Extended Gari-Krümpelmann model fits to nucleon electromagnetic form factors

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Significant new nucleon electromagnetic form factor data has accumulated since the Gari-Krümpelmann model (GK) was last adjusted in 1992. The model includes the major vector meson pole contributions and at high momentum transfer conforms to the predictions of QCD. Its simple analytic form is very useful for incorporating the nucleon form factors into predictions for many-nucleon systems, including the deuteron. We consider different parameterizations of the model’s hadronic form factors and the effect of including the width of the \( \rho \) meson and the addition of the next (in mass) isospin 1 vector meson. These alternatives are fitted to the world data. We obtain much better fits to the data than obtained with the original GK parameters. The sensitivity to the particular parameterization is discussed. Projections are made to higher momentum transfers which are now accessible. The projections vary little for the preferred models.

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I. INTRODUCTION

The electromagnetic form factors (emff) of the neutron and proton contain all the information about the charge and current distribution of these baryons, providing strong constraints on the fundamental theory of strong interactions. In addition the predictions for the emff of many-nucleon systems are sensitive to the input nucleon emff, as well as the many-body effects one would like to determine. For the first aspect one would like to have an accurate description of the data in a form closely linked to the fundamental theory. For the second it is convenient to have a simple analytic form to embed in the many-body calculation.

Accepting QCD as the fundamental theory of strong interactions, the emff can be described by perturbative QCD (PQCD) at very high momentum transfers. At low momentum transfers the confinement property of QCD implies an effective hadronic description with vector meson dominance (VMD, the coupling of the photon to vector mesons that couple in turn to the nucleons). Early models of the nucleon emff were based on VMD alone [1,2] including the \( \rho \), \( \omega \), and \( \phi \) poles and the cut associated with the \( \rho \) width, but with several phenomenological higher mass poles added. Gari and Krümpelmann [3] restricted the VMD contribution to the \( \rho \), \( \omega \), and \( \phi \) poles, but added factors and terms which explicitly constrained the asymptotic momentum transfer behavior to the scaling behavior of PQCD. These additional factors, specified in the next section, are, in effect, hadronic form factors.
We fit a series of GK type models (varying only in the details of the hadronic form factors) to the present data set. In addition to the GK type models we consider a group of models (generically designated DR-GK) that use the analytic approximation of \cite{4} to the dispersion integral approximation for the $\rho$ meson contribution, modified by the hadronic form factors of the type we use with the GK model, and the addition of the $\rho'$ (1450) pole. These additions result in a better fit to the data than we obtain with the GK model \cite{3} and minor variants of the hadronic form factors.

In this paper we fit the world data set for $G_{Ep}$, $G_{Mp}$, $G_{En}$, $G_{Mn}$ and $R_p = \mu_p G_{Ep}/G_{Mp}$. The last quantity, $R_p$, is a direct result of a recent measurement \cite{5} with a polarized electron beam. We find similar results with the GK type models for three different parameterizations of the hadronic form factors, all of the fits being reasonable when the inconsistency of the data, particularly for the neutron, is taken into account (some of the data sets must have large systematic errors, unless the emff oscillate over unnaturally small momentum transfer scales). With the extended type models described above, qualitatively better fits are obtained for four parameterizations of the hadronic form factors.

In Section II we will specify the models and parameters. Section III will summarize the data set and the optimization procedure, while Section IV will present the results in comparison with each other and the original GK fit. We extrapolate to higher momentum transfer and comment on the differences between the models in the extended range. For the three models with the lowest (nearly equal) $\chi^2$ fits to the data and parameters most consistent with other reactions, the differences are small.

If these models are used as input in many-nucleon form factor calculations they will provide stable results consistent with the nucleon data. Discrepancies with the many-nucleon data can then be attributed to deficiencies in the many-body wave function, meson exchange currents or relativistic corrections.

