Magneto–Fluid Coupling — Eruptive Events in the Solar Corona

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I. INTRODUCTION

The latest TRACE and SOHO/EIT observations have brought the solar corona into sharp focus. The observations reveal: 1) the structures that constitute the solar corona are in constant motion; they are full of fast–moving gases, and are heated primarily at their foot–points (base) very close to the solar surface. The heating occurs in a few minutes in the first ten to twenty thousand kilometers above the surface, i.e., in a rather small fraction of the bright part of the anchored structure. In direct contradiction to the predictions of some theories, the heating is neither uniform (throughout the loops) nor does it happen preferentially near the top. A direct quote sums up the situation aptly: "Moreover, not only heat is deposited low down, but the gas is often actually thrust upward very rapidly. It does not merely 'evaporate' into the coronal structures, it is often actually thrown up there. Exactly how that happens is still a puzzle" [1]. 2) the loops are composed of clusters of filamentary structures which are not, as believed before, static bodies supported by interior gas pressure and heated along their lengths [2]. They fill and drain so quickly that the gas in them must be moving nearly ballistically (see latest TRACE news and e.g. [1]) along the substructures, rather than being "quiescently heated". From a detailed study of the loops with different characteristic parameters one concludes that the heating process is quite non–uniform [3].

Transient brightenings, with their associated flows of cool and hot material, are also a very common phenomenon in the TRACE movies. These relatively fast (violent) happenings vary from small events in the quiet Sun to major flares in active regions; brightenings which are more than $10^5$ km apart often occur within the same exposure that typically lasts for 10 to 30 s [1]. This kind of a coincidence in the events at distant locations is suggestive of fast particle beams propagation along separate magnetic loops which come together at the flaring site. The flaring sites are generally assumed to be reconnection sites although observations have not establish a causal connection: "Direct evidence for reconnection in flares is difficult to find, despite the fact that it is thought to be the primary process behind flares" [1]. It is remarkable that often the post–flare loop systems begin to glow at the TRACE EUV wavelengths without substantial distortion: reconnection that probably took place appears to be (largely) completed by the time the loops are detected.

These observations pose a new challenge for the theories of quiescent as well as not so quiescent coronal structures and events. In this paper we examine the conjecture that the formation and primary heating of the coronal structures as well as the more violent events (possibly flares, erupting prominences and coronal mass ejections (CMEs)) are the expressions of different aspects of the same general global dynamics that operates in a given coronal region [4]. It is stipulated that the coronal structures are created from the evolution and re–organization of a relatively cold plasma flow emerging from the subcoronal region and interacting with the ambient solar magnetic field. The plasma flows, the source of both the particles and energy (part of which is converted to heat), in their interaction with the magnetic field, also become dynamic determinants of a wide variety of plasma states; it is likely that this interaction may be the cause of the immense diversity of the observed coronal structures [4–6]. Preliminary results from this magneto–fluid approach reproduce several of the salient observational features of the typical loops: the structure creation and primary heating are simultaneous – the heating takes place (by the viscous dissipation of the flow kinetic energy) in a few minutes, is quite non–uniform, and the base of the hot structure is hotter than the rest.
Within the framework of our approach, there are two distinct scenarios for eruptive events: a) when a "slowly" evolving structure finds itself in a state of no equilibrium, and b) when the process of creating a long-lived hot structure is prematurely aborted; the flow shrinks/distorts the structure which suddenly shines and/or releases energy or ejects particles. The latter mechanism requires a detailed time-dependent treatment and is not the subject matter of this paper. The following semi-equilibrium, collisionless magneto-fluid treatment pertains only to the former case.

A given structure is supposed to correspond to the equilibrium solutions of the two-fluid system

$$\frac{\partial}{\partial t} \omega_j - \nabla \times (U_j \times \omega_j) = 0 \quad (j = 1, 2) \quad (1)$$

written in terms of a pair of generalized vorticities \( \omega_1 = B, \omega_2 = B + \nabla \times V \), and effective flows \( U_1 = V - \nabla \times B, U_2 = V \), with the following normalizations: the magnetic field \( B \) to an appropriate measure of the magnetic field \( B_0 \), the fluid velocity \( V \) to the corresponding Alfvén speed, and the distances to the collisionless ion skin depth \( l_i \).

