Higgs Mechanism with a Topological Term

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Abstract

In cases of both abelian and nonabelian gauge groups, we consider the Higgs mechanism in topologically massive gauge theories in an arbitrary space-time dimension. It is shown that the presence of a topological term makes it possible to shift mass of gauge fields in a nontrivial way compared to the conventional value at the classical tree level. We correct the previous misleading statement with respect to the counting of physical degrees of freedom, where it is shown that gauge fields become massive by ‘eating’ the Nambu-Goldstone boson and a higher-rank tensor field, but a new massless scalar appears in the spectrum so the number of the physical degrees of freedom remains unchanged before and after the spontaneous symmetry breakdown. Some related phenomenological implications and applications to superstring theory are briefly commented.

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1 Introduction

Despite its experimental success, the Standard Model based on $SU(3)_C \times SU(2)_L \times U(1)_Y$ still leaves many questions unsolved. For instance, why is a particular gauge group observed at low energy, together with the multiplicity of generations? Can we explain the origin of the proliferation of parameters in the flavor sector from a fundamental theory? How do we incorporate gravity into the theory? It seems that the resolution of these problems requires new conceptional framework and tools of the underlying fundamental theory, that is, superstring theory.

In addition to these theoretical questions, the Higgs sector in the Standard Model has not been observed experimentally and the least understood. It is then of theoretical interest to ask what modification superstring theory provides for the Higgs sector. However, superstring phenomenology, the study of how superstring theory makes contact with physics at accessible energy, is still in its infancy, so we have no quantitative predictions, as yet, from superstring theory. Nevertheless, there are a number of important qualitative implications and insights. In particular, superstring theory predicts the existence of many new particles such as a dilaton, an axion and perhaps other scalar moduli. In them, a bunch of antisymmetric tensor fields also naturally appear in the spectrum and play an important role in the non-perturbative regime, such as D-branes and various dualities, in superstring theory [1]. Thus it is natural to inquire if such antisymmetric tensor fields yield a new phenomenon to the still mysterious Higgs sector in the Standard Model.

In fact, it has been known that when there is a topological term, antisymmetric tensor fields (including gauge field) exhibit an ingenious mass generation mechanism, which we call, the ‘topological Higgs mechanism’ in the sense that antisymmetric tensor fields acquire masses and spins without breaking the local gauge invariance explicitly. (Of course, in this case, unlike the conventional Higgs mechanism, we do not have the Higgs particle in general, though.) This interesting mass generation mechanism is first found within the framework of three-dimensional gauge theory with Chern-Simons term [2, 3]. Afterwards, this three-dimensional topological Higgs mechanism has been generalized to an arbitrary higher-dimension in cases of both abelian [4, 5, 6] and non-abelian gauge theories [7]. More recently, a new type of the topological massive nonabelian gauge theories with the usual Yang-Mills kinetic term has been constructed [8, 9, 10].

In this paper, we would like to study the mass generation mechanism in the abelian gauge theories [4, 5, 6] and the nonabelian gauge theories [8, 9, 10] with the usual Higgs potential in addition to a topological term. Since in the Weinberg-Salam theory the Higgs doublets plus their Yukawa couplings are one of the key ingredients with great experimental success, we do not want to dismiss the framework of the conventional Higgs mechanism. Instead, we wish to consider how mass of gauge fields is modified if the conventional Higgs mechanism coexists with the topological Higgs mechanism. About ten years ago, Yahikozawa and the present author have studied such a model, but it is a pity that there is some misleading statement about the counting of the degrees of freedom and only abelian gauge theories are taken into consideration [11].
The purpose of the present paper is threefold. First of all, we wish to correct our previous misleading statement with respect to the counting of physical degrees of freedom. We show that gauge fields become massive by 'eating' both the Nambu-Goldstone boson and a higher-rank tensor field, but a new massless scalar appears in the spectrum so the number of the physical degrees of freedom remains unchanged before and after the spontaneous symmetry breakdown. Secondly, this phenomenon is generalized to the nonabelian gauge theories. Finally, we wish to comment on some phenomenological implications and applications to superstring theory of our results.

