Testing the principle of equivalence by supernova neutrinos

M. M. Guzzo\textsuperscript{1}, H. Nunokawa\textsuperscript{1} and R. Tomàs\textsuperscript{2}

\textsuperscript{1} Instituto de Física Gleb Wataghin
Universidade Estadual de Campinas, UNICAMP
13083-970 Campinas SP, Brazil

\textsuperscript{2} Institut de Física Corpuscular (IFIC), CSIC - U. de València,
Edifici Instituts d’Investigació, Apartado de Correos 22085
E-46071–València, Spain

Abstract

We study the possible impact of the neutrino oscillation which could be induced by a tiny violation of equivalence principle (VEP) for neutrinos emitted from supernova driven by gravitational collapse. Due to the absence of any significant indication of neutrino oscillation in the SN1987A data, we obtain sever bounds on relevant VEP parameters $\delta \gamma \lesssim O(10^{-31})$ for massless or degenerated neutrinos and $\delta \gamma \lesssim O(10^{-16}) \times \left[ \Delta m^2 / 10^{-5} \text{ eV}^2 \right]$ for massive neutrinos.

I. INTRODUCTION

Principle of equivalence, one of the fundamental bases in Einstein’s general relativity, has been tested for many years with great accuracy. One can state the possible violation of equivalence principle (VEP) in several ways. For massive objects, existence of VEP implies that gravitational mass is not equal to inertial mass. For macroscopic objects, by the torsion balance experiment, Eötvös and his collaborators \textsuperscript{1} obtained the stringent bound $\eta \lesssim 3 \times 10^{-9}$, where $\eta$ is the parameter which measures the difference between gravitational and inertial mass. Forty years later, Dicke \textit{et al.} \textsuperscript{2} improved the bound to $\eta \lesssim 3 \times 10^{-11}$ by removing some systematic uncertainties making use of the influence of Sun’s gravitational field which could produce a torque with 24 hour periodicity in the experiment in the presence of VEP. With similar apparatus, Braginski and Panov \textsuperscript{3} obtained the bound $\eta \lesssim 0.9 \times 10^{-12}$. 
Equivalence principle has been tested also for microscopic objects. Using the neutron free fall refractometry, the bound $\eta < 3 \times 10^{-4}$ is obtained [4] for inertial and gravitational mass difference of neutrons.

For particles like photon and neutrinos, one can not perform such a comparison of inertial and gravitational masses. One can use, instead, another parameter, $\delta \gamma$, in order to test VEP, which measures the difference of the gravitational coupling between the two particles involved, $\delta \gamma_{\beta \alpha} \equiv (G_{\beta} - G_{\alpha})/(G_{\beta} + G_{\alpha})$. A mild bound $|\delta \gamma_{\gamma \nu}| \lesssim 10^{-3}$ for violation of the equivalence principle for photons and neutrinos have been derived [5] by using the small arrival time differences between photons and neutrinos from the supernova SN1987A observed by Kamiokande and IMB detectors [6].

For neutrinos, more sensitive test of VEP can be performed by using neutrino oscillation which can be induced by VEP even if neutrinos are massless [7,8]. Based on this proposal, many works have been performed [9–23]. In Ref. [24] such VEP induced neutrino oscillation was shown to be phenomenologically equivalent to neutrino oscillations induced by a possible violation of Lorentz invariance [25].

Some theoretical insight on the type of gravitational potential that could violate the weak equivalence principle can be found in Ref. [26]. Some discussions on the possibility that violation of equivalence principle comes from string theory are found in Ref. [27]. A discussion on the departure from exact Lorentz invariance in the standard model Lagrangian is developed in Ref. [28].

A possibility to test VEP by the future long-base line neutrino oscillation experiment has been discussed in Ref. [19] whereas some bounds from the current accelerator based neutrino oscillation experiments were obtained in Ref. [21]. Recently such neutrino oscillation mechanisms have been investigated [17,18] in the light of the experimental results from Super-Kamiokande (SK) on the atmospheric neutrino anomaly, obtaining stringent limits for the $\nu_{\mu} \to \nu_{\tau}$ channel.