The simplest and perhaps the most fundamental equilibrium solution to (1) is given by the "Beltrami conditions", which imply the alignment of the vorticities and the corresponding flows \( \omega_j//U_j \),

$$B = a(V - \nabla \times B), \quad (2)$$

$$B + \nabla \times V = bV, \quad (3)$$

with \( a, b = \text{const}. \) We have used constant density assumption for simplicity - extension to varying density is straightforward [4]. Equations (2) and (3) combine to yield:

$$\begin{align*}
\text{curl} - \lambda_+ \text{curl} - \lambda_- B &= 0, \\
\lambda_{\pm} &= \frac{1}{2} \left[ (b - \tilde{a}) \pm \sqrt{(b + \tilde{a})^2 - 4} \right].
\end{align*} \quad (4)$$

For sub-Alfvénic flows (the flows we generally encounter in the solar atmosphere), the length scales \( \lambda_{\pm} \) are quite disparate. We assume \( \lambda_+^{-1} \gg \lambda_-^{-1} \) without loss of generality. The general solution to the "double Beltrami equations" (4) is a linear combination of the single Beltrami fields \( G_{\pm} \) satisfying \( \text{curl} - \lambda G = 0 \). Thus, for arbitrary constants \( C_{\pm} \), the sum

$$B = C_+ G_+ + C_- G_- \quad (6)$$

solves (4), and the corresponding flow is given by

$$V = (\lambda_+ + \tilde{a}) C_+ G_+ + (\lambda_- + \tilde{a}) C_- G_-.$$
The DB field encompasses a wide class of steady states of mathematical physics – from the force-free paramagnetic field to the fully diamagnetic field. The Beltrami conditions also demand "generalized Bernoulli conditions" which allow pressure confinement when an appropriate flow is driven [5] (and references therein).

The DB field is characterized by four parameters: \( \lambda_+ \), \( \lambda_- \) (eigenvalues), and \( C_+, C_- \) (amplitudes). The three invariants [13]: the helicity \( h_1 \), the generalized helicity \( h_2 \),

\[
 h_1 = \frac{1}{2} \int (A \cdot B) \, \text{d}r,
\]

\[
 h_2 = \frac{1}{2} \int (A + V) \cdot (B + \nabla \times V) \, \text{d}r,
\]

where \( A \) is the vector potential, and the total energy

\[
 E = \frac{1}{2} \int (B^2 + V^2) \, \text{d}r
\]

will provide three algebraic relations connecting them [14]. To predict the possibility of an eruptive event, interpreted as the termination of an equilibrium sequence (for solar flares, this kind of an approach, albeit in different contexts, has been followed in numerous investigations, (see e.g. [15]-[16] and references therein), we analytically investigate this system using the macro-scale \( (\lambda_{+}^{-1}) \) of the closed structure as a control parameter. This choice is physically sensible and is motivated by observations because in the process of structure-structure interactions, "initial" shapes do undergo deformations/distortions with rates strongly dependent on the initial and boundary conditions.

For simplicity we explicitly work out the system in a Cartesian cube of length \( L \). We take \( G_\pm \) to be the simple 2-D Beltrami ABC field [17],

\[
 G_\pm = g_{x\pm} \left( \begin{array}{c}
 0 \\
 \sin \lambda_{\pm} x \\
 \cos \lambda_{\pm} x 
\end{array} \right) + g_{y\pm} \left( \begin{array}{c}
 \cos \lambda_{\pm} y \\
 0 \\
 \sin \lambda_{\pm} y 
\end{array} \right),
\]

with the normalization \((g_{x\pm})^2 + (g_{y\pm})^2 = 1\). For real \( \lambda_\pm \), (10) represents an arcade–magnetic field structure resembling interacting coronal loops [in Fig. 1]. Assuming \( L = n_+(2\pi/\lambda_+) = n_-(2\pi/\lambda_-) \) \((n_{\pm} \text{ are integers})\), \( G_\pm \) satisfy the following relations:

\[
 \int G_\pm^2 \, \text{d}r = L^2,
\]

\[
 \int G_+ \cdot G_- \, \text{d}r = 0,
\]

where \( \int \text{d}r = \int_0^L \int_0^L \, \text{d}x \, \text{d}y \).

The invariants can now be readily evaluated and the results can be displayed in several equivalent forms. We find the following three equations to be the most convenient for further analysis: \( h_2 = h_1 + h_2, \ h_2 = bE - \lambda_+ \lambda_- h_1 \):

\[
 \tilde{h}_2 = \frac{E}{2} \left[ (\lambda_+ + \lambda_-) \pm \sqrt{(\lambda_+ - \lambda_-)^2 + 4} \right] - \lambda_+ \lambda_- h_1,
\]

\[
 C_2^+ = D^{-1} \left[ E - [1 + (\lambda_+ + \tilde{a})^2] \lambda_- h_1 \right] \lambda_+,
\]

\[
 C_2^- = -D^{-1} \left[ E - [1 + (\lambda_+ + \tilde{a})^2] \lambda_+ h_1 \right] \lambda_-,
\]

where we have removed the common factor \( L^2/2 \), and

\[
 D = \left[ 1 + (\lambda_+ + \tilde{a})^2 \right] \lambda_+ - [1 + (\lambda_- + \tilde{a})^2] \lambda_- = (\lambda_+ - \lambda_-) b(b + \tilde{a}).
\]