II. NUCLEON EMFF MODELS

The emff of a nucleon are defined by the matrix elements of the electromagnetic current $J_\mu$

$$\langle N(p') \mid J_\mu \mid N(p) \rangle = e \bar{u}(p') \left\{ \gamma_\mu F_1^N(Q^2) + \frac{i}{2m_N} \sigma_{\mu\nu} Q^\nu F_2^N(Q^2) \right\} u(p) \quad (1)$$

where $N$ is the neutron, $n$, or proton, $p$, and $-Q^2 = (p' - p)^2$ is the square of the invariant momentum transfer. $F_1^N(Q^2)$ and $F_2^N(Q^2)$ are respectively the Dirac and Pauli form factors, normalized at $Q^2 = 0$ as

$$F_1^p(0) = 1, \quad F_1^n(0) = 0, \quad F_2^p(0) = \kappa_p, \quad F_2^n(0) = \kappa_n \quad . \quad (2)$$

Expressed in terms of the isoscalar and isovector electromagnetic currents

$$2F_i^p = F_i^{is} + F_i^{iv}, \quad 2F_i^n = F_i^{is} - F_i^{iv}, \quad (i = 1, 2) \quad . \quad (3)$$

The Sachs form factors, most directly obtained from experiment, are then

$$G_{EN}(Q^2) = F_1^N(Q^2) - \tau F_2^N(Q^2)$$

$$G_{MN}(Q^2) = F_1^N(Q^2) + F_2^N(Q^2), \quad \tau = \frac{Q^2}{4m_N} \quad . \quad (4)$$
The model of Gari and Krümpelmann [3] prescribes the following form for the four emff:

\[ F_1^i (Q^2) = \frac{g_\rho}{f_\rho} \frac{m_\rho^2}{m_\rho^2 + Q^2} F_1^i (Q^2) + \left( 1 - \frac{g_\rho}{f_\rho} \right) F_1^D (Q^2) \]

\[ F_2^i (Q^2) = \kappa \frac{g_\rho}{f_\rho} \frac{m_\rho^2}{m_\rho^2 + Q^2} F_2^\rho (Q^2) + \left( \kappa - \kappa \frac{g_\rho}{f_\rho} \right) F_2^D (Q^2) \]

\[ F_1^{is} (Q^2) = \frac{g_\omega}{f_\omega} \frac{m_\omega^2}{m_\omega^2 + Q^2} F_1^{\omega} (Q^2) + \frac{g_\phi}{f_\phi} \frac{m_\phi^2}{m_\phi^2 + Q^2} F_1^{\phi} (Q^2) + \left( 1 - \frac{g_\rho}{f_\omega} \right) F_1^D (Q^2) \]

\[ F_2^{is} (Q^2) = \kappa \frac{g_\omega}{f_\omega} \frac{m_\omega^2}{m_\omega^2 + Q^2} F_2^{\omega} (Q^2) + \kappa \frac{g_\phi}{f_\phi} \frac{m_\phi^2}{m_\phi^2 + Q^2} F_2^{\phi} (Q^2) + \left( \kappa_s - \kappa_s \frac{g_\omega}{f_\omega} - \kappa_\phi \frac{g_\phi}{f_\omega} \right) F_2^D (Q^2) \]

where the pole terms are those of the \( \rho \), \( \omega \), and \( \phi \) mesons, and the final term of each equation is determined by the asymptotic properties of PQCD. The \( F_i^\alpha \), \( \alpha = \rho, \omega, \phi \) are the meson-nucleon form factors, while the \( F_i^D \) are effectively quark-nucleon form factors.

In the final form used by GK, called Model 3 in [3], the above hadronic form factors are parameterized in the following way:

\[ F_1^{\alpha, D} (Q^2) = \frac{\Lambda_{1,D}^2}{\Lambda_{1,D}^2 + \bar{Q}^2 \Lambda_2^2 + \bar{Q}^2} \]

\[ F_2^{\alpha, D} (Q^2) = \left( \frac{\Lambda_{1,D}^2}{\Lambda_{1,D}^2 + \bar{Q}^2} \right)^2 \frac{\Lambda_3^2}{\Lambda_2^2 + \bar{Q}^2} \]

\[ F_1^\phi (Q^2) = F_1^\alpha \left( \frac{Q^2}{\Lambda_1^2 + Q^2} \right)^{1.5}, \quad F_1^\phi (0) = 0 \]

\[ F_2^\phi (Q^2) = F_2^\alpha \left( \frac{\Lambda_1^2 Q^2 + \mu_\phi^2}{\mu_\phi^2 \Lambda_1^2 + Q^2} \right)^{1.5} \]

\[ \bar{Q}^2 = Q^2 \frac{\ln[(\Lambda_2^2 + Q^2)/\Lambda_2^2]}{\ln(\Lambda_2^2/\Lambda_{QCD}^2)}, \quad \text{where} \quad \alpha = \rho, \omega. \]