2 Abelian gauge theories

Let us start by reviewing the 'topological Higgs mechanism' in abelian gauge theories in an arbitrary space-time dimension [4]:

\[ S = -\frac{1}{2} \int dA \wedge *dA - \frac{1}{2} \int dB \wedge *dB + \mu \int A \wedge dB, \]  

(1)

where \( A \) and \( B \) are respectively an \( n \)-form and a \((D-n-1)\)-form in \( D \)-dimensional space-time with metric signature \((-,+,+,...,+\)), "\(*\)" is the Hodge dual operator, and \( \wedge \) is the Cartan’s wedge product, which we will omit henceforth for simplicity. The equations of motion are easily derived to

\[ d* \text{d}A + (-1)^n \mu dB = 0, \]
\[ d* \text{d}B + (-1)^{n(D-1)+1} \mu dA = 0. \]  

(2)

From these equations of motion, we obtain

\[ (\Delta + \mu^2) \text{d}A = 0, \]
\[ (\Delta + \mu^2) \text{d}B = 0, \]  

(3)

where \( \Delta \) is the Laplace-Beltrami operator. Note that these equations are equations of motion for the transverse components of \( A \) and \( B \) fields, and they clearly represent that two massless antisymmetric tensor fields \( A \) and \( B \) have gained the same mass \( \mu \) through the topological term \( \mu \int A \wedge dB \), whose phenomenon we call the "topological Higgs mechanism". It is worthwhile to count the degrees of freedom of physical states before and after the topological Higgs mechanism occurs. Originally, we have two massless fields, so the total number of the degrees of freedom is given by \( \binom{D-2}{n} + \binom{D-2}{D-n-1} \), which is rewritten as

\[ \binom{D-2}{n} + \binom{D-2}{D-n-1} = \binom{D-1}{n} = \binom{D-1}{D-n-1}. \]  

(4)
This equation clearly indicates that the massless $n$-form field $A$ has become massive by ‘eating’ the massless $(D - n - 1)$-form field $B$, and vice versa.

Now we would like to introduce the conventional Higgs potential for only $A$ field, for which we have to restrict $A$ field to be a 1-form since the exterior derivative couples with only 1-form to make the covariant derivative. Thus we are led to consider [11]

$$ S = -\frac{1}{2} \int dA \ast dA - \frac{1}{2} \int dB \ast dB + \mu \int A dB $$

$$ - \int \left[ (D\phi)^\dagger \ast D\phi + \lambda(|\phi|^2 - \frac{1}{2}v^2) \ast (|\phi|^2 - \frac{1}{2}v^2) \right], $$

where $B$ is now a $(D - 2)$-form field, and $\phi$ and $v$ are respectively a 0-form complex field and a real number. The covariant derivative $D$ is defined in a usual way as

$$ D\phi = d\phi - igA\phi, $$

$$ (D\phi)^\dagger = d\phi^\dagger + igA\phi^\dagger. $$

The gauge transformations are given by

$$ \phi(x) \rightarrow \phi'(x) = e^{-i\alpha(x)}\phi(x), $$

$$ A(x) \rightarrow A'(x) = A(x) - \frac{1}{g}d\alpha(x), $$

$$ B(x) \rightarrow B'(x) = B(x) - d\beta(x), $$

where $\alpha(x)$ and $\beta(x)$ are a 0-form and a $(D - 3)$-form gauge parameters, respectively. And $g$ denotes a $U(1)$ gauge coupling constant. Note that there are still off-shell reducible symmetries for $B$ field when $D > 3$.

The minimum of the potential is achieved at

$$ |\phi| = \frac{v}{\sqrt{2}}, $$

which means that the field operator $\phi$ develops a vacuum expectation value $|<\phi>| = \frac{v}{\sqrt{2}}$.