The possibility of solving the solar neutrino problem (SNP) by these gravitationally induced neutrino oscillations have been investigated in Refs. [8–14]. On the other hand, possibilities to constrain VEP parameters by using astrophysical neutrino sources are discussed in Ref. [15] for solar neutrinos and in Ref. [16] for very high energy neutrinos from AGN. The possible effect of neutrino oscillation induced by the violation of equivalence principle (VEP) as an explanation of the pulsar kick velocity has been considered in Refs. [22,23]. In Ref. [11], it was discussed that supernova (SN) explosion dynamics as well as heavy element nucleosynthesis can be significantly affected by VEP induced neutrino oscillation.

In this paper, extending the discussions in Ref. [11], we consider the possible impact of VEP induced neutrino oscillation for SN neutrinos, in particular for the observation of $\bar{\nu}_{e}$ signal. In this work, we do not consider the possible effect of neutrino oscillation induced by VEP on heavy element nucleosynthesis in SN [29] as the bounds on VEP parameters from this effect is expected to be weaker. We show that one can try to constrain the relevant VEP parameters using the observed anti-electron neutrinos events from supernova SN1987A following the same argument used in Refs. [30,31]. In Sec. II, we discuss the formalism we use in this work. In Sec. III we describe some features of SN neutrinos and discuss how to
use supernova neutrino data to constrain oscillation parameters. In Sec. IV we present our results. In Sec. V we give conclusions.

II. NEUTRINO PROPAGATION IN THE PRESENCE OF VIOLATION OF EQUIVALENCE PRINCIPLE

Here we describe the framework we will use in this work. Let us assume that neutrinos of different species are not universally coupled to gravity, or neutrinos of different flavor propagate with different gravitational red shift in a given gravitational field. In such case, the weak interactions eigenstates $|\nu_W\rangle$ and gravitational eigenstates $|\nu_G\rangle$, the bases with which gravitational part of the energy is diagonalized, can, in general, be different, $|\nu_W\rangle \neq |\nu_G\rangle$.

If neutrinos are massive, the mass eigenstates $|\nu_M\rangle$, can be different from both of these two states, $|\nu_M\rangle \neq |\nu_W\rangle \neq |\nu_G\rangle$. For two neutrino system, $\nu_e - \nu_x \ (x = \mu \ or \ \tau)$, we can describe such a general situation as follows,

$$
\begin{bmatrix}
\nu_e \\
\nu_\mu
\end{bmatrix} = \begin{bmatrix}
\cos \theta_G & \sin \theta_G \\
-\sin \theta_G & \cos \theta_G
\end{bmatrix}
\begin{bmatrix}
\nu_{G1} \\
\nu_{G2}
\end{bmatrix},
$$

(1)

$$
\begin{bmatrix}
\nu_e \\
\nu_\mu
\end{bmatrix} = \begin{bmatrix}
\cos \theta_M & \sin \theta_M \\
-\sin \theta_M & \cos \theta_M
\end{bmatrix}
\begin{bmatrix}
\nu_{M1} \\
\nu_{M2}
\end{bmatrix},
$$

(2)

where $\theta_G$ is the mixing angle due to the presence of VEP which is in general different from the usual flavor mixing $\theta_M$ between weak and mass eigenstates, $\theta_G \neq \theta_M$, implying $|\nu_M\rangle \neq |\nu_G\rangle$.

For simplicity, let us consider, the system of two neutrino flavors. The evolution equation of massive neutrino for one generation without flavor mixing in the gravitational field in the supernova can be described by,

$$
i \frac{d}{dr} \nu = E \left[ 1 - \frac{m^2}{2E^2} - \phi(r) \right] \nu,
$$

(3)

where the gravitational potential $\phi(r)$ describes the red shift of neutrinos and it is given, up to the first order in Newton’s gravitational constant $G$, assuming the spherically symmetric metric and taking the center of the star as origin of the coordinate, as follows,

$$
\phi(r) = -G \left[ \frac{M(r)}{r} + \int_r^\infty \frac{M(r)}{r^2} dr \right] + \phi_{sc},
$$

(4)

where

$$
M(r) \equiv \int_0^r 4\pi r'^2 \rho(r')
$$

is the total mass contained in the volume with radius $r$, and $\phi_{sc}$ is the background potential which can be taken as the one coming from the local Super-cluster [32]. For definiteness, in this work, we take $\phi_{sc} = -3 \times 10^{-5}$, the value estimated in Ref. [32].