This parameterization, together with Eq. (5), guarantees that the normalization conditions of Eq. (2) are met and that asymptotically

\[ F_1^i \sim [Q^2 \ln(Q^2/\Lambda_{QCD}^2)]^{-2} \]

\[ F_2^i \sim F_1^i / Q^2 \]

\[ i = is, iv \]

as required by PQCD. When fitted to the data set described in section III, the result is here called Model GK(3).

In their Model 1 (fitted only to the proton data) GK associated the helicity flip hadronic form factors, \( F_2 \), with the quark-gluon scale cut-off \( \Lambda_2 \). However in model 3, in fitting to the available data, they chose to associate the helicity flip with the meson scale cut-off \( \Lambda_1 \), as incorporated in Eq. (6). To investigate the effect of this change we also fit our data set, in Model GK(1), with the hadronic form factors of GK Model 1, for which

\[ F_2^{\alpha, D} = \frac{\Lambda_{1,D}^2}{\Lambda_{1,D}^2 + \bar{Q}^2} \left( \frac{\Lambda_2}{\Lambda_2^2 + Q^2} \right)^2 \]

\[ F_2^{\alpha, D} = \frac{\Lambda_{1,D}^2}{\Lambda_{1,D}^2 + \bar{Q}^2} \left( \frac{\Lambda_2}{\Lambda_2^2 + Q^2} \right)^2 \]
replaces the expressions in Eq. (6).

In both of the above parameterizations the logarithmic $Q^2$ dependence of PQCD is approached through a form factor determined by the $\Lambda_2$ and $\Lambda_{QCD}$ cut-offs. In our Model GK’(1) we replace $\Lambda_2$ with $\Lambda_D$ for that factor which relates to the quark-nucleon vertex:

$$\tilde{Q}^2 = Q^2 \frac{\ln[(\Lambda_D^2 + Q^2)/\Lambda_{QCD}^2]}{\ln(\Lambda_D^2/\Lambda_{QCD}^2)}.$$  \hspace{1cm} (9)

Otherwise Model GK’(1) is the same form as Model GK(1). A similar replacement was attempted for Model GK(3), but the best fit was substantially worse with the modification.

The next group of models replaces the $\rho$ meson pole terms in $F_{1iv}^i$ and $F_{2iv}^i$ (Eq. 5) with the $\rho'$ (1450) meson pole term, and adds the $\rho$ meson term from the dispersion relation in approximate analytic form [4]:

$$F_{1iv}^i(Q^2) = N \frac{1.0317 + 0.0875(1 + Q^2/0.3176)^{-2}}{(1 + Q^2/0.5496)} F_1^\rho(Q^2)$$

$$+ \frac{g_\rho}{f_\rho} \frac{m_\rho^2}{m_{\rho'}^2 + Q^2} F_1^\rho(Q^2) + (1 - 1.1192 N - \frac{g_\rho}{f_\rho}) F_1^D(Q^2)$$

$$F_{2iv}^i(Q^2) = N \frac{5.7824 + 0.3907(1 + Q^2/0.1422)^{-1}}{(1 + Q^2/0.5362)} F_2^\rho(Q^2)$$

$$+ \frac{\kappa_\rho g_\rho}{f_\rho} \frac{m_\rho^2}{m_{\rho'}^2 + Q^2} F_2^\rho(Q^2) + (\kappa_\nu - 6.1731 N - \kappa_\rho \frac{g_\rho}{f_\rho}) F_2^D(Q^2) \hspace{1cm} (10)$$

For $N = 1$ the numerical values in Eq. 10 are those of [4] and are similar to those of [2]. They are determined by pion form factor and pion-nucleon p-wave phase shift input into the dispersion relation [4]. Because this input has uncertainties and is truncated at high momentum transfer, we considered the effect of an overall normalization factor $N$ (the same for $F_{1iv}^i$ and $F_{2iv}^i$).

