Neutrino masses, anomalous U(1) gauge symmetry 
and doublet-triplet splitting

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Abstract

We propose an attractive scenario of grand unified theories in which doublet-triplet splitting is beautifully realized in \(SO(10)\) unification using Dimopoulos-Wilczek mechanism. The anomalous \(U(1)_A\) gauge symmetry plays essential roles in the doublet-triplet splitting mechanism. It is interesting that the anomalous \(U(1)_A\) charges determine the unification scale and mass spectrum of additional particles as well as the order of Yukawa couplings of quarks and leptons. For the neutrino sector, bi-maximal mixing angles are naturally obtained and proton decay via dimension 5 operators is suppressed. It is suggestive that the anomalous \(U(1)_A\) gauge symmetry motivated by superstring theory excellently solves the two biggest problems in grand unified theories, fermion mass hierarchy problem and doublet-triplet splitting problem.

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1 Introduction

The standard model is consistent with all the present experiments. However, there are many reasons for thinking that it is not the final theory, for examples, it does not explain the miracle of anomaly cancellation between quarks and leptons, hierarchies of gauge and Yukawa couplings, charge quantization, etc. Therefore we have a strong motivation for examining the idea of grand unified theories (GUT) [1] in which the quarks and leptons are beautifully unified in several multiplets in a simple gauge group. Three gauge groups in the standard model are unified to a simple gauge group at a GUT scale which is considered as just below the Planck scale. Once we accept the higher scale than the weak scale, it is one of the most promising way to introduce supersymmetry(SUSY) around the weak scale to stabilize the weak scale. We are thus led to examine SUSY GUT [2].

However, it is not so easy to obtain a realistic SUSY GUT. One of the reasons is that it is difficult to obtain realistic fermion mass pattern in a simple way because unified multiplet gives strong constraints on the Yukawa couplings of quarks and leptons. Moreover, one of the hardest obstacles in building a realistic GUT is “Doublet-triplet(DT) splitting problem”. Generally a fine tuning is required to obtain the light $SU(2)_{L}$ doublet Higgs multiplet of weak scale while keeping the triplet Higgs heavy enough to suppress the dangerous proton decay.

For the former problem, by using the clue on neutrino masses from recent progress on neutrino experiments [3], there are several impressive papers [4, 5, 6, 7, 8] to explain the order of the Yukawa couplings though most of them need tuning parameters to explain the large mixing angle for the atmospheric neutrino. It is natural to examine $SO(10)$ or higher gauge group because every quarks and leptons including right-handed neutrino can be unified in a single multiplet, which is important to discuss neutrino masses.

There are several attempts to evade the latter problem [9, 10]. One of the most promising ways to realize DT splitting in the $SO(10)$ SUSY GUT is Dimopoulos-Wilczek(DW) mechanism [10, 11, 12, 13]. If the adjoint field $A$ of $SO(10)$ has a vacuum expectation value (VEV) $\langle A \rangle = i \tau_2 \times \text{diag}(v, v, v, 0, 0)$, then $SO(10)$ is broken to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and the VEV can give masses to the triplet Higgs but not to the doublet Higgs. Unfortunately, in order to realize the DW mechanism, rather complicated Higgs structure is required [13]. The reason is simple. The DW mechanism works essentially in a larger rank unified gauge group like $SO(10)$ GUT than that of the standard gauge group. For example, since the adjoint field in $SU(5)$ GUT is traceless and the rank is the same as that of the standard gauge group, it is impossible to realize DW mechanism. On the other hand, we have to introduce VEVs of spinors $C$ and $\bar{C}$ or other multiplets to break the remaining gauge group to the standard gauge group, because the adjoint VEV does not reduce the rank of $SO(10)$ gauge group. If the VEV of the spinor appears in the equation of motion which determines the adjoint VEV, the VEV of the adjoint field generally deviates from the form required for DW mechanism. On the other hand, if the adjoint and spinor Higgs sectors are not coupled to each other in the superpotential, then pseudo Nambu-Goldstone (PNG) fields $(3, 2)_{1/6} + (3, 1)_{-2/3} + \text{h.c.}$ of $SU(3)_C \times SU(2)_L \times U(1)_Y$ appear.