It is straightforward to generalize the evolution equation (2) for more than one generation with mixing. Since, as we will discuss in detail in the following sections, we are most
interested in the oscillation effect for anti-electron neutrinos, $\bar{\nu}_e$, from now on we will consider only $\bar{\nu}_e - \bar{\nu}_x$ ($x = \mu$ or $\tau$) system unless otherwise stated. The evolution equation for massive anti-neutrinos in the presence of VEP can be written as follows [7],

$$\frac{d}{dt} \begin{bmatrix} \bar{\nu}_e \\ \bar{\nu}_x \end{bmatrix} = \begin{bmatrix} -V_e - V_G \cos 2\theta_G - \Delta \cos 2\theta_M & \frac{1}{2}(V_G \sin 2\theta_G + \Delta \sin 2\theta_M) \\ \frac{1}{2}(V_G \sin 2\theta_G + \Delta \sin 2\theta_M) & 0 \end{bmatrix} \begin{bmatrix} \bar{\nu}_e \\ \bar{\nu}_x \end{bmatrix},$$

(5)

where $V_e \equiv \sqrt{2}G_F \rho Y_e/m_N$ is the standard matter potential, $\Delta \equiv \Delta m^2/2E$, $\theta_M$ is the usual flavor mixing induced by mass which can be different from $\theta_G$ and $V_G(r) \equiv 2\delta \gamma E\phi(r)$. For neutrinos system, $\nu_e - \nu_x$, the same equation hold by replacing $-V_e$ by $V_e$.

We assume that as a reasonably good approximation the density profile in the relevant region of the star takes the following form,

$$\rho(r) = \rho_0 \left(\frac{R_0}{r}\right)^m,$$

(6)

where $m = 5 - 7$ in the inner part and $m = 3$ in the outer layer of the star. For definiteness, we take $\rho_0 = \rho_1 = 8 \times 10^{14}$ g/cm$^3$ and $R_0 = R_1 = 10^6$ cm for the inner part and $\rho_0 = \rho_2 = 4 \times 10^4$ g/cm$^3$ and $R_0 = R_2 = 10^9$ for the outer part of the star. The radius which defines inner/outer part has been taken to be the distance at which both definitions merge, $R_{12} = 4.5 \times 10^6$ cm.

Under above assumptions, $\phi(r)$ can be explicitly computed as,

$$\phi(r) = -G \frac{M_1}{r} \left[ 5 - 2 \left(\frac{R_1}{r}\right)^2 \right] - 2 \frac{GM_2}{R_2} + \phi_{sc} \text{ for } R_1 < r < R_{12},$$

(7)

$$\phi(r) = -G \frac{M_1}{r} \left[ 5 + 6 \left(\frac{R_1}{R_2}\right)^2 \ln \left(\frac{r}{R_2}\right) \right] + \phi_{sc} \text{ for } R_{12} < r,$$

(8)

where $M_1 \equiv 4\pi \rho_1 R_1^3/3$ and $M_2 \equiv M_1 (R_1/R_2)^2$. In Fig. 1 we show the profiles of $V_e + \Delta \cos 2\theta_M$ and $V_G$ in eV for some values of mass and mixing parameters, and VEP parameters, as a function of distance from the center of the star.

A resonantly enhanced conversion can occur if the the resonance condition as well as the adiabaticity conditions are satisfied. The resonance condition is given by,

$$V_e = -V_G \cos 2\theta_G - \Delta \cos 2\theta_M,$$

(9)

Note that depending on the sign of $V_G$ and $\Delta$, resonance condition can be satisfied either for neutrino or anti-neutrino.