For given \( h_1 \), \( E \), \( \tilde{h}_2 \) and \( \lambda_+ \) (control parameter), we can solve the preceding system to determine the physical quantities \( \lambda_- \), and \( C_\pm \) which must all remain real for an equilibrium. Before we give an analytic derivation for the bifurcation conditions (leading to loss of equilibrium), we display in Fig. 2 the plots of \( \lambda_- \) and \( C_\pm \) as functions of \( \lambda_+ \) for two distinct sets for the values of the invariants: we choose \( h_1 = 1, \ h_2 = 1.5, \ E = 0.4 \) for Fig. 2(a), and \( h_1 = 1, \ h_2 = 1.5, \ E = 1.3 \) for Fig. 2(b) (dashed lines correspond to the region of imaginary \( C_- \)). We find that the behavior of the solution changes drastically with \( E \). For the parameters of Fig. 2(a), \( \lambda_- \) and \( C_- \) remain real and change continuously with varying \( \lambda_+ \) implying that as the macroscopic scale of the structure \( (1/\lambda_+) \) changes continuously, the equilibrium expressed by (10) persists – there is no catastrophic or qualitative change. For Fig. 2(b) with \( E \) changing from 0.4 to 1.3 (with same \( h_1 \), \( h_2 \)) we arrive at a fundamentally different situation; when \( \lambda_+ \) exceeds a critical value \( \lambda_+^{\text{crit}} \), i.e., the macro-scale becomes smaller than a critical size, the physical determinants of the equilibrium cease to be real; the sequence of equilibria is terminated.

The condition for catastrophe turns out to be a constraint involving \( h_1 \), \( h_2 \) and \( E \), which will allow the vanishing of \( C_2^+ \) for positive \( C_2^- \), and real \( \lambda_+ \). It is straightforward to show that the system has a critical point if

\[
 E^2 \geq E_c^2 = 4 \left( h_1 \pm \sqrt{h_1 h_2} \right)^2
\]

and the critical \( \lambda \) is determined by a simultaneous solution of (11) and

\[
 E - [1 + (\lambda_+ + \tilde{a})^2] \lambda_+ h_1 = 0 \quad \text{giving:}
\]

\[
 \lambda_+^{\text{crit}} = \frac{1}{2h_1} \left( E \pm \sqrt{E^2 - E_c^2} \right).
\]

Thus, for \( E > E_c \) (determined by helicities \( h_1 \) and \( h_2 \)), when the macroscopic size of a structure shrinks below a critical value, it can go through a severe reorganization.

At the critical point, an expected but most remarkable transition occurs. Using the value of \( \lambda_+^{\text{crit}} \), we find from equation(13) that the coefficient \( C_- \), which measures the strength of the short scale fields, identically vanishes, and the equilibrium changes from Double Beltrami to a single Beltrami state defined by \( \lambda_+ = \lambda_+^{\text{crit}} \), i.e., \( \mathbf{B} = C_+ \mathbf{G}_+ \) \((\nabla \times \mathbf{B} = \lambda_+ \mathbf{B})\) with \( \mathbf{V} \) parallel to \( \mathbf{B} \). The transition leads to a magnetically more relaxed state with the magnetic energy reaching its minimum with appropriate gain in the flow kinetic energy (see Fig. 3).

III. CONCLUSIONS

By modelling quasi-equilibrium, slowly evolving coronal structures as a sequence of Double–Beltrami mag-
netofluid states in which the magnetic and the velocity field are self-consistently coupled, we have shown the possibility of, and derived the conditions for catastrophic changes leading to a fundamental transformation of the initial state. The critical condition comes out as an inequality involving three invariants of the collisionless magneto-fluid dynamics. When the total energy exceeds a critical energy the DB equilibrium suddenly relaxes to a single Beltrami state corresponding to the large macroscopic size. All of the short-scale magnetic energy is lost having been transformed to the flow energy and partly to heat via the viscous dissipation of the flow energy.

This general mechanism in which the flows (and their interactions with the magnetic field) play an essential role could certainly help in advancing our understanding of a variety of sudden (violent) events in the solar atmosphere like the flares, the erupting prominences, and the coronal mass ejections. The connection of flows with eruptive events is rather direct: it depends on their ability to deform (in specific cases distort) the ambient magnetic field lines to temporarily stretch (shrink, destroy) the closed field lines so that the flow can escape the local region with a considerable increase in kinetic energy in the form of heat/bulk motion.

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