as $100 \text{ GeV} < m_i < 1000 \text{ GeV}$, the trilinear terms as $0 < |A_i| < m_H^2 + m_{\tilde{q}_L}^2 + m_{\tilde{q}_R}^2$ with arbitrary phases and $2 < \tan \beta < 50$. It is interesting to notice in this plot the very strong correlation among the chargino and stop masses. In fact, this correlation can be easily understood with the help of the one–loop RGE (18). Neglecting for the moment the so–called D-terms and the small radiatively generated intergenerational squark mixing, we get for the stop masses:

$$m_{\tilde{t}_{1,2}}^2 = \frac{1}{2} \left( M_{Q_3}^2 + M_{U_3}^2 + 2m_t^2 \mp \sqrt{(M_{Q_3}^2 - M_{U_3}^2)^2 + 4m_t^2(A_t - \mu \cot \beta)^2} \right), \quad (5)$$

in terms of the soft parameters at the electroweak scale. Thanks to the proximity of the top quark mass to its quasi–fixed point and the relative smallness of $\mu \cot \beta$ for $\tan \beta \geq 2.5$, we can express Equation 5 as a function of the initial parameters at $M_{GUT}$ with only a small variation of the coefficients with $\tan \beta$. In the CMSSM case we find:

$$m_{\tilde{t}_{1,2}}^2 = 0.43M_0^2 + 4.55M_{1/2}^2 + m_t^2 + 0.19 \Re(M_{1/2}^* A_0)$$

$$\pm \frac{1}{2} \sqrt{2.25M_{1/2}^4 + 1.13 M_0^2 M_{1/2}^2 + 20.2 m_t^2 M_{1/2}^2} \quad (6)$$

Moreover, in the CMSSM $|\mu| \gtrsim \sqrt{3} m_{1/2}$ is always larger than $M_2 \simeq 0.81 m_{1/2}$ and hence, the lightest chargino is predominantly gaugino. Then we can replace the initial gaugino mass in terms of the lightest chargino mass and finally get,

$$m_{\tilde{t}_{1,2}}^2 = 0.43M_0^2 + 6.93m_{\chi_1^+}^2 + m_t^2 + 0.23 \Re(m_{\chi_1^+} A_0)$$

$$\pm \frac{1}{2} \sqrt{5.23m_{\chi_1^+}^4 + 1.72 M_0^2 m_{\chi_1^+}^2 + 30.8 m_t^2 m_{\chi_1^+}^2} \quad (7)$$

From this equation, we obtain for $100 \text{ GeV} < m_0 < 1 \text{ TeV}$ and with $m_\chi = 100 \text{ GeV}$ a maximal allowed range for the lightest stop mass of $230 \text{ GeV} \lesssim m_{\tilde{t}_1} \lesssim 660 \text{ GeV}$. As Figure 2 shows, this correlation is maintained for larger chargino masses. In
the case of $SU(5)$, the main difference is the fact that the Higgs masses are not tied to the other scalar masses and now may be quite different. This has important effects on the radiative symmetry breaking and in fact, lower values of $\mu$ are possible such that the lightest chargino can have a predominant higgsino component. In the rare scenarios where $|\mu| \lesssim M_2$, the stop masses are somewhat lower than for the CMSSM case. Anyhow, as we can see in Figure 2, a strong correlation is still maintained. We must emphasize that, due to gluino dominance in the soft–term evolution, this kind of correlation is general in any RGE evolved MSSM from some GUT initial conditions, assuming that gaugino masses unify as well. In summary, this implies that the “light stop and chargino” scenario in any GUT evolved MSSM must be shifted to stop masses on the range of 250 GeV and chargino masses of 100 GeV. A very similar correlation can be obtained for other squark masses, roughly,

$$m_{\tilde{q}} \simeq 9.3 \times m_{\chi}^2 + m_0^2$$

Finally, we discuss the charged Higgs–boson mass. In Figure 3, we show a scatter plot of the mass of the charged Higgs boson versus the value of tan $\beta$ for the CMSSM and the SU(5) examples. The main features here are the fact that, for low tan $\beta$, the masses of the charged Higgs–boson are above 400 GeV, and that specially in the CMSSM case, most of these light Higgs–boson masses are eliminated by the $b \rightarrow s\gamma$ constraint. For larger values of tan $\beta$ slightly lighter masses are allowed, but it is still true that we seldom find charged Higgs masses below 300 GeV in any case. The reason for this is again the gluino dominance in the RGE. For instance, at $\tan \beta = 5$ the charged Higgs is $m_{H^+}^2 \simeq 1.23m_0^2 + 3.31m_{1/2}^2$ and at $\tan \beta = 30$, $m_{H^+}^2 \simeq 0.72m_0^2 + 1.98m_{1/2}^2$, from the one–loop RGE (24). Taking into account the relevant features of the MSSM spectrum
discussed above, we can discuss the SUSY contribution in a flavor blind scenario to the different CP violating observables. First, the $b \rightarrow s \gamma$ CP asymmetry has already been discussed in the presence of large supersymmetric phases that survive the EDM constraints through a cancellation mechanism. In that case, the asymmetry could reach a few percent; however, with vanishing SUSY phases we again obtain an asymmetry in the range of the SM value, well below 1% (28,29). A similar situation is found in $\varepsilon'/\varepsilon$ where the SUSY contributions tend to lower the SM prediction (43,44).