If we write $\phi$ in terms of two real scalar fields $\phi_1$ and $\phi_2$ as

$$ \phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2), $$

we can select

$$ <\phi_1> = v, <\phi_2> = 0. $$

With the shifted fields

$$ \phi'_1 = \phi_1 - v, \phi'_2 = \phi_2, $$

$$ \phi'_1 = \phi_1 - v, \phi'_2 = \phi_2, $$
we have
\[ <\phi'_1> = <\phi'_2> = 0. \tag{12} \]

Note that \( \phi'_2 \) corresponds to the massless Goldstone boson. At this stage, let us take the unitary gauge to remove the mixing term between \( A \) and \( \phi'_2 \) in the action:
\[
\phi^u(x) = e^{-i\xi(x)}\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x)),
\]
\[
G_\mu(x) = A_\mu(x) - \frac{1}{gv} \partial_\mu \xi(x). \tag{13}
\]

In this gauge condition, \( \xi(x) \) and \( \eta(x) \) correspond to \( \phi'_2(x) \) and \( \phi'_1(x) \), respectively. Also note that the unitary gauge corresponds to the gauge transformation with a fixed gauge parameter \( \alpha(x) = \frac{1}{v} \xi(x) \).

Then, the action (5) reduces to the form
\[
S = -\frac{1}{2} \int dG^* dG - \frac{1}{2} (gv)^2 \int G^* G - \frac{1}{2} \int dB^* dB
+ \mu \int G dB - \int \left[ \frac{1}{2} \partial_\eta^* d\eta + \frac{1}{2} (\sqrt{2} \lambda v)^2 \eta^* \eta \right]
- \int \left[ \frac{1}{2} g^2 \eta(2v + \eta)G^* G + \lambda (v \eta^* \eta^2 + \frac{1}{4} \eta^2 \eta^2) \right]. \tag{14}
\]

From the above action, we can easily read off that the Higgs field \( \eta(x) \) becomes a massive field with mass \( \sqrt{2} \lambda v \) and the would-be Goldstone boson \( \xi(x) \) is absorbed into the gauge field \( G_\mu \), as in the conventional Higgs mechanism without a topological term.

In order to clarify the mass generation mechanism for the gauge field \( G \) and the antisymmetric tensor field \( B \), it is enough to consider only the quadratic terms with respect to fields in the action (14). In other words, neglecting the interaction terms in the action (14), we derive the equations of motion for \( G \) and \( B \) fields whose concrete expressions are given by
\[
-d^* dG - (gv)^2 G + \mu dB = 0. \tag{15}
\]
\[
(-)^D d^* dB + \mu dB = 0. \tag{16}
\]

From these equations, we can obtain
\[
[\Delta + \mu^2 + (gv)^2]dG = 0. \tag{17}
\]
\[
[\Delta + \mu^2 + (gv)^2] \delta dB = 0. \tag{18}
\]
\[
\delta G = 0, \tag{19}
\]

where \( \delta \) denotes the adjoint operator. Eqs. (17) and (18) reveal that fields \( G \) and \( B \) have become massive fields with the same mass \( \sqrt{\mu^2 + (gv)^2} \) through the conventional and topological Higgs mechanisms. Also note that since Eq. (19) holds only when \( gv \neq 0 \), the existence of Eq. (19) reflects a characteristic feature in the case at hand.
Now let us consider how the conventional and topological Higgs mechanisms have worked. For this, it is useful to count physical degrees of freedom in the present model. Before spontaneous symmetry breaking, we have two real scalar fields $\phi_1$ and $\phi_2$, and two massless fields $A_\mu$ and $B_{\mu_1 \cdots \mu_{D-2}}$. The total number of the degrees of freedom is

$$2 + \binom{D-2}{1} + \binom{D-2}{D-2} = D + 1.$$  \hspace{1cm} (20)

On the other hand, after the symmetry breaking, we have one real scalar field and one massive field $G_\mu$ (or equivalently, $B_{\mu_1 \cdots \mu_{D-2}}$), so it appears that the total number of the degrees of freedom is now given by

$$1 + \binom{D-1}{1} = 1 + \binom{D-1}{D-2} = D.$$  \hspace{1cm} (21)

However, we encounter a mismatch of one degree of freedom before and after spontaneous symmetry breaking, which is precisely a question raised in our previous paper [11]. It is a pity that we have proposed a misleading answer to this question in the previous paper, so we wish to give a correct answer in this paper.