To avoid this problem, the adjoint field must couple to the spinor to obtain the mass of the PNG fields keeping DW mechanism. Is it possible? The answer is Yes. This is because for the equation of motion the first derivative of the superpotential is important, while for the mass term, the second derivative is essential. However, usually it requires rather complicated Higgs sector. Recently Chacko and Mohapatra proposed a simpler model [12] in which they introduce two 45, one 54, a pair of 16 and $\overline{16}$ and two 10 just for Higgs sector. Several years ago, Barr
and Raby [11] examined a minimal DT splitting model which includes a single 45, two pairs of 16 and \( \overline{16} \) and two 10 for the Higgs sector. This simple model is very attractive. However, they must introduce many singlets whose VEVs are not determined classically and must be given by hand. Moreover, in their model, dangerous terms are not forbidden by symmetry. Once the mass term \( A^2 \) and the non-renormalizable term \( A^4 \), which are essential for their model, are allowed, there is no reason to forbid higher power terms \( A^{2n} \). With these terms, since a lot of (infinite number of) degenerate undesired vacua appears, it is unnatural to obtain the desired DW vacuum.

In this paper, we propose a more attractive DT splitting scenario in which GUT scale is automatically determined and the higher terms are naturally forbidden. In this scenario, the anomalous \( U(1)_A \) gauge symmetry plays essential roles. Using this mechanism, a GUT with realistic Yukawa couplings can be constructed in a simple way. This model has interesting structure of quark and lepton mass matrices, which predicts bi-maximal mixing in neutrino sector.

2 Anomalous \( U(1)_A \) gauge symmetry and neutrino masses

First let us recall the anomalous \( U(1)_A \) gauge symmetry. It is well-known that some low energy effective theories of the string theory include the anomalous \( U(1)_A \) gauge symmetry which has non-zero anomalies, such as the pure \( U(1)_A^3 \) anomaly, mixed anomalies with the other gauge groups \( G_a \), and the mixed gravitational anomaly [14]. These anomalies are canceled by combining the nonlinear transformation of the dilaton chiral supermultiplet \( D \) with the gauge transformation of the \( U(1)_A \) vector supermultiplet \( V_A \),

\[
V_A \rightarrow V_A + \frac{i}{2} \left( \Lambda - \Lambda^\dagger \right),
\]

\[
D \rightarrow D + \frac{i}{2} \delta_{GS} \Lambda,
\]

where \( \Lambda \) is a parameter chiral superfield. This cancellation occurs because the gauge kinetic functions for \( V_A \) and the other vector supermultiplets \( V_a \) are given by

\[
\mathcal{L}_{\text{gauge}} = \frac{1}{4} \int d^2 \theta \left[ k_A D W_A^\alpha W_A^\alpha + k_a D W_a^\alpha W_a^\alpha \right] + \text{h.c.},
\]

where \( W_A^\alpha \) and \( W_a^\alpha \) are super field-strength of \( V_A \) and \( V_a \), respectively, and \( k_A \) and \( k_a \) are Kac-Moody levels of \( U(1)_A \) and \( G_a \), respectively. And the square of the gauge coupling is written in terms of the inverse of the VEV of the dilaton, i.e., \( k_a \langle D \rangle = 1/g_a^2 \).

The parameter \( \delta_{GS} \) in Eq.(2.2) is related to the conditions for the anomaly cancellations,

\[
2\pi^2 \delta_{GS} = \frac{C_a}{k_a} = \frac{1}{3k_A} \text{tr} Q_A^3 = \frac{1}{24} \text{tr} Q_A.
\]

The last equality is required by the cancellation of the mixed gravitational anomaly. These anomaly cancellations are understood in the context of the Green-Schwarz mechanism [15].