The adiabaticity parameter $\kappa(r)$ at position $r$ can be defined as

$$\kappa(r) \equiv \Delta H(r) \left| \frac{d\theta_m}{dr} \right|^{-1},$$

(10)

where $\Delta H(r)$ is the energy splitting between two energy eigenvalues of the Hamiltonian, $\Delta H(r) \equiv E_{m2} - E_{m1}$. Explicitly, $\kappa(r)$ can be written as follows,
\[
\kappa(r) = \left( \frac{\sin 2\theta_M + \frac{V_e}{\Delta} \sin 2\theta_G}{|\sin 3\theta_m|} \right)^2 \frac{4\pi \ell_0 \ell_{\rho}^{\text{eff}}}{\ell_{\rho}^2},
\]

(11)

where \( \ell_0 \equiv 2\pi m_N/\sqrt{2}G_F\rho Y_e \) is the matter refraction length, \( \ell_{\nu} \equiv 4\pi E/\Delta m^2 \) is the oscillation length in vacuum without VEP and \( \ell_{\rho}^{\text{eff}} \) is defined as,

\[
\ell_{\rho}^{\text{eff}} \equiv \left| \ell_{\rho}^{-1} \frac{\sin 2\theta_G + \frac{V_e}{\Delta} \sin 2(\theta_G - \theta_M)}{\sin 2\theta_G + \frac{V_e}{\Delta} \sin 2\theta_M} - \ell_{\rho}^{-1} \right|^{-1},
\]

(12)

where \( \ell_{\rho} \equiv |d \ln V_e/dr|^{-1} \) and \( \ell_{G} \equiv |d \ln V_G/dr|^{-1} \) are the density and gravitational scale height, respectively. Mixing angle in matter \( \theta_m \) in the presence of VEP is given by,

\[
\tan 2\theta_m = \frac{2H_{12}}{H_{22} - H_{11}} = \frac{\Delta \sin 2\theta_M + V_G \sin 2\theta_G}{V_e + \Delta \cos 2\theta_M + V_G \cos 2\theta_G}.
\]

(13)

For the case where resonant conversion occur, at resonance point, \( \kappa \) takes somewhat simpler form,

\[
\kappa(r_{\text{res}}) = 2\left( \frac{\Delta \sin 2\theta_M + V_G(r_{\text{res}}) \sin 2\theta_G}{|dV_e/dr + dV_G/dr \cos 2\theta_G|_{\text{res}}} \right)^2
\]

(14)

from which it is possible to derive the known cases, when \( \Delta m^2 \to 0 \) or \( \delta\gamma \to 0 \).

We compute the survival probability of \( \bar{\nu}_e \) at the surface of the outer region of the supernova \( P_{ee}^{\text{SN}} \), by using the following formula,

\[
P_{ee}^{\text{SN}} = P_{ee}(\bar{\nu}_e \to \bar{\nu}_e) = \frac{1}{2} + \left( \frac{1}{2} - P_c \right) \cos 2\theta_i \cos 2\theta_f,
\]

(15)

where \( \theta_i \) and \( \theta_f \) are the initial and final mixing angle which can be taken as \( \cos 2\theta_i = 1 \) and \( \cos 2\theta_f = \cos 2\theta_0 \) where \( \theta_0 \) is defined as

\[
\tan 2\theta_0 = \frac{\Delta \sin 2\theta_M + V_G^{\text{vac}} \sin 2\theta_G}{\Delta \cos 2\theta_M + V_G^{\text{vac}} \cos 2\theta_G}.
\]

(16)

The crossing probability \( P_c \) is computed as [33],

\[
P_c = \frac{\exp(-\frac{\pi}{4}K_F) - \exp(-\frac{\pi}{4}K_F/\sin^2\theta_0)}{1 - \exp(-\frac{\pi}{4}K_F/\sin^2\theta_0)},
\]

(17)

where, as an approximation, following Ref. [33], we use \( F \equiv |1 - \tan^2\theta_0| \) which is valid for exponentially decreasing density profile. When \( \bar{\nu}_e \) does not undergo resonance, as an approximation, we use the same formulas but with \( \kappa \) estimated at the resonance point for \( \nu_e \) channel.