To find a missing one physical degree of freedom, we have to return to the original equations of motion (15) and (16). From Eq. (16), we have

$$*dB + (-)^D \mu G = d\Lambda,$$

where $\Lambda$ is a 0-form. Then, using (19) we obtain

$$\Delta \Lambda = 0,$$  \hspace{1cm} (23)

which means that $\Lambda$ is a massless real scalar field that we have sought. By counting this one degree of freedom, we have $D + 1$ degrees of freedom after the symmetry breaking which coincides with the number of the degrees of freedom before the symmetry breaking. In order to understand this degree of freedom further, using Eqs. (15), (19) and (22), let us derive equations of motion for $G$ and $dB$. The result reads

$$\left[ \Delta + \mu^2 + (gv)^2 \right] G - (-1)^D \mu d\Lambda = 0,$$

$$\left[ \Delta + \mu^2 + (gv)^2 \right] dB - (-1)^D (gv)^2 \delta * \Lambda = 0.$$  \hspace{1cm} (24)

In Eq. (17), the last term involving $\Lambda$ in the former equation of (24) is projected out by the differential operator $d$. Similarly, in Eq. (18), the last term in the latter equation of (24) is also projected out owing to the adjoint operator $\delta$. As a final remark, it is worth emphasizing that $\Lambda$ is not the Goldstone boson $\xi$ but a new boson whose fact can be understood by comparing (13) with (22). Roughly speaking, $G_\mu$ (or $B_{\mu_1 \cdots \mu_{D-2}}$) has become massive by 'eating' both the Nambu-Goldstone boson $\xi$ and $B_{\mu_1 \cdots \mu_{D-2}}$ (or $G_\mu$), but it has eaten too much more than its capacity! In consequence, a new scalar field has been vomitted.
3 Nonabelian gauge theories

We now turn to nonabelian theories. Let us start by reviewing the topologically massive nonagelian gauge theories \[8, 9, 10\]. The action reads

\[
S = \int Tr \left[ -\frac{1}{2} F \ast F - \frac{1}{2} \mathcal{H} \ast \mathcal{H} + \mu BF \right],
\]

where we use the following definitions and notations: \( F = dA + gA^2, H = DB = dB + g[A, B], \mathcal{H} = H + g[F, V] \) and the square bracket denotes the graded bracket \([P, Q] = P \wedge Q - (-1)^{|P||Q|} Q \wedge P\). And \( A, B \) and \( V \) are respectively a 1-form, a \((D-2)\)-form, a \((D-3)\)-form. All the fields are Lie group valued, for instance, \( A = A^aT^a \) where \( T^a \) are the generators. This action is invariant under the gauge transformations

\[
\begin{align*}
\delta A &= D\theta + g[A, \theta], \\
\delta B &= D\Omega + g[B, \theta], \\
\delta V &= -\Omega + g[V, \theta],
\end{align*}
\]

where \( \theta \) and \( \Omega \) are a 0-form and a \((D-3)\)-form gauge parameters. Recall that a new field strength \( \mathcal{H} \) together with an auxiliary field \( V \) has been introduced to compensate for the non-invariance of the usual kinetic term \( TrH^2 \) under the tensor gauge transformations associated with \( B \) \[12\]. From now on, we shall set a coupling constant \( g \) to be 1 for simplicity since we can easily recover it whenever we want.

The equations of motion take the forms

\[
\begin{align*}
D \ast F &= -[B, \ast \mathcal{H}] + D[V, \ast \mathcal{H}] + \mu DB, \\
D \ast \mathcal{H} &= (-1)^{D-1} \mu F, \\
[F, \ast \mathcal{H}] &= 0.
\end{align*}
\]

From these equations, we can derive the following equations

\[
\begin{align*}
D \ast D \ast F + (-1)^D \mu^2 F &= -\mu D \ast [F, V] - D \ast [B, \ast \mathcal{H}] + D \ast D[V, \ast \mathcal{H}], \\
D \ast D \ast \mathcal{H} + (-1)^D \mu^2 \mathcal{H} &= -(-1)^D \mu \left( -\mu [F, V] - [B, \ast \mathcal{H}] + D[V, \ast \mathcal{H}] \right).
\end{align*}
\]