One of the most interesting features of the anomalous \( U(1)_A \) gauge symmetry is that it induces the Fayet-Iliopoulos \( D \)-term (F-I term) radiatively[14]. Since the Kähler potential \( K \)

\[
\text{tr} G_a T(R) Q_A.
\]
for the dilaton $D$ must be a function of $D + D^\dagger - \delta_{GS} V_A$ for the $U(1)_A$ gauge invariance, the F-I term can be given as follows.

$$\int d^4 \theta K(D + D^\dagger - \delta_{GS} V_A) = \left(-\frac{\delta_{GS} K'}{2}\right) D_A + \cdots \equiv \xi^2 D_A + \cdots,$$

(2.5)

where we take the sign of $Q_A$ so that $\xi^2 > 0$. If the Kähler potential for the dilaton is given by $K = -\ln(D + D^\dagger - \delta_{GS} V_A)$, which can be induced by a stringy calculation at tree level, $\xi^2$ can be estimated as

$$\xi^2 = \frac{g_s^2 \text{tr} Q_A}{192 \pi^2},$$

(2.6)

where $g_s^2 = 1/\langle D \rangle$. Notice that since the $\xi^2$ is induced radiatively, the parameter $\xi$ is expected to be smaller than the Planck scale.

When some superfields $\Phi_i$ have anomalous $U(1)_A$ charges $\phi_i$, the scalar potential becomes

$$V_{\text{scalar}} = \frac{g_A^2}{2} \left( \sum_i \phi_i |\Phi_i|^2 + \xi^2 \right)^2,$$

(2.7)

where $1/g_A^2 = k_A \langle D \rangle$. If one superfield has a negative anomalous $U(1)_A$ charge, it gets the VEV. Below we assume the existence of the field $\Phi$ with the negative charge and normalize the anomalous $U(1)_A$ charges so that $\Phi$ has the charge $-1$. Then the VEV of the scalar component $\Phi$ is given by

$$\langle \Phi \rangle = \xi \equiv \lambda M_P,$$

(2.8)

which breaks the anomalous $U(1)_A$ gauge symmetry. ($M_P$ is some gravity scale and usually taken as the reduced Planck mass, $1/\sqrt{8 \pi G_N}$. In the followings, we take the unit $M_P = 1$.)

Next we discuss the fermion masses. In general, the Yukawa hierarchy can be explained by introducing a flavor dependent $U(1)$ symmetry [16, 17, 18, 19]. We can adopt the anomalous $U(1)_A$ gauge symmetry as this $U(1)$ symmetry. Suppose that the standard model matter fields $Q_i$, $U_i^c$, $D_i^c$, $L_i$, $E_i^c$, $H_u$ and $H_d$ have the anomalous $U(1)_A$ charges $q_i$, $u_i$, $d_i$, $l_i$, $e_i$, $h_u$ and $h_d$, respectively, which are taken as non-negative integers here. If the field $\Phi$ with the charge $-1$ is a singlet under the standard model gauge symmetry, the superpotential can be written as

$$W \sim \Phi^{q_i + u_j + h_u} H_u Q_i U_j^c + \cdots.$$  

(2.9)

In this paper, for simplicity, we usually do not write $O(1)$ coefficients explicitly. Since the scalar component of $\Phi$ has the VEV in (2.8), we get the hierarchical mass matrices,

$$(M_u)_{ij} \sim \lambda^{q_i + u_j + h_u} \langle H_u \rangle = V_L^u \begin{pmatrix} m_u \\ m_c \\ m_t \end{pmatrix} V_R^{u\dagger},$$

(10.10)

$$(M_d)_{ij} \sim \lambda^{q_i + d_j + h_d} \langle H_d \rangle = V_L^d \begin{pmatrix} m_d \\ m_s \\ m_b \end{pmatrix} V_R^{d\dagger},$$

(10.11)

where $V_{L,R}^u,d$ are $3 \times 3$ unitary diagonalizing matrices, and $(V_L^u)_{ij} \sim \lambda^{q_i - q_j}$, $(V_R^u)_{ij} \sim \lambda^{u_i - u_j}$ and so on. The diagonalized masses of quarks, $m_f$, are given as follows: $(m_u)_i \sim \lambda^{q_i + u_i + h_u} \langle H_u \rangle$ and