We compute the final survival probability at the detector as follows [34],

\[
P_{D}(\bar{\nu}_e) = \frac{P_{ee}^{\text{SN}} - \sin^2\theta_0 + P_E^2(1 - 2P_{ee}^{\text{SN}})}{\cos 2\theta_0} + P_{coh},
\]

(18)
where $P^{SN}_{ee}$ is the averaged survival probability of $\bar{\nu}_e$ at the surface of supernova, computed by taking into account the energy spectrum (see eq. (23)) and $P^{E}_{\bar{e}e}$ is the conversion probability of $\bar{\nu}_2 \to \bar{\nu}_e$ after crossing the Earth. We define new eigenstates, $\nu_1$ and $\nu_2$, which are neither the mass nor gravitational eigenstates in the presence of mass and VEP, by using the mixing angle defined in eq. (16) as follows,

$$
\begin{bmatrix}
\nu_e \\
\nu_\mu \\
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_0 & \sin \theta_0 \\
-\sin \theta_0 & \cos \theta_0 \\
\end{bmatrix}
\begin{bmatrix}
\nu_1 \\
\nu_2 \\
\end{bmatrix}
$$

(19)

$P_{coh}$ is the “coherent” part of the probability which is given by,

$$
P_{coh} = 2\sqrt{P^{SN}_{\bar{e}e}(1 - P^{SN}_{\bar{e}e})P^{E}_{\bar{e}e}(1 - P^{E}_{\bar{e}e})} \cos(\alpha_{SN} + \alpha_E + \alpha),
$$

(20)

where $P^{SN}_{\bar{e}e}$ and $P^{E}_{\bar{e}e}$ denote the conversion probability of $\bar{\nu}_e \to \bar{\nu}_1$ in the supernova and in the Earth, respectively, $\alpha_{SN}$ is the relative phase difference between the $\bar{\nu}_1$ and $\bar{\nu}_2$ states at the surface of the supernova, $\alpha_E$ is the phase difference between the amplitude of $\bar{\nu}_1 \to \bar{\nu}_e$ and $\bar{\nu}_2 \to \bar{\nu}_e$ after crossing the Earth, and

$$
\alpha = \left[ \Delta^2 + (\delta \gamma \phi_{sc})^2 + \frac{1}{2} \Delta \cdot \delta \gamma \phi_{sc} \sin 4\theta_G \sin 4\theta_M \right]^{1/2} L,
$$

(21)

where $L$ is the distance between supernova and the Earth. In our analysis we will use $L = 50$ kpc for the distance between SN1987A and the Earth. We note that $P_{coh}$ could be important if sum of these phases is not so large. If one of them is very large, then the coherent term will be averaged out, after we take into account the energy spread of supernova neutrinos. The largest contribution is coming from $\alpha$ and when $\alpha$ is small, the other phases are expected to be very small compared to $\alpha$, and therefore, we can simply neglect other two phases $\alpha_{SN}$ and $\alpha_E$, as a good approximation, to compute $P_{coh}$.

Assuming that the density is constant, $P^{E}_{2e}$ can be computed as follows [30],

$$
P^{E}_{2e} \equiv P^e(\bar{\nu}_2 \to \bar{\nu}_e) = \sin^2 \theta_0 + \sin 2\theta_\oplus \sin 2(\theta_\oplus - \theta_0) \sin^2 \left( \frac{\pi}{2} \frac{\ell}{\ell_\oplus} \right),
$$

(22)

where $\ell$ is the distance traveled by neutrino inside the Earth, $\theta_\oplus$ and $\ell_\oplus$ are the mixing angle and the neutrino oscillation length, respectively, in the Earth.

### III. POSSIBLE IMPACT OF OSCILLATION FOR SUPERNOVA NEUTRINOS

Let us consider the influence of neutrino oscillation between $\bar{\nu}_e$ and $\bar{\nu}_x$ ($x = \mu$ or $\tau$) for SN neutrinos. From the viewpoint of observation, influence on $\bar{\nu}_e$ signal is most important because the cross section for absorption reaction $\bar{\nu}_e p \to e^- + n$ is much larger than elastic scattering processes $\nu_x e^- \to \nu_x e^-$ ($x = e, \mu, \tau$). In fact it is considered that observed events at Kamiokande-II and IMB detectors [6] from SN1987A are induced by $\bar{\nu}_e$.