Second, the $\Delta F = 2$ observables, i.e $\varepsilon_K$ and $B^0-\bar{B}^0$ mixing which play a fundamental role in the unitarity triangle fit, are also modified by new SUSY contributions. Taking into account the new SUSY contributions, the SM fit of the unitarity triangle is modified and one obtains different restrictions on the $\rho$ and $\eta$ parameters of the CKM matrix. Moreover this fit has to be compatible with the new direct measurements of the $B^0 CP$ asymmetries (3,4). As explained elsewhere (24), given that the SUSY contributions tend to interfere constructively with the SM with a factorized CKM dependence, this implies that for a given SUSY contribution the values of $\eta$ required to saturate $\varepsilon_K$ are now smaller. The value of $|V_{td}V_{tb}^*|$ required to saturate $\Delta M_{B_d}$ is analogously decreased. Hence, it is evident that the addition of SUSY tends to lower the values of $\eta$ and increase the values of $\rho$ in the fit, therefore reducing the actual value of $\beta$ in the direction of the recent experimental results (3,4). However, as shown by Buras & Buras (45), in any minimal flavor violation model at the electroweak scale, a strong correlation exists among the $\Delta F = 2$ contributions to $\varepsilon_K$ and $\Delta M_{B_d}$, and this allows only a small departure of $\sin 2\beta$ from the SM prediction. Still, different values of $\alpha$ and $\gamma$ are, in principle, allowed. Nevertheless, as shown above, the
relative heaviness \((24, 46)\) of the SUSY spectrum implies that the deviation from
the standard model fit in these models tends to be small for these angles. In
summary, a flavor blind MSSM can not generate large deviations from the SM
expectations in the \(B^0\) \(CP\) asymmetries \((3, 4)\).

3.2 \(CP\) Violation in the Presence of New Flavor Structures

Flavor universality of the soft SUSY breaking is a strong assumption and is
known not to be true in many supergravity and string–inspired models \((47–
50)\). In these models, a non trivial flavor structure in the squark mass matrices
or trilinear terms is generically obtained at the supersymmetry breaking scale.
Hence, sizeable flavor off–diagonal entries appear in the squark mass matrices,
and new FCNC and \(CP\) violation effects can be expected. In fact, most of
these flavor off–diagonal entries are severely constrained or even ruled–out by
low energy FCNC and \(CP\) violation observables.

A very convenient parameterization of the SUSY effects in these rare processes
is the so–called mass insertion approximation \((51)\). It is defined in the super CKM
(SCKM) basis at the electroweak scale, where all the couplings of sfermions to
neutral gauginos are flavor diagonal. In this basis, the sfermion mass matrices
are not diagonal. The sfermion propagators, now flavor off–diagonal, can be
expanded as a series in terms of \(\delta = \Delta / \tilde{m}^2\), where \(\Delta\) denotes the off–diagonal
terms in the sfermion mass matrices, with \(\tilde{m}^2\) an average sfermion mass \((52)\).
As a result, FCNC and \(CP\) violation constraints can be expressed as model–
independent upper bounds on these mass insertions at the electroweak scale and
they can readily be compared with the corresponding mass insertions calculated
in a well defined SUSY model.
A complete analysis of this kind was performed (see (52,53) for a more complete discussion) and the constraints from $\Delta S = 2$ processes were later updated (54). In the following, we present the phenomenological constraints from (52,54).

The main constraints on $CP$ violating mass insertions (with non–vanishing phases) come from $\varepsilon_K$, $\varepsilon'/\varepsilon$ and EDMs, although “indirect” constraints from $b \to s\gamma$ and $B-\bar{B}$ mixing are also relevant. These constraints are presented in tables 1 to 5. It is important to emphasize the strong sensitivity of $\varepsilon'/\varepsilon$ to $(\delta_{LR})_{12}$ and of $\varepsilon_K$ to $(\delta_{LL})_{12}$, which implies that it is difficult to saturate both simultaneously with a single mass insertion (55,56).

What message should we draw from the constraints in tables 1–5? First, it is apparent that FCNC and especially $CP$ violating processes represent a significant test for SUSY extensions of the SM. Taking arbitrary sfermion mass matrices completely unrelated to the fermion mass matrices would lead to mass insertions of order unity. Consequently, the first conclusion we draw from the small numbers in tables 1–5 is that there must be some close relation between the flavor structures of the sfermion and fermion sectors. Large portions of the parameter spaces of minimal SUSY models are completely ruled out thanks to the severity of the FCNC and $CP$ constraints.