A linear approximation for fields in (28) leads to equations

\[
\begin{align*}
(\Delta + \mu^2) dA &= 0, \\
(\Delta + \mu^2) dB &= 0,
\end{align*}
\]

which imply that the fields \( A \) and \( B \) become massive by the topological Higgs mechanism as in the abelian gauge theories.
Next let us couple the Higgs potential to the model. For simplicity and definiteness, we shall take the gauge group to be \( G = SU(2) \), with generators \( T^i \) satisfying

\[
[T^i, T^j] = i\varepsilon^{ijk} T^k, \\
Tr(T^i T^j) = \frac{1}{2} \delta^{ij}, \\
T^i = \frac{1}{2} \tau^i,
\]

where \( \tau^i \) are the Pauli matrices. Then, the action is given by

\[
S = \int Tr \left[ - F \ast F - H \ast H + 2\mu BF \right] \\
- \int \left[ (D\phi^u) \ast D\phi^u + \lambda (|\phi|^2 - \frac{1}{2} v^2) \right] \ast \left( v^2 - \frac{1}{2} \right),
\]

where \( F_{\mu\nu} = \partial_{\mu} A_{\nu}^i - \partial_{\nu} A_{\mu}^i + \varepsilon^{ijk} A_{\mu}^j A_{\nu}^k \) and \( D_{\mu} \phi = (\partial_{\mu} - i\frac{1}{2} \tau^i A_{\mu}^i) \phi \).

As in the abelian theories, let us take the unitary gauge given by

\[
\phi^u(x) = U(x)\phi(x) = \left( \begin{array}{c} 0 \\ \sqrt{2}(v + \eta(x)) \end{array} \right), \\
G(x) = U(x)A(x)U(x)^{-1} + iU(x)dU(x)^{-1}, \\
B'(x) = U(x)B(x)U(x)^{-1}, \\
V'(x) = U(x)V(x)U(x)^{-1},
\]

where we have defined as \( U(x) = e^{-i\frac{1}{2} \tau^i \xi^i(x)} \). It then turns out that the action (31) reduces to the form

\[
S = \int Tr \left[ - F \ast F - H \ast H + 2\mu BF \right] \\
- \int \left[ \frac{1}{2} d\eta \ast d\eta + \frac{1}{2} \left( \frac{g v}{2} \right)^2 G^i \ast G^i + \lambda v^2 \eta \ast \eta \right],
\]

where we have rewritten \( B' \) and \( V' \) as \( B \) and \( V \), respectively. Namely, we now have the expressions like \( F = dG + G^2 \), \( D\phi^u = d\phi^u - iG\phi^u \). In order to study the mass generation mechanism, it is sufficient to examine only the quadratic action, which is given by

\[
S_0 = \int Tr \left[ - dG \ast dG - dB \ast dB + 2\mu BdG \right] \\
- \int \left[ \frac{1}{2} d\eta \ast d\eta + \frac{1}{2} \left( \frac{g v}{2} \right)^2 G^i \ast G^i + \lambda v^2 \eta \ast \eta \right].
\]

Here we have recovered the coupling constant \( g \). From this action (34), it is easy to obtain the equations of motion

\[
-d \ast dG^i - \frac{(g v)}{2} G^i + \mu dB^i = 0, \\
(-)^p d \ast dB^i + \mu G^i = 0, \\
d \ast d\eta - (\sqrt{2} \lambda v)^2 \ast \eta = 0.
\]
From these equations, we can obtain

\[
\left[ \Delta + \mu^2 + \left( \frac{gv}{2} \right)^2 \right] dG^i = 0. \tag{38}
\]

\[
\left[ \Delta + \mu^2 + \left( \frac{gv}{2} \right)^2 \right] \delta dB^i = 0. \tag{39}
\]

\[
\delta G^i = 0. \tag{40}
\]

\[
\left[ \Delta + (\sqrt{2\lambda}v)^2 \right] \eta = 0. \tag{41}
\]

Eq. (41) shows that the field \( \eta \) is indeed the Higgs particle with mass \( \sqrt{2\lambda}v \) as in the conventional Higgs mechanism. Also note that Eqs. (38)-(40) are the same equations as Eqs. (17)-(19) except the \( SU(2) \) index \( i \) and the replacement \( gv \to \frac{2v}{\sqrt{2}} \), so we can show that a completely similar mass generation mechanism to that in the abelian case occurs also in this case. Thus the original \( SU(2) \) gauge symmetry is completely broken and all gauge fields (or antisymmetric tensor field) acquire the same mass \( \sqrt{\mu^2 + \left( \frac{gv}{2} \right)^2} \) via the conventional and topological Higgs mechanisms. At the same time, we have a massless scalar with an \( SU(2) \) index.