The energy spectrum of supernova neutrinos can be described by “pinched” Fermi-Dirac distribution where pinched form can be parametrized by the “effective” chemical potential $\eta_i$ as follows [35],
\[ f_i(E_\nu) \propto \frac{E_\nu^2}{\exp(E_\nu/T_i - \eta_i) + 1}, \]  

where \( T_i \) and \( \eta_i \) are the temperature and effective chemical potential, respectively, for \( i \)-th neutrino species. Because of the different interaction rates of neutrinos in the proto-neutron star, it is expected that there exist the following hierarchy relation between temperatures,

\[ T_{\nu_e} < T_{\bar{\nu}_e} < T_{\nu_{\mu,\tau}} \approx T_{\bar{\nu}_{\mu,\tau}}. \]  

Due to such temperature difference, average neutrino energies in typical numerical SN simulations, which are approximately related with temperatures as \( \langle E_\nu \rangle \approx 3T_\nu \), are [36–38],

\[
\begin{align*}
\langle E_{\nu_e} \rangle &\approx 11 - 12 \text{ MeV}, \\
\langle E_{\bar{\nu}_e} \rangle &\approx 14 - 17 \text{ MeV}, \\
\langle E_{\nu_{\mu,\tau}} \rangle &\approx \langle E_{\bar{\nu}_{\mu,\tau}} \rangle \approx 24 - 27 \text{ MeV}.
\end{align*}
\]  

Such energy hierarchy is of crucial importance for our discussion as we will see below.

If neutrino oscillation between \( \bar{\nu}_e \) and \( \bar{\nu}_x \) \((x = \mu, \tau)\) occurs in SN and/or in the space between SN and Earth, expected neutrino flux at the Earth can be given by,

\[ F_{\bar{\nu}_e} = (1 - p)F_{\bar{\nu}_e}^0 + pF_{\bar{\nu}_x}^0, \]  

where \( p \) is the permutation factor, or the energy averaged conversion probability for \( \bar{\nu}_e \leftrightarrow \bar{\nu}_x \) which can be computed by convoluting the probability in eq. (18) in the previous section with the Fermi-Dirac energy distribution in eq. (23), and \( F_{\bar{\nu}}^0 \) denotes the time integrated original neutrino flux in the absence of oscillation. For simplicity, we will set the “effective chemical potential, \( \eta_i \), in eq. (23), equal to zero in this work.

Because of the energy hierarchy in eq. (25), large oscillation between \( \bar{\nu}_e \leftrightarrow \bar{\nu}_{\mu,\tau} \) would make the observable energy of \( \bar{\nu}_e \) larger than the theoretical predictions. However, the observed SN1987A neutrino data by Kamiokande and IMB detectors [6] imply rather smaller average energy, which are in the lower side of the theoretical expectations, indicating that there was no significant oscillation between \( \bar{\nu}_e \) and \( \bar{\nu}_{\mu,\tau} \). Based on this argument, it was obtained in Ref. [30] the bound on permutation parameter as \( p < 0.35 \) at 99 % C. L., which disfavors large oscillation between \( \bar{\nu}_e \) and \( \bar{\nu}_{\mu,\tau} \). The same argument was used in Ref. [31] and most recently in Refs. [39–43], to constrain large oscillation for neutrinos from SN1987A.

**IV. BOUNDS ON VEP PARAMETERS FROM SUPERNOVA**

In this section we discuss our results which are obtained using the formulas described in the previous section.
A. massless neutrinos case

First let us discuss the case without neutrino mass, or the case where masses are degenerated. In this case, the relevant formulas can be obtained by putting $\Delta m^2 \rightarrow 0$ in the previous section. In order to cover all the physical case, we fix $\delta \gamma$ to be positive but consider $\theta_G$ in the range from 0 to $\pi/2$, analogous to the case of usual MSW effect [44]. Here we try to constrain the VEP parameters from the observation of the supernova SN1987A neutrinos by Kamiokande and IMB following the argument discussed in Refs. [30,31].

In Fig. 2 we show the iso-contours of permutation factor for anti electron neutrinos from supernova SN1987A on the $\tan^2 \theta_G - \delta \gamma$ plane. For parameters in the inner region of these curves, the neutrino conversion is efficient so that the energy spectrum of $\bar{\nu}_e$ gets harder which would lead to worse agreement of the data with the predictions from SN simulations. If we use the criteria adopted in Ref. [30], shaded region could be disfavored at 99 % C.L. In the same plot, for the purpose of comparison, we also show the region (inside the dashed curves) which can be excluded by the laboratory experiments obtained in Ref. [21].