But then an even more important question emerges after one stares at tables 1–5: Given the strong constraints from FCNC and $CP$ violating processes that we have already observed, can we still hope to see SUSY signals in other rare processes? In particular, restricting this question to $CP$ violation, can we still hope to find a significant disagreement with the SM expectations when we measure $CP$ violation in various $B$ decay channels? Fortunately for us, the answer to this last question is yes. For example, it has been shown (57) that considering
the $CP$ asymmetry in several $B$ decay channels, which in the SM would give just the same answer (the angle $\beta$ of the unitarity triangle), it is possible to obtain different values when SUSY effects are switched on. SUSY contributions to some of the decay amplitudes can be as high as 70% with respect to the SM contribution, whereas other decay channels are not affected at all by the SUSY presence. Hence, assuming large $CP$ violating phases in SUSY, one could find discrepancies with the SM expectations that are larger than any reasonable theoretical hadronic uncertainty in the SM computation. We refer the interested reader to Reference (57) for a detailed discussion.

It is worth emphasizing that the above example shows that there is still room for sizeable SUSY signals in $CP$ violating processes, but this represents some kind of “maximal hope” of what we can expect from SUSY. In other words, one takes the maximally allowed values of relevant $\delta$’s to maximize the possible SUSY deviations from SM on $CP$ observables. A different question is what we can “typically” expect in a SUSY model. As we stressed in the introduction, no “typical” SUSY model exists; what we call low–energy SUSY represents a vast class of models. Yet it makes sense to try to identify some features of minimal SUSY models where no drastic departures from flavor universality are taken and to consider in this more restricted context what we can expect.

In the following, we analyze a “realistic” non–universal MSSM, and compute the “reasonable” expectations for the different mass insertions in this context. In first place, we define our generic MSSM through a set of four general conditions:

1. **Minimal particle content**: we consider the MSSM, with no additional particles from $M_W$ to $M_{\text{GUT}}$.

2. **Arbitrary Soft–Breaking terms $O(m_{3/2})$**: The supersymmetry soft–
breaking terms as given at the scale $M_{GUT}$ have a completely general flavor structure, but all of them are of the order of a single scale, $m_{3/2}$.

3. **Trilinear couplings originate from Yukawa couplings:** Although trilinear couplings are a completely new flavor structure they are related to the Yukawas in the usual way: $Y_{ij}^A = A_{ij} \cdot Y_{ij}$, with all $A_{ij} \simeq \mathcal{O}(m_{3/2})$.

4. **Gauge coupling and gaugino unification at $M_{GUT}$ and RGE evolution** of the different parameters from that scale.

In this framework, any particular MSSM is completely defined, once we specify the soft–breaking terms at $M_{GUT}$. We specify these soft–breaking terms in the basis in which all the squark mass matrices, $M_Q^2, M_U^2, M_D^2,$ are diagonal. In this basis, the Yukawa matrices are, $v_1 Y_d = K^{D_L\dagger} \cdot M_d \cdot K^{D_R}$ and $v_2 Y_u = K^{D_L\dagger} \cdot K^{U_R}$, with $M_d$ and $M_u$ diagonal quark mass matrices, $K$ the CKM mixing matrix and $K^{D_L}, K^{U_R}, K^{D_R}$ unknown, completely general, $3 \times 3$ unitary matrices.

Although our analysis is completely general within this scenario (58), we prefer to discuss a concrete example based on type–I (50) string theory, (see (59) for definition).\footnote{The case where large flavor–independent soft phases may give a dominant contribution to $CP$ violation has been discussed elsewhere (56)} In this particular example, gaugino masses, right–handed squarks, and trilinear terms are non–universal. Gaugino masses are,

$$M_3 = M_1 = \sqrt{3} m_{3/2} \sin \theta e^{-i \alpha_S},$$
$$M_2 = \sqrt{3} m_{3/2} \cos \theta \Theta_1 e^{-i \alpha_1},$$

whereas the $A$–terms are obtained as

$$A_1 = -\sqrt{3} m_{3/2} (\sin \theta e^{-i \alpha_S} + \cos \theta [\Theta_1 e^{-i \alpha_1} - \Theta_3 e^{-i \alpha_3}])$$
\[
A_2 = -\sqrt{3} m_{3/2} \left[ \sin \theta e^{-i\alpha_S} + \cos \theta (\Theta_1 e^{-i\alpha_1} - \Theta_2 e^{-i\alpha_2}) \right]
\]
\[
A_3 = -\sqrt{3} m_{3/2} \sin \theta e^{-i\alpha_S} = -M_3,
\]