$\footnotesize{\text{Throughout this paper we denote all the superfields with uppercase letters and their anomalous } U(1)_A \text{ charges with the corresponding lowercase letters.}}$
\((m_d)_i \sim \lambda^{q_i+d_i+h_d} \langle H_d \rangle\). The Cabbibo-Kobayashi-Maskawa matrix [20] is given by

\[
V_{\text{CKM}} = V_L^\dagger V_L \sim \begin{pmatrix}
1 & \lambda^{q_1-q_2} & \lambda^{q_1-q_3} \\
\lambda^{q_2-q_1} & 1 & \lambda^{q_2-q_3} \\
\lambda^{q_3-q_1} & \lambda^{q_3-q_2} & 1
\end{pmatrix},
\]

which is determined solely by the charges of the left-handed quarks, \(q_i\). The relation \(V_{12} V_{23} \sim V_{13}\) can naturally be understood with this mechanism and if we take \(q_i = (3, 2, 0)\) and \(\lambda \sim 0.2\), we can reproduce the measured value.

If there are right-handed neutrinos \(N_i^e\) with \(U(1)_A\) charges \(n_i\), Dirac and Majorana neutrino masses are also given by

\[
(M_D)_{ij} \sim \lambda^{l_i+n_j+h_u} \langle H_u \rangle, \quad (M_R)_{ij} \sim M_m \lambda^{n_i+n_j}.
\]

Through the see-saw mechanism [21] the left-handed neutrino mass matrix is given by

\[
(M_\nu)_{ij} \sim \lambda^{l_i+l_j+2h_u} \frac{\langle H_u \rangle^2}{M_m}.
\]

The mixing matrix for the lepton sector [22] is induced as for the quark sector:

\[
V_{\text{MNS}} = V_L^\dagger V_L^\dagger \sim \begin{pmatrix}
1 & \lambda^{|l_1-l_2|} & \lambda^{|l_1-l_3|} \\
\lambda^{|l_2-l_1|} & 1 & \lambda^{|l_2-l_3|} \\
\lambda^{|l_3-l_1|} & \lambda^{|l_3-l_2|} & 1
\end{pmatrix}.
\]

This matrix is also determined only by the charges of the left-handed leptons, \(l_i\). If we take \(l_i = (2, 2, 2)\), it generally gives large mixing angles among three generations and can give the bi-maximal mixing angles. It is called anarchy solution for large mixing angles in neutrino sector [24].

Until now, we have examined only terms with the non-negative total anomalous \(U(1)_A\) charge, but what happens if the total charge becomes negative? The terms with negative total anomalous \(U(1)_A\) charge are forbidden because of the anomalous \(U(1)_A\) gauge symmetry, while the terms with positive or zero charge are allowed because the negative charge of the singlet \(\Phi\) can compensate the positive charge as discussed above. The vanishing coefficients because of the anomalous \(U(1)_A\) gauge symmetry are called “SUSY zero”. This feature plays an essential role in our mechanism of DT splitting.