We note that for $\delta \gamma$ larger than $10^{-20}$, strong VEP induced resonant conversion [8], which is similar to the well known MSW effect, can occur inside the supernova, and the permutation parameter can be larger than 0.5 even for very small values of gravitational mixing angle $\theta_G$. We see that significantly large parameter region can be disfavored which can not be tested by laboratory experiments (see dashed curve).

On the other hand, for $\delta \gamma$ smaller than $\sim 10^{-20}$, the dominant effect is the VEP induced vacuum like-oscillation with the probability,

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta_G \sin^2 [\delta \gamma \phi_{sc} EL]. \tag{27}$$

In this case the permutation parameter can be large if the mixing angle $\theta_G$ is large and $\delta \gamma \lesssim 10^{-31}$ and therefore, such parameter region could be constrained. The reason why we can obtain such stringent bound on $\delta \gamma$, far below than the existing experimental limit [21], $\delta \gamma \lesssim 10^{-19} [|\phi_{sc}|/3 \times 10^{-5}]$, is simply because of the fact that the distance between the observed supernova and Earth is much larger (about 50 kpc, for the case of SN1987A) compared to the baseline of laboratory neutrino oscillation experiments or even compared to the baseline for solar neutrinos, Sun-Earth distance.

B. massive neutrinos case

Next let us discuss the case where neutrinos are massive. Let us consider how the result presented in the previous section can be modified due to mass and usual flavor mixing. For the sake of demonstration, we take some particular sets of mixing parameter implied by the MSW solutions to the solar neutrino problem [45]. Basically, the main effect is that the sensitivity to VEP is weakened due to the presence of usual mass and mixing terms. It is easy to see this. When the magnitudes of mass and usual flavor mixing terms are larger than those by VEP, oscillation effect is essentially induced or dominated by the usual mass and flavor mixing, and vice versa, as they mainly govern the evolution of neutrinos.
In Fig. 3 we show our results obtained by assuming, particular set of the mass and flavor mixing implied by the large mixing angle (LMA) MSW solution to the solar neutrino problem, $\Delta m^2 = 3.2 \times 10^{-5}$ eV$^2$ and $\tan^2 \theta_M = 0.33$ in addition the VEP effect. For $\delta \gamma$ smaller than $\sim O(10^{-16})$ the effect of neutrino conversion is essentially dominated by the usual mass induced oscillation. We note that new disfavored region appears for $\delta \gamma \sim 10^{-16} - 10^{-14}$ for any small values of $\theta_G$. This is due to the combined effects of the usual mass and VEP induced terms in the neutrino evolution Hamiltonian. In these parameter regions of $\delta \gamma$ around $\sim O(10^{-16})$, resonant conversion is mainly satisfied by the VEP parameters whereas the adiabaticity condition is met due to the presence of mass induced mixing term or $\frac{1}{2} \Delta \sin 2\theta_M$, even if $\theta_G$ is zero.

In Fig. 4 we show the similar plot but for the small mixing angle (SMA) MSW solution with $\Delta m^2 = 5.0 \times 10^{-6}$ eV$^2$ and $\tan^2 \theta_M = 5.8 \times 10^{-4}$. In this case, unlike in Fig. 3, no new disfavored region appears for $\delta \gamma \sim O(10^{-16})$. This is because for these values of $\delta \gamma$ with small $\theta_G \ll 1$, the adiabaticity condition, which was satisfied in the case of LMA solution, is not satisfied anymore since $\frac{1}{2} \Delta \sin 2\theta_M$ is much smaller than the case of LMA.

Some comments are in order. We showed in Fig. 3 and 4 some parameter region which can be disfavored by SN1987A data assuming the mass and mixing suggested by LMA and SMA solutions to the solar neutrino problem. We should note, however, that if $\delta \gamma$ is much larger than $\sim 10^{-16}$, solar neutrino flux can be significantly affected by the VEP effect and hence LMA and SMA solutions must be affected accordingly, as discussed in Ref. [15]. Therefore, the parameter regions allowed in Fig. 3 and 4 by the SN1987A data are not necessarily implying the parameter region allowed by the solar neutrino data.