for the trilinear terms associated to the first, second and third generation right-handed squarks respectively. Here \( m_{3/2} \) is the gravitino mass, \( \alpha_S \) and \( \alpha_i \) are the \( CP \) phases of the \( F \) terms of the dilaton field \( S \) and the three moduli fields \( T_i \), and \( \theta \) and \( \Theta_i \) are goldstino angles with the constraint, \( \sum \Theta_i^2 = 1 \). Hence, the trilinear SUSY breaking matrix, \((Y^A)_{ij} = (Y)_{ij}(A)_{ij}\), itself is obtained as,

\[
Y^A = \begin{pmatrix} Y_{ij} \end{pmatrix} \cdot \begin{pmatrix} A_{C_3^0} & 0 & 0 \\ 0 & A_{C_2^0} & 0 \\ 0 & 0 & A_{C_1^0} \end{pmatrix},
\]

in matrix notation (60).

In addition, universal soft scalar masses for quark doublets and the Higgs fields are obtained,

\[
m_{L_i}^2 = m_{3/2}^2 \left[ 1 - \frac{3}{2} \cos^2 \theta (1 - \Theta_1^2) \right].
\]

And finally, the soft scalar masses for quark singlets are non-universal,

\[
m_{R_i}^2 = m_{3/2}^2 (1 - 3 \cos^2 \theta T_i),
\]

with \( T_i = (\Theta_3^2, \Theta_2^2, \Theta_1^2) \).

To complete the definition of the model, we need to specify as well the Yukawa textures. The only available experimental information is the CKM mixing matrix and the quark masses. We choose our Yukawa texture following two simple assumptions: (a) the CKM mixing matrix originates from the down-type Yukawa couplings (as done in Reference (61)) and (b) our Yukawa matrices are hermitian (62). With these two assumptions we get \( K^{D_L} = K \) and \( K^{U_L} = 1 \). However, it is important to emphasize that given that now \( K^{D_L} \) and \( K^{U_L} \) measure the flavor
misalignment between quarks and squarks, and that we already use the rephasing invariance of the quarks to make $K_{CKM}$ real, we can expect new observable (unremovable) phases in the quark–squark mixings, and in particular in the first two generation sector. That is,

$$K^{DL} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda \, e^{i\alpha} & A \, \rho' \, \lambda^3 \, e^{i\beta} \\ -\lambda \, e^{-i\alpha} & 1 - \lambda^2/2 & A \, \lambda^2 \, e^{i\gamma} \\ A \, \lambda^3 \, (e^{-i(\alpha+\gamma)} - \rho' \, e^{-i\beta}) & -A \, \lambda^2 \, e^{-i\gamma} & 1 \end{pmatrix}$$

(14)

to $O(\lambda^4)$; $A$ and $\rho' = |\rho + i\eta|$ are the usual parameters in the Wolfenstein parameterization, both $O(1)$. We must emphasize here that the observable phase in the CKM mixing matrix corresponds to the combination $\delta_{CKM} = \beta - \alpha - \gamma$; hence it is transparent that we can have a vanishing $\delta_{CKM}$ while being left with large observable phases in the SUSY sector (20). Hence, the Yukawa matrices are, $v_1 Y_d = K^{DL \dagger} \cdot M_d \cdot K^{DL}$ and $v_2 Y_u = M_u$. It is important to remember that this is the simplest structure consistent with all phenomenological constraints.

Now, the next step is to use the MSSM RGEs (18,19) to evolve these matrices down to the electroweak scale. The main RGE effects from $M_{GUT}$ to $M_W$ are those associated with the gluino mass and the large third generation Yukawa couplings. Regarding squark mass matrices, it is well–known that diagonal elements receive important RGE contributions proportional to gluino mass that dilute the mass eigenstate non–degeneracy, $m_{\tilde{D}_A}^2 (M_W) \simeq c_A^i \cdot m_{\tilde{g}}^2 + m_{\tilde{D}_A}^2 (18,19,22)$, with $c_L^{1,2} \simeq (6.5, 6.5)$, $c_L^3 \simeq (5.5, 4.6)$, $c_R^{1,2} \simeq (6.1, 6.1)$ and $c_R^3 \simeq (6.1, 4.3)$ for $(\tan\beta = 2.5, \tan\beta = 40)$ (24). In the SCKM basis, the off–diagonal elements in the sfermion mass matrices are given by $(K^A \cdot M_{\tilde{D}_A}^2 \cdot K^{A \dagger})_{ij} \neq \delta_{ij}$ up to smaller RGE corrections. Similarly, gaugino effects in the trilinear RGE are always proportional to the Yukawa matrices, not to the trilinear matrices themselves.
and so they are always diagonal to extremely good approximation in the SCKM basis. Once more, the off–diagonal elements will be approximately given by \( (K^{QL} \cdot Y^A_Q \cdot K^{QR}^\dagger)_{i \neq j} \).