Because of the dispersion relation $\rho$ meson term, these models are labeled by DR-GK. Model DR-GK(3) has the hadronic form factors of Model GK(3) (Eq. (6)). Model DR-GK(1) uses the hadronic form factors of Model GK(1) (Eq. (8)). Model DR-GK’(1) and DR-GK’(3) are like Models DR-GK(1) and DR-GK(3) respectively but use the $\tilde{Q}^2$ of Eq. (9).

The best fit value of $N$ varied between 0.78 and 0.94 for these models, but $\chi^2$ decreased substantially only for model DR-GK(3). Consequently we present the results for the other three models with $N = 1$, only introducing the extra parameter for Model DR-GK(3), now called DRN-GK(3).

**III. DATA BASE AND FITTING PROCEDURE**

The data for $G_{M_p}$ is from [6–13]. The $G_{E_p}$ data is that of [6,7,10,12–14].

The data sources for $G_{M_n}$ are [12,15–24]. The $G_{E_n}$ data is derived from [12,18,22,23,25–32]. Recent small revisions in the published values of [27,29,31] are included [33]. Quasi-elastic deuteron and $^3$He data has been included, but the elastic deuteron data
has been omitted because of its great sensitivity to the deuteron wave function. Another datum is the slope \( dG_{En}/dQ^2 (Q^2 = 0) = 0.0199 \pm 0.0003 \text{ fm}^2 \), as determined by thermal neutron scattering [34].

The data set for the ratio \( R_p \) includes not only [5], which measures the ratio directly in a polarization experiment, but also the data of [11], which extracts the ratio from unpolarized data dominated by the magnetic scattering.

There are 11 free parameters in each of the models; the three \( g_m/f_m \) and the three \( \kappa_m \) for the \( \rho \) or \( \rho' \), \( \omega \) and \( \phi \) mesons, \( \Lambda_1, \Lambda_2, \Lambda_D, \Lambda_{QCD} \) and \( \mu_\phi \). Model DRN-GK(3) has a 12th parameter, \( N \). They were fitted by minimizing the value of \( \chi^2 \) for all the data using a Mathematica program that incorporates the Levenberg-Marquardt method.

### IV. RESULTS

Table I presents the “best fit” parameters to the present data set for the above 7 models. The parameters of [3] as fitted to the data set used in that reference are included in parentheses for Models GK(1) and GK(3). For all but two of the seven models “best fit” implies, as usual, the lowest local minimum in the search over the parameters. However for Models DRN-GK(3) and DR-GK(3) the minimum is associated with indefinitely increasing negative values of \( \kappa'_\rho \). But \( \chi^2 \) decreases negligibly (<1%) after reaching reasonable values of \( \kappa'_\rho \) which we choose to represent those two models.

As the models are simplifications of the actual physical situation, it is not required that the fitted parameters correspond to the values expected of them from measurement of independent observables. However those models for which the parameters are near those expectations are most self consistent. Only four of the models, GK(1), DRN-GK(3), DR-GK(1) and DR-GK(1) have \( \Lambda_{QCD} \) in the range of 100–300 MeV consistent with high energy experiment. The value of \( \kappa_\rho \) is only a free parameter for the three GK models. Its value is reasonable for GK(3) and GK(3) but is much too large for GK(1). The combination of \( \Lambda_{QCD} \) and \( \kappa_\rho \) leaves only the above three DR-GK models. Unfortunately none of the models have the expected small negative value of \( \kappa_\omega \). This is probably indicative that at least one higher mass isoscalar meson is important to the form factor description (the \( \omega \) and \( \phi \) meson widths are too small to require a modification of the pole representation). Rather than further complicating the models, the isoscalar pole terms are to be regarded as effectively representing the more complicated situation. The stability and adequacy of the fits is an indication that the form factors with more poles would be similar to those already obtained.