V. CONCLUSIONS

We have studied the impact of the neutrino oscillation which could be induced by a tiny breakdown of the equivalence principle. We have obtained regions of parameters which are disfavored by the SN1987A neutrino data and showed that supernova can prove some region of parameter space which can not be tested by laboratory experiments. We show that supernova neutrinos can be sensitive to disfavor very tiny values of $\delta \gamma$ as small as $O(10^{-31})$, which is far below the laboratory bound, if neutrino mass can be neglected. We also showed that much smaller gravitationally induced mixing angle, $\tan^2 \theta_G \ll 10^{-4}$, which can not be tested by the laboratory experiments, can be proved by SN neutrinos. For massive neutrinos, we showed that the sensitivity to these bounds will become worse. The effect of VEP in oscillation is lost or weakened when mass induced terms become larger than VEP terms. Let us note that, compared to the bounds on VEP obtained by using the macroscopic objects, $\eta \lesssim 10^{-12}$, mentioned in the introduction, these bounds we obtained are much more stringent.

Let us finally try to compare the sensitivity to VEP parameter for neutrinos from supernova with that for neutrinos from other astrophysical sources. With solar neutrinos, it was discussed that effect of VEP can be tested for $\delta \gamma$ as small as $\sim 10^{-20}$ for massless
neutrinos [13] and for $\delta \gamma$ as small as $\sim 10^{-16} - 10^{-15}$ for massive neutrinos if we assume the MSW solutions to the solar neutrino problem [15]. Much better sensitivity can be obtained if we can observe very high energy neutrino from AGN [16]. Due to much higher energy and larger distance, it is discussed in Ref. [16] that one can test VEP for $\delta \gamma$ as small as $10^{-41}$ if neutrinos are massless or degenerated neutrinos and $\delta \gamma \sim 10^{-28} \times (\Delta m^2/1\text{eV}^2)$ for massive neutrinos. However, let us stress that in the absence of such ANG neutrino data, currently, SN1987A data seem to provide better bound on VEP parameters than any existing terrestrial experiments.

ACKNOWLEDGMENTS

This work was supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). H.N. would like to thank Jose F. W. Valle for his hospitality and support during his visit at Univ. of València where part of this work was done. We thank O.L.G.Peres and H. Minakata for useful discussions and comments. R. T. has been supported by a grant from the Generalitat Valenciana. R. T. would like to thank the department of cosmic ray at Gleb Wataghin physics institute and also P.C. de Holanda and M. Queiroz for their hospitality during his visit to UNICAMP.
REFERENCES


FIG. 1. $V_G(r)$ are plotted as a function of the distance from the center of the star for various values of $\delta \gamma$ by dotted, dot-dashed, short-dashed, long-dashed curves. Also $V_{mat}(r) + \Delta \cos^2 \theta$ is plotted for massless (or degenerate) case and $E = 10$ MeV, $\Delta m^2 = 10^{-5}$ eV$^2$ and $\sin^2 2\theta_M = 0.9.$
FIG. 2. Iso-permutation factor contours for anti electron neutrinos from supernova on the $\tan^2 \theta_G - \delta \gamma$ plane. The curves correspond, from inside to outside, $p = 0.9, 0.75, 0.35, 0.23, 0.1$. The shaded region corresponds to $p > 0.35$. Inside the dashed curve indicates the excluded region obtained from laboratory experiment obtained in Ref. [21].
FIG. 3. Same as in Fig. 2 but in the presence of mass and mixing, $\Delta m^2 = 3.2 \times 10^{-5} \text{ eV}^2$ and $\tan^2 \theta_M = 0.33$, suggested by the LMA MSW solution to the solar neutrino problem.
FIG. 4. Same as in Fig. 2 but in the presence of mass and mixing, $\Delta m^2 = 5.0 \times 10^{-6} \text{ eV}^2$ and $\tan^2 \theta_M = 5.8 \times 10^{-4}$, suggested by the SMA MSW solution to the solar neutrino problem.