The LR and RR mass insertions are defined as \( (\delta^Q_{LR})_{ij} = (M^2_{Q_{LR}})_{ij}/m^2_{\tilde{q}} \) and \( (\delta^Q_{RR})_{ij} = (M^2_{Q_{RR}})_{ij}/m^2_{\tilde{q}} \) respectively. Hence, in our example defined in Equations 9–14, we have LR and RR off–diagonal mass insertions, which can be estimated as,

\[
(\delta^d_{LR})_{ij} = \frac{1}{m^2_{\tilde{q}}} m_i \left[ K_{i2}^{DL} K_{j2}^{DL*} (A_2^* - A_1^*) + K_{i3}^{DL} K_{j3}^{DL*} (A_3^* - A_1^*) \right] \tag{15}
\]

and

\[
(\delta^d_{R})_{ij} = \frac{1}{m^2_{\tilde{q}}} \left[ K_{i2}^{DL} K_{j2}^{DL*} (m^2_{R_2} - m^2_{R_1}) + K_{i3}^{DL} K_{j3}^{DL*} (m^2_{R_3} - m^2_{R_1}) \right] \tag{16}
\]

Equation 15 reveals an important feature of the LR mass insertions. Because of the trilinear terms structure in generic models of soft–breaking, the LR sfermion matrices are always suppressed by \( m_{qi}/m_{\tilde{q}} \), with \( m_{qi} \) the mass of one of the quarks involved in the coupling and \( m_{\tilde{q}} \) the average squark mass \( (60) \). In any case, this suppression is necessary to avoid charge and color breaking and directions unbounded from below \( (39) \). We can easily estimate the different mass insertions with these formulas. First we must take into account that, owing to the gluino dominance in the squark eigenstates at \( M_W, m^2_{\tilde{q}}(M_W) \approx 6 \ m^2_{\tilde{g}}(M_{GUT}) \). In the kaon system, we can neglect \( m_d \); replacing the values of masses and mixings in Equations 9–14 we obtain,

\[
(\delta^d_{LR})_{12} \approx \frac{m_s}{m_{\tilde{q}}} \frac{(A_2 - A_1)}{m_{\tilde{q}}} K_{12}^{DL} K_{22}^{DL*} \approx 2.8 \times 10^{-5} \cdot (\Theta_2 e^{-i\alpha_2} - \Theta_3 e^{-i\alpha_3}) \cdot \left( \frac{100 \text{ GeV}}{m_{3/2}} \right) \tag{17}
\]

where we have used \( \theta \approx 0.7 \) as in Reference \( (59) \). Comparing this value with the bounds in Table 2, we see that it could indeed give a very sizeable contribution.
to $\varepsilon'/\varepsilon$ \(^{(59,61,63)}\). The phases $\alpha_2$ and $\alpha_1$ are actually unconstrained by EDM experiments as emphasized in (59). This important result means that even if the relative quark–squark flavor misalignment is absent and the only flavor mixing is provided by the usual CKM matrix, i.e. $K^{DL} = K_{CKM}$, the presence of non–universal flavor–diagonal trilinear terms is enough to generate large FCNC effects in the kaon system.

Similarly, in the neutral $B$ system, $(\delta_{LR}^d)_{13}$ contributes to the $B_d - \bar{B}_d$ mixing parameter, $\Delta M_{B_d}$. However, in our minimal scenario, $K^{DL} \approx K$, we obtain,

$$
(\delta_{LR}^d)_{13} \simeq \frac{m_b}{m_{\tilde{q}}} \frac{(A_3 - A_1)}{m_{\tilde{q}}} K^{DL}_{13} K^{DL*}_{33}
$$

$$
\simeq 2.5 \times 10^{-5} \cdot (\Theta_1 e^{-i\alpha_1} - \Theta_3 e^{-i\alpha_3}) \cdot \left(\frac{100 \text{ GeV}}{m_{3/2}}\right), \quad (18)
$$

clearly too small to generate sizeable $\tilde{b} - \tilde{d}$ transitions, as the bounds in Table 5 show. Notice that larger effects are still possible in a more “exotic” scenario with a large mixing in $K^{DL}_{13}$. For instance, with a maximal value, $|K^{DL}_{13} K^{DL*}_{33}| = 1/2$, we would get $(\delta_{LR}^d)_{13} \simeq 2 \times 10^{-3} \cdot (100 \text{ GeV}/m_{3/2})$. Even in this limiting situation, this result is roughly one order of magnitude too small to saturate $\Delta M_{B_d}$, though it could still be observed through the $CP$ asymmetries. Hence in the $B$ system we reach a very different result: it is not enough to have non–universal trilinear terms, large flavor misalignment among quarks and squarks is also required.