Finally, we wish to address a few topics related to the mechanism clarified in the present paper. Firstly, we would like to consider how our model sheds some light on the Standard Model. For definiteness, let us consider the Weinberg-Salam model on the basis of \( SU(2)_L \times U(1)_Y \) even if it is easy to extend the model at hand to the Standard Model based on \( SU(3)_C \times SU(2)_L \times U(1)_Y \). Note that we can construct a Weinberg-Salam model with topological terms by unifying the abelian model treated in the previous section and the \( SU(2) \) nonabelian model in this section in an \( SU(2)_L \times U(1)_Y \)-invariant way. Then, we can observe the following facts: In the conventional Weinberg-Salam model, mass of weak bosons is given by \( M_W = \frac{gv}{2} \), whereas in our model it is given by \( m_W = \sqrt{\mu^2 + \left( \frac{v^2}{2} \right)^2} \). Similarly, in our model mass of \( Z \) boson receives a contribution from a topological term. Concerning the Higgs particle, we have the same mass \( \sqrt{2\lambda}v \) in both the models. Fermion masses are also the same in both the models. Thus, we can conclude that compared to the conventional Standard Model, in our model with topological terms we can in general introduce additional parameters stemming from topological terms in the mass formulas of gauge bosons without violating the local gauge symmetries explicitly and changing the overall structure of the Standard Model. In turn, provided that experiment would predict \( \mu \approx 0 \) in future it seems that we need to propose some mechanism to suppress the contribution to mass of gauge bosons, since there is \textit{a priori} no local symmetry prohibiting the appearance of topological terms. Moreover, our present model predicts the existence of a new massless scalar, which is in a sharp contrast to the Weinberg-Salam model where there is no such a massless boson. These distinct features in the model at hand will be testable by future experiments.

Secondly, as mentioned in the introduction, many of antisymmetric tensor fields naturally appear in the spectrum in superstring theory. For instance, the action of a supersymmetric \( D3 \)-brane in Type IIB superstring theory includes topological terms among antisymmetric tensor fields in the Wess-Zumino term as well as their kinetic terms in the Born-Infeld action.
[13, 14, 15, 16, 17]. Hence, it is expected that our model would have some implications in the non-perturbative regime of superstring theory. We wish to stress again that antisymmetric tensor fields and their topological coupling play a critical role in string dualities [1].

Thirdly, as another implication to superstring theory, note that we have the term $\int_{M_{10}} B \wedge X_8$ in the effective action of superstring theory for the Green-Schwarz anomaly cancellation. This term yields a topological term upon compactification to four dimensions. If the Higgs potential appears in addition to the topological term via a suitable compactification, the mass generation mechanism discussed in the paper would work nicely [11].

4 Conclusion

In this paper, we have investigated a physical situation where a topological term coexists with the Higgs potential. We have found that the gauge field becomes massive by ‘eating’ the Nambu-Goldstone boson and the antisymmetric tensor field, and ‘vomits’ a new massless scalar field. Moreover, we have pointed out that when our model is extended to a more realistic model such as the Weinberg-Salam model and the Standard Model, it gives us some distinct results, those are, the shift of mass of gauge fields and the presence of a new massless boson, which would be checked in future by experiment. Although the experiment might preclude our model, it would be necessary to propose some mechanism for suppressing the effects coming from a topological term since there is no symmetry prohibiting the existence of such a topological term. This problem becomes more acute in superstring-inspired phenomenology since superstring theory gives rise to many antisymmetric tensor fields with a topological term at low energy.

To make the present model viable, we need to give the proof of unitarity and renormalizability. We wish to attack this proof within the framework of perturbation theory in future.

References