In Table II the values of \( \chi^2 \) are listed for all the models and the contribution from each of the five form factor classes of measurement (see beginning of Section III) are detailed. For the GK type models \( 348.5 < \chi^2 < 352.8 \) and for the DR-GK type \( 322.5 < \chi^2 < 327.1 \). So the quality of the fit is essentially the same within a model type, but the models that add the \( \rho \) meson contribution determined by dispersion relations (and substitute the parameterized \( \rho' \) contribution for the parameterized \( \rho \)) are significantly better fits to the data. Within a model type there are large differences in the fitted parameters and important differences in the distribution of \( \chi^2 \) contributions among the different form factors, in spite of the small variation of the total values of \( \chi^2 \). But the \( \chi^2 \) contributions differ little for the 3 models, DRN-GK(3), DR-GK(1) and DR-GK(1), favored by their physical values of \( \Lambda_{QCD} \) and dispersion representation of the \( \rho \) meson contribution. We also note that while Model...
DRN-GK(3) has the smallest value of $\chi^2$, it is the only one incorporating a 12th parameter, the $\rho$ normalization $N$. With $N = 1$ the best $\chi^2$ for this model is 375.1. By contrast the value of $\chi^2$ only decreases by 3 if $N$ is allowed to vary in the other 3 DR-GK type models.

We note that with the parameters of [3] the value of $\chi^2$ with respect to the present data set is 2.4 times larger than the best fit value for GK(1) and 3.0 times larger for GK(3). Therefore the data accumulated since 1992 has made an important difference. We also note that while the best fit values of $\chi^2$ are about twice the number of degrees of freedom, this excess is mostly due to clear inconsistencies in the data sets, most particularly for $G_{Mn}$ at $Q^2 < 0.8 \text{ GeV}/c^2$. The displacement of nearby data points well beyond their given error bars is evident in the figures below. [4] quotes a $\chi^2$/datum of 1.1. As their fit is similar to those given here, this disparity may be due not only to the data accumulated since 1995 but also to the compactification in their case of many low momentum transfer points into slopes of the form factors at the origin. Indeed, for the DRN-GK(3) model two points, at 0.33 and 0.81 GeV$^2$/c$^2$, deviate in opposite directions for $G_{En}$ contributing 43.6 to a $\chi^2$ of 63.9. For the same model 8 points, ranging from 0.24 to 0.81 GeV$^2$/c$^2$, deviate from $G_{Mn}$ with both signs and contribute 89.6 to a $\chi^2$ of 120.1. The results are similar for the other models. This makes it clear that without the severe fluctuations of the experimental values outside their stated errors the fits presented here have achieved a value of $\chi^2$ close to the number of degrees of freedom.

The following Figs. 1–5 display the results for $G_{Mp}$, $G_{Ep}$, $R_p$, $G_{Mn}$, and $G_{En}$, in that order. $G_{Ep}$ and $G_{En}$ are normalized to the dipole form factor $G_d = (1 + Q^2/0.71)^{-2}$. $G_{Mp}$ ($G_{Mn}$) are normalized to the product of $G_d$ and $\mu_p$ ($\mu_n$). The Models GK(3), GK(1) and GK′(1) as fitted to the present data are compared in Figs. 1(a)–5(a), while in Figs. 1(b)–5(b) the same GK(3) and GK(1) are compared to those models with the parameters originally obtained in [3]. Figs. 1(c)–5(c) compare the results of Models DRN-GK(3), DR-GK(1), DR-GK′(1) and DR-GK′(3) with the data. Figs. 6(a)–(c) show how all 7 models extrapolated up to $Q^2 = 8 \text{ GeV}^2/c^2$ for $R_p$ and the neutron form factors, for which data is now restricted to $Q^2 < 4 \text{ GeV}^2/c^2$. For those observables we may expect data at higher momentum transfers in the near future.

For $G_{Mp}$ (Fig. 1) all the models agree closely over the very large momentum transfer range up to 31 GeV/c$^2$. As shown in Fig. 1(b) even the substantial change in the fitted parameters from those of [3], which cause major differences in other form factors, make only a moderate difference here. But it should be noted that GK(3)-original is substantially lower at the peak near 2.5 GeV/c$^2$ than all the other models. In this same momentum transfer region there is also a dichotomy in the experimental points. There are some that peak near 1.06 ([6,11]) and others that peak near 1.03 ([7,8]). The fits of all the present models favor the higher values.