A similar analysis can be maid with the chirality conserving mass insertions. From Equation 16, in the kaon system, we get,

$$
(\delta_{R}^d)_{12} \simeq \frac{\cos^2 \theta (\Theta_1^2 - \Theta_2^2)}{6 \sin^2 \theta} K^{DL}_{12} K^{DL*}_{22} + \frac{\cos^2 \theta (\Theta_1^2 - \Theta_3^2)}{6 \sin^2 \theta} K^{DL}_{13} K^{DL*}_{23}
$$

$$
\simeq \frac{\cos^2 \theta (\Theta_1^2 - \Theta_2^2)}{6 \sin^2 \theta} \lambda e^{i\alpha} \quad \text{ (19)}
$$

This value has to be compared with the mass insertion bounds required to saturate $\varepsilon_K$ \(^{(51)}\), which in this case are, $(\delta_{R}^d)_{12}^{\text{bound}} \leq 0.0032$. Using $\theta \simeq 0.7$, we
get,

\[(\delta^d_R)_{12} \simeq 0.035(\Theta_1^2 - \Theta_2^2) \sin \alpha \lesssim 0.0032. \tag{20}\]

Hence, it is clear that we can easily saturate \(\varepsilon_K\) without any special fine-tuning.

Indeed, this constraint which is one of the main sources of the so-called Supersymmetric flavor problem, in this generic MSSM amounts to the requirement that \((\Theta_1^2 - \Theta_2^2) \sin \alpha \lesssim 0.1\) with all the different factors in this expression \(\Theta_1^2, \Theta_2^2, \sin \alpha \leq 1\) \(^{(20)}\).

Now we turn to the \(CP\) asymmetries in the \(B\) system. Once more, with Equation 15 we have,

\[
(\delta^d_R)_{13} \simeq \frac{\cos^2 \theta (\Theta_3^2 - \Theta_1^2)}{6 \sin^2 \theta} K_{12}^{D_L} K_{32}^{D_L*} + \frac{\cos^2 \theta (\Theta_2^2 - \Theta_1^2)}{6 \sin^2 \theta} K_{13}^{D_L} K_{33}^{D_L*}
\]

\[
\simeq A \lambda^3 \frac{\cos^2 \theta}{6 \sin^2 \theta} \left[ - (\Theta_2^2 - \Theta_1^2) e^{i(\alpha + \gamma)} + (\Theta_3^2 - \Theta_1^2) (e^{-i(\alpha + \gamma)} - \rho e^{-i\beta}) \right] \lesssim 10^{-3}, \tag{21}\]

to be compared with the mass insertion bound \((\delta^d_R)^{\text{bound}}_{12} \leq 0.098\) required to not over-saturate the \(B^0\) mass difference.

We conclude that large effects are expected in the kaon system in the presence of non-universal squark masses even with a “natural” CKM-like mixing both for chirality changing and chirality conserving transitions. The \(B\) system is much less sensitive to supersymmetric contributions, so observable effects are expected only with approximately maximal \(\tilde{b} - \tilde{d}\) mixings.

Recently, the arrival of the first measurements of \(B^0\) \(CP\) asymmetries from the \(B\) factories has caused a great excitement in the high energy physics community.

\[
a_{J/\psi} = \begin{cases} 
0.34 \pm 0.20 \pm 0.05 & \text{(Babar (3))} \\
0.58_{-0.34}^{+0.32+0.09} \pm 0.10 & \text{(Belle (4))} \\
0.79_{-0.44}^{+0.41} & \text{(CDF (5))}
\end{cases} \tag{22}\]
The errors are still too large to draw any firm conclusion. Still, these measurements, and specially the BaBar value which is the most precise one, leave room for an asymmetry considerably smaller than the standard model expectations corresponding to $0.59 \leq a_{J/\psi}^{SM} = \sin (2\beta) \leq 0.82$. This possible discrepancy, if confirmed, would be a first sign of the presence of new physics in $CP$ violation experiments. Several papers have discussed the possible implications of a small $CP$ asymmetry (58, 64) and pointed out two possibilities. A small asymmetry can be due to a large new physics contribution in the $B$ system and/or to a new contribution in the $K$ system modifying the usual determination of the unitarity triangle. Taking into account the results above, in a non-universal MSSM it is realistic to reproduce the $CP$ violation in the kaon system through SUSY effects, while being left with a small $a_{J/\psi}$ in the $B$ system. Indeed the role of the CKM phase could be confined to the SM fit of the charmless semileptonic $B$ decays and $B_d^0 - \bar{B_d}^0$ mixing, while predominantly attributing to SUSY the $K$ $CP$ violation ($\varepsilon_K$ and $\varepsilon'/\varepsilon$). In this case the CKM phase can be quite small, leading to a lower $a_{J/\psi} CP$ asymmetry (20).

4 $CP$ VIOLATION AND OTHER SM EXTENSIONS

So far, we have reviewed $CP$ violation in SUSY as a particularly appealing extension of the SM. In this last section we briefly outline other SM extensions (see (6,65) for a more complete discussion).