In Eq. (2.14), we have to introduce the Majorana mass scale \(M_m\) smaller than \(M_P\) by hand. If we simply take \(M_m \sim M_P\), which is the unique scale in this model, the upper bound of the neutrino mass becomes \(O(10^{-5}\text{eV})\), which is much smaller than the expected values \(0.04 \sim 0.07\) eV for the atmospheric neutrino anomaly. Here we naively expect that in the effective term (2.15), the factor \(\lambda^{l_i+l_j+2h_u}\) cannot be larger than 1, because terms with negative total \(U(1)_A\) charge \((l_i + l_j + 2h_u < 0)\) must be forbidden by SUSY zero mechanism. If we adopt the anomalous \(U(1)_A\) gauge symmetry as flavor symmetry which induces quark and lepton masses, we have to explain why the mass of the right-handed neutrinos is much smaller than that expected from the anomalous \(U(1)_A\) charges, or to find an loop hole to avoid the “SUSY zero” mechanism. One might think that introducing a singlet whose VEV gives the mass of the right-handed neutrino can evade this problem. Unfortunately, this solution does not work well if the \(F\)-flatness condition determines the VEV. This is because as we will discuss in the next section,
the VEV of the singlet $S$ is generally determined by the anomalous $U(1)_A$ charge $s$ as $\langle S \rangle = \lambda^{-s}$, which does not improve the situation. Of course we can always give the right-handed neutrino scale determined by some other conditions, for example, by $D$-flatness conditions or by SUSY breaking terms or by some dynamical mechanism. In this paper, however, we examine more attractive solutions. One of them is very simple. Notice that even if we shift the anomalous $U(1)_A$ charges $(q_i, u_i, d_i, l_i, e_i, n_i, h_u, h_d)$ to $(q_i+n, u_i+n, d_i+n, l_i+n, e_i+n, n_i+n, h_u-2n, h_d-2n)$, the Dirac mass matrices of quarks and leptons remain unchanged. On the other hand the right-handed neutrino masses become smaller by factor $\lambda^{2n}$ for positive $n$. Then the neutrino masses can be enhanced by factor $\lambda^{-2n}$. Notice that even if the total charge $l_i + l_j + 2h_u$ is negative, the term in Eq. (2.15) is allowed. It means that the “SUSY zero ” mechanism does not work in the effective interaction. In the effective theory, which is obtained by integrating heavy fields with positive anomalous $U(1)_A$ charges, terms with negative total charges can be induced. It is easily shown that the induced terms with negative total charges do not contribute the $F$-flatness conditions if the heavy fields have vanishing VEVs. This observation is important for DT splitting models discussed in this paper, because SUSY zero mechanism plays an essential role to determine VEVs. Notice that integrating heavy fields with masses of the Planck scale do not induce such terms, because the total $U(1)_A$ charge of the mass term is zero. This solution inevitably leads to the negative charge of the Higgs field, which will be required also by DT splitting mechanism proposed in this paper. For the other solution, which can give a smaller mass to right-handed neutrino than that expected from the anomalous $U(1)_A$ charge, it is essential that the right-handed neutrinos have charges of a gauge interaction. We will return to this point in the next section.

3 Relation between VEVs and anomalous $U(1)_A$ charges, and neutrino masses

In this section, we argue how VEVs are determined by the anomalous $U(1)_A$ quantum numbers. First of all, the VEV of a gauge invariant operator with positive anomalous $U(1)_A$ charge must vanish. Otherwise, the mechanism of SUSY zero does not work since such a VEV can compensate the negative $U(1)_A$ charge of the term. At this stage, such an undesired vacuum is not forbidden. However, it is shown soon that such a vacuum requires VEV larger than the Planck scale. If the cutoff is rigid and the VEV larger than the cutoff is not allowed by some reasons, then the condition for SUSY zero mechanism is automatically satisfied.

Next we show that the VEV of a gauge invariant operator $O$ is generally determined by its $U(1)_A$ charge $o$ as $\langle O \rangle = \lambda^{-o}$ if the $F$-flatness condition determines the VEV. For simplicity, we examine this relation using singlets fields $Z_i$ with the anomalous $U(1)_A$ charge $z_i$. The general superpotential is written as

$$W = \sum_i \lambda^{z_i} Z_i + \sum_{i,j} \lambda^{z_i+z_j} Z_i Z_j + \cdots$$

$$= \sum_i \tilde{Z}_i + \sum_{i,j} \tilde{Z}_i \tilde{Z}_j + \cdots,$$

where $\tilde{Z}_i \equiv \lambda^{z_i} Z_i$. The equations of $F$-flatness of $Z_i$ fields require

$$\lambda^{z_i}(1 + \sum_j \tilde{Z}_j + \cdots) = 0,$$
which generally lead to solutions \( \tilde{Z}_j \sim O(1) \). Notice that at least one field of a term in the superpotential must have positive or zero anomalous \( U(1)_A \) charge, otherwise we cannot write down the term satisfying \( U(1)_A \) gauge invariance. As noted before, keeping SUSY zero feature requires that the VEV of a gauge invariant operator with positive anomalous \( U(1)_A \) charge vanishes. Therefore under this requirement, usually it is sufficient to examine the \( F \)-flatness of the gauge invariant operator with positive or zero anomalous \( U(1)_A \) charges. ³ (Therefore the \( F \)-flatness condition for the \( \Phi \) is automatically satisfied because \( \Phi \) has negative charge -1.)