The three GK type models are very close for $G_{Ep}$, while the 4 DR-GK type models have more spread at $Q^2 > 5 \text{ GeV}/c^2$ (but still insignificant compared to experimental errors in that region). GK(3)-original is remarkable for its divergence from the present fits and GK(1)-original. This may be due to an emphasis on fitting the data of [7] at $Q^2$ of 2.003, 2.497, and 3.007 GeV$^2$/c$^2$, which were published shortly before [3] and in part motivated the variation of the GK(3) parameterization from that of GK(1). This data is substantially higher in value than other data sets in the same range of momentum transfer [6,11,13] that were published earlier.
The presented $R_p$ data in Figs. 3 is independent of the $G_{Mp}$ and $G_{Ep}$ data of Figs. 1 and 2. The experiment of [11] and the polarization data of [5], which measure this ratio directly, are included only in these figures. It is noticeable in comparing Figs. 3(a) and 3(c), and evident from the $\chi^2$ values (Table II), that the DR-GK model fits are somewhat better than those of the GK models. Fig. 3(b) shows, as in the case of $G_{Ep}$, that the GK(3)-original model was too constrained by one particular set of data. The extrapolation of this fit to 8 GeV$^2$/c$^2$ for $R_p$, Fig. 6(a), shows that this observable may be able to discriminate between the models at the higher $Q^2$ if the experimental errors do not increase at the higher momentum transfers. Even the models preferred for their parameters, DRN-GK(3), DR-GK(1) and DR-GK'(1), differ by as much as 0.1 at 8 GeV$^2$/c$^2$.

Examining Figs. 4(a) and 4(c) one notes that while the overall fit to the $G_{Mn}$ data is about the same for all models, the GK models converge near $Q^2 = 4$ GeV$^2$/c$^2$ while the DR-GK models diverge there. Extrapolating to 8 GeV$^2$/c$^2$, Fig. 6(b), the parameter favored models differ by almost 0.2, an accuracy that may be more achievable than the above mentioned split for $R_p$.

For $G_{En}$ Figs. 5(a) and 5(c) show the improved fit of the DR-GK over the GK type models is most evident at the higher $Q^2$. Extrapolating to 8 GeV$^2$/c$^2$, Fig. 6(c), there is little difference among the three parameter favored models.

V. CONCLUSIONS

Good fits to the nucleon electromagnetic form factor data are achieved for seven variations and extensions of the Gari-Krümpelmann type model [3] which preserves VMD at low momentum transfers and PQCD behavior at high momentum transfers. The models all have simple analytic forms which are easily incorporated into few-nucleon form-factor predictions.

The four models which include the width of the $\rho$ meson, by use of dispersion relations, and the $\rho'(1450)$ meson pole are a substantially better fit to the data than the $\rho$, $\omega$, and $\phi$ meson pole only GK models. The fitted parameters of three of the four, DRN-GK(3), DR-GK(1), and DR-GK'(1), have values most compatible with independent evaluations. For these three models the predictions for the nucleon electromagnetic form factors are not only quantitatively similar over the range of the present experimental data, but differ little when $R_p$, $G_{Mn}$, and $G_{En}$ are extrapolated to 8 GeV$^2$/c$^2$. Consequently only small differences due to the nucleon form factors are expected in predictions of deuteron and other few-nucleon electromagnetic form factors. This will eliminate a major ambiguity in the extraction of information about the few-nucleon wave functions and meson-exchange current effects. Precise data in the $Q^2 = 4$–8 GeV$^2$/c$^2$ range may eventually further narrow the uncertainty.