We classify other extensions of the SM with respect to its low energy spectrum. In this way, we distinguish between models with an extended fermion sector, an extended scalar sector and an extended gauge sector.

The fermion sector can be extended either by a complete fourth generation
or simply by the addition of extra nonsequential quarks (66). A chiral fourth generation model faces strong restrictions from neutrino masses and electroweak precision data (40). On the other hand, extensions with additional down quarks in a vector–like representation of the SM are free from these problems and are especially interesting from the $CP$ phenomenology point of view. These models naturally arise, for instance, as the low energy limit of an $E_6$ grand unified theory. At a more phenomenological level, models with isosinglet quarks provide the simplest self–consistent framework to study deviations of $3 \times 3$ unitarity of the CKM matrix, as well as FCNC at tree level. The extra vector–like quarks mix with the three ordinary down quarks giving rise to a $4 \times 4$ mixing matrix, out of which the $3 \times 4$ upper submatrix corresponds to the CKM matrix in charged currents. In this extended matrix, new observable phases arise and, through unitarity, produce complex tree level FCNC couplings in the $Z$ and Higgs vertices. The fact that the experimental bounds on CKM mixings allow a larger mixing with the third generation quarks implies that despite the tight constraints from other FCNC processes (67), these tree level FCNCs are able to modify significantly the SM prediction for $B^0 CP$ asymmetries (68). Indeed, these models are even able to reproduce low asymmetries in the lower range of the recent BaBar result (69).

Many SM extensions, include additional scalar multiplets (70). These models have additional Yukawa couplings, as well as scalar self–interactions that introduce many new sources of $CP$ violation. However, a generic model suffers from severe phenomenological problems that are solved with the requirement of extra symmetries (71–73). For example, the Higgs sector of the MSSM, discussed in previous sections is an example of a two–Higgs doublet model with natural fla-
NEW PHYSICS IN CP VIOLATION EXPERIMENTS

vor conservation (71), this means that each Higgs doublet couples to a different fermion sector. In this case, the only new ingredient is the presence of a physical charged Higgs with CKM couplings. In some cases, it is also possible that the neutral scalars are mixtures of $CP$ even and $CP$ odd fields (74). This mixing shows up mainly in EDM’s, $b \to s\gamma$, and top quark decays.

An enlargement of the gauge group to include, for instance a left–right symmetric theory is also a very attractive extension of the SM (75). Concerning $CP$ violation, it is especially interesting that these models can accommodate spontaneous $CP$ violation (76). The relative phases among neutral Higgs vacuum expectation values are the only source of $CP$ violation here. Actually, a single phase enters the charged fermion mass matrices giving rise to six physical phases in these matrices. This is the only source for $CP$ violation at low energy, in a very predictive scheme. In fact, the rephasing invariant phase of the (left–handed) CKM matrix is small (77). Hence, an extra contribution from right–handed bosons is required to saturate the experimental bound in $\varepsilon_K$. This, in turn, translates into an upper bound on the mass of the right–handed boson. In a similar way, there can be sizeable new contributions in $B\bar{B}$ mixing from the phases in the right–handed mixing matrix and consequently, the $CP$ asymmetries in $B$ decays may be significantly different from the SM expectations (78).

5 CONCLUSIONS AND OUTLOOK

Here we summarize the main points in our review:

- There exist strong theoretical and “observational” reasons to go beyond the SM.
- The gauge hierarchy and coupling unification problems favor the presence of
low–energy SUSY (either in its minimal version, CMSSM, or more naturally, in some less constrained realization).

- Flavor and $CP$ problems constrain low–energy SUSY, but, at the same time, provide new tools to search for SUSY indirectly.

- In general, we expect new $CP$ violating phases in the SUSY sector. However, these new phases are not going to produce sizeable effects as long as the SUSY model we consider does not exhibit a new flavor structure in addition to the SM Yukawa matrices.

- In the presence of a new flavor structure in SUSY, large contributions to $CP$ violating observables are indeed possible.

- $CP$ violation is also very sensitive to the presence of other low energy extensions of the SM.

Maybe a reader who has followed us up to this point would like to have a final assessment about the chances of having some signals of the presence of low energy SUSY in FCNC and $CP$ violating processes. It should be clear from our discussion that the answer heavily depends on the presence or absence of a link between the mechanism which is responsible for the flavor structure of the theory and the mechanism originating the breaking and transmission of SUSY.