Let us return to the problem on the mass of the neutrino, which is discussed in the previous section. From the above argument, it is shown that introducing singlet field \( S \) with VEV and interaction \( \lambda^{s+2n_c}S N^c_R N^c_R \) cannot improve the situation because the VEV of the singlet is written as \( \langle S \rangle = \lambda^{-s} \) if \( F \)-flatness conditions determine the VEV. Of course it is obvious that this problem can be avoided if the VEV is determined dynamically or by some other conditions, for example, \( D \)-flatness conditions [18]. However, we argue another simple way to avoid this problem. Let us introduce additional gauge freedom ⁴ which transforms the right-handed neutrino fields non-trivially, for examples, an additional \( U(1)_A \) gauge symmetry or larger gauge group than the standard gauge group like \( SO(10) \). If the gauge variant field couples to the mass term of the right-handed neutrino and the VEV of the field breaks the additional gauge symmetry, then the coefficient can be changed. For examples, if we introduce additional singlets under the standard gauge group \( \Theta(1, -6) \) and \( \tilde{\Theta}(-1, 0) \) as well as the right-handed neutrinos \( N^c_R, (1, 1) \) under the gauge group \( U(1)_X \times U(1)_A \), then the VEV of the gauge invariant operator \( \langle \Theta\Theta \rangle \) is determined by the anomalous \( U(1)_A \) charge -6 and given as \( \langle \Theta\Theta \rangle = \lambda^6 \). The mass term of the right-handed neutrino is given from the term \( \lambda^2(N^c_R, (1, 1)\tilde{\Theta}(-1, 0))^2 \) with the VEV \( \langle \Theta(1, 6) \rangle = \langle \tilde{\Theta}(-1, 0) \rangle \sim \lambda^3 \), which is required by \( D \)-flatness condition of \( U(1)_X \). This mass term is of order \( \lambda^8 \) which is much smaller than the naively expected value \( \lambda^2 \). The fact that the additional gauge freedom is required to obtain the correct size of the mass of the right-handed neutrino implies that the GUT, if any, must have the rank more than 4. \( SO(10) \) gauge group is one of the most promising candidate because they also unify one generation of quarks and leptons including right-handed neutrino in a single multiplet (spinor) \( \Psi \). Actually if we adopt \( SO(10) \) gauge group which is broken by the VEV of spinor \( \langle C \rangle = \langle \bar{C} \rangle \sim \lambda^{-(c+\bar{c})/2} \), the mass term of the right-handed neutrino is given from the term \( \lambda^2(\psi^c \bar{c})(\bar{\Psi} \bar{\bar{C}})^2 \). The mass \( \lambda^{2c} \) can be smaller than the naively expected value \( \lambda^{2\psi} \). The model which is proposed by Bando and Kugo [6] has such a structure in \( E_6 \) unification.

Such a solution introducing a larger unification group is attractive, but GUT is generally suffering from the DT splitting problem. In the next section we point out that the DT splitting is beautifully realized in \( SO(10) \) unification using the anomalous \( U(1)_A \) gauge symmetry.

## 4 Doublet-triplet splitting with anomalous \( U(1)_A \) gauge symmetry

In the previous section, we have emphasized that introducing the \( SO(10) \) grand unified group or larger group can naturally explain the mass scale of the right-handed neutrino. However, if we proceed to the unified theory with a simple group, we have to solve the doublet-triplet

³Notice that the \( F \)-flatness conditions of gauge variant fields with negative charge can be important to determine the VEVs. This is because gauge variant fields with positive charge may have non-vanishing VEV.

⁴A global symmetry can play the same role if the Nambu-Goldstone fields are phenomenologically allowed.
splitting problem. In this section, we will propose an attractive $SO(10)$ unified model which naturally realizes the doublet-triplet splitting.

The content of the Higgs sector under $SO(10) \times U(1)_A$ gauge symmetry is given in Table 1, where $\pm$ denote a parity quantum number.

\[
\begin{align*}
45 & : A(a = -2, -), \quad A'(a' = 6, -) \\
16 & : C(c = -1, +), \quad C'