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FIG. 1. $G_{M_P}$ normalized to $\mu_p G_d$. (a) Comparison of the models GK(3)[solid], GK(1)[dotted] and GK'(1)[dash-dotted] with the data. (b) Comparison of GK(3)[solid] and GK(1)[dotted] with the same models and the parameters of [3], GK(3)-original[dash-dotted] and GK(1)-original[dashed]. (c) Comparison of models DRN-GK(3)[solid], DR-GK(1)[dotted], DR-GK'(1)[dash-dotted] and DR-GK'(3)[dashed] with the data.
FIG. 2. $G_{Ep}$ normalized to $G_d$. (a) Comparison of the models GK(3)[solid], GK(1)[dotted] and GK'(1)[dash-dotted] with the data. (b) Comparison of GK(3)[solid] and GK(1)[dotted] with the same models and the parameters of [3], GK(3)-original[dash-dotted] and GK(1)-original[dashed]. (c) Comparison of models DRN-GK(3)[solid], DR-GK(1)[dotted], DR-GK'(1)[dash-dotted] and DR-GK'(3)[dashed] with the data.
FIG. 3. $R_p$, the ratio $\mu_p G_{Ep}/G_{Mp}$. (a) Comparison of the models GK(3)[solid], GK(1)[dotted] and GK'(1)[dash-dotted] with the data. (b) Comparison of GK(3)[solid] and GK(1)[dotted] with the same models and the parameters of [3], GK(3)-original[dash-dotted] and GK(1)-original[dashed]. (c) Comparison of models DRN-GK(3)[solid], DR-GK(1)[dotted], DR-GK'(1)[dash-dotted] and DR-GK'(3)[dashed] with the data.
FIG. 4. $G_{Mn}$ normalized to $\mu_n G_d$. (a) Comparison of the models GK(3)[solid], GK(1)[dotted] and GK'(1)[dash-dotted] with the data. (b) Comparison of GK(3)[solid] and GK(1)[dotted] with the same models and the parameters of [3], GK(3)-original[dash-dotted] and GK(1)-original[dashed]. (c) Comparison of models DRN-GK(3)[solid], DR-GK(1)[dotted], DR-GK'(1)[dash-dotted] and DR-GK'(3)[dashed] with the data.
FIG. 5. $G_{En}$ normalized to $G_d$. (a) Comparison of the models $GK(3)[$solid$]$, $GK(1)[$dotted$]$ and $GK'(1)[$dash-dotted$]$ with the data. (b) Comparison of $GK(3)[$solid$]$ and $GK(1)[$dotted$]$ with the same models and the parameters of [3], $GK(3)-original[$dash-dotted$]$ and $GK(1)-original[$dashed$]$. (c) Comparison of models $DRN-GK(3)[$solid$]$, $DR-GK(1)[$dotted$]$, $DR-GK'(1)[$dash-dotted$]$ and $DR-GK'(3)[$dashed$]$ with the data.
FIG. 6. Extrapolation to $Q^2 = 8 \text{ GeV}^2/c^2$. Comparison of the models DRN-GK(3)[solid], DR-GK(1)[dotted], DR-GK’(1)[dash-dotted], DR-GK’(3)[dashed], GK(3)[dash-double dotted], GK(1)[long dashes] and GK’(1)[double dash-dotted]. (a) $R_p$, the ratio $\mu_p G_{Ep}/G_{Mp}$. (b) $G_{Mn}$ normalized to $\mu_n G_d$. (c) $G_{En}$ normalized to $G_d$. 

$Q^2 (\text{GeV}^2/c^2)$
TABLE I. Model parameters. Common to all models are $\kappa_v = 3.706$, $\kappa_s = -0.12$, $m_\rho = 0.776$ GeV, $m_\omega = 0.784$ GeV, $m_\phi = 1.019$ GeV and $m_{\rho'} = 1.45$ GeV. Parentheses contain the values of [3].

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<td>$N$</td>
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<td>1.0 (b)</td>
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(a) $\rho^{(\prime)}$ signifies the $\rho$ meson for the GK models and the $\rho'(1450)$ meson for the DR-GK models. (b) not varied
TABLE II. Contributions to the standard deviation, $\chi^2$, from each data type for each of the models. The number of data points, $n$, is listed for each data type. Parentheses contain results of ref. [3] parameters.

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<th>Data type</th>
<th>$n$</th>
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<th>GK(1)</th>
<th>GK'(1)</th>
<th>DRN-GK(3)</th>
<th>DR-GK(3)</th>
<th>DR-GK'(3)</th>
<th>DR-GK(1)</th>
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<td>(71.9)</td>
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