If no link is present (flavor blind SUSY), there exists quite a few especial places where we can hope to “see” SUSY in action: the EDMs, the $A^{b\rightarrow s\gamma}_{CP}$, and, as it emerged recently, the anomalous magnetic moment of the muon. On the other hand, in the more general (and, in our view, also more likely) case where, indeed, SUSY breaking is not insensitive to the flavor mechanism, there exist a rich variety of FCNC and $CP$ violation potentialities for SUSY to show up. As we
have seen, $K$ and $B$ physics offer appealing possibilities: $\varepsilon_K$, $\varepsilon'/\varepsilon$, $CP$ violating rare kaon decays, $CP$ asymmetries in $B$ decays, rare $B$ decays. In fact, we think that the relevance of SUSY searches in rare processes is not confined to the usually quoted possibility that indirect searches can arrive “first”, before direct searches (Tevatron and LHC), in signaling the presence of SUSY. Even after the possible direct production and observation of SUSY particles, the importance of FCNC and $CP$ violation in testing SUSY remains of utmost relevance. They are and will be complementary to the Tevatron and LHC establishing low energy supersymmetry as the response to the electroweak breaking puzzle.

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Figure 1: $CP$ asymmetry in % versus branching ratio in the decay $b \to s\gamma$. Black dots respect (or grey open circles violate) electric dipole moment constraints.

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<th>$x$</th>
<th>$\sqrt{\text{Im} (\delta^d_{A})_{12}^2}$</th>
<th>$\sqrt{\text{Im} (\delta^d_{LR})_{12}^2}$</th>
<th>$\sqrt{\text{Im} (\delta^d_{L})<em>{12} (\delta^d</em>{R})_{12}}$</th>
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Table 1: Limits from $\varepsilon_K$ on $\text{Im} (\delta^d_{A})_{12}$, $\text{Im} (\delta^d_{LR})_{12}$, with $A, B = (L, R, LR)$ including next-to-leading-order QCD corrections and lattice $B$ parameters (54), for an average squark mass $m_{\tilde{q}} = 500$ GeV and for different values of $x = m_\tilde{g}^2/m_{\tilde{q}}^2$. 


Figure 2: Chargino mass versus lightest stop mass for the parameter space described in the text in the CMSSM and SU(5) cases. Black dots and open circles represent points satisfying or violating the $b \to s \gamma$ constraint respectively.
Figure 3: Charged Higgs–boson mass as a function of tan β for the parameter space described in the text in the CMSSM and SU(5) cases. Black dots and open circles represent points satisfying or violating the $b \to s\gamma$ constraint respectively.
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Table 2: Limits from $\varepsilon' / \varepsilon < 2.7 \times 10^{-3}$ on $\text{Im}(\delta_{12}^d)$, for an average squark mass $m_{\tilde{q}} = 500$ GeV and different values of $x = m_{\tilde{g}}^2 / m_{\tilde{q}}^2$.

| $x$ | $\text{Im}(\delta_{LR}^d)_{11}$ | $|\text{Im}(\delta_{LR}^u)_{11}|$ | $|\text{Im}(\delta_{LR}^l)_{11}|$ |
|-----|-------------------------------|---------------------------------|-------------------------------|
| 0.3 | $2.4 \times 10^{-6}$          | $4.9 \times 10^{-6}$            | $3.0 \times 10^{-7}$          |
| 1.0 | $3.0 \times 10^{-6}$          | $5.9 \times 10^{-6}$            | $3.7 \times 10^{-7}$          |
| 4.0 | $5.6 \times 10^{-6}$          | $1.1 \times 10^{-5}$            | $7.0 \times 10^{-7}$          |

Table 3: Limits on $\text{Im}(\delta_{LR})_{11}$ from electric dipole moments, for $m_{\tilde{q}} = 500$ GeV and $m_{\tilde{l}} = 100$ GeV.

| $x$ | $|\delta_{23}^d|$ | $|\delta_{23}^L|$ |
|-----|-----------------|-----------------|
| 0.3 | $4.4$           | $1.3 \times 10^{-2}$ |
| 1.0 | $8.2$           | $1.6 \times 10^{-2}$ |
| 4.0 | $26$            | $3.0 \times 10^{-2}$ |

Table 4: Limits on the $|\delta_{23}^d|$ from $b \rightarrow s \gamma$ decay for an average squark mass $m_{\tilde{q}} = 500$ GeV and different values of $x = m_{\tilde{g}}^2 / m_{\tilde{q}}^2$. 
$$\sqrt{\text{Re} \left( \delta_{13}^A \right)_{LL}^2}$$
$$\sqrt{\text{Re} \left( \delta_{13}^A \right)_{LR}^2}$$
$$\sqrt{\text{Re} \left( \delta_{13}^A \right)_{RR} \left( \delta_{13}^B \right)_{RR}}$$

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<td>$2.5 \times 10^{-2}$</td>
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Table 5: Limits on $\text{Re} \left( \delta_{A}^A \right)_{13} \left( \delta_{B}^B \right)_{13}$, with $A, B = (L, R, LR)$ from $B^0 - \bar{B}^0$ mixing, for an average squark mass $m_{\tilde{q}} = 500$ GeV and for different values of $x = m_{\tilde{g}}^2 / m_{\tilde{q}}^2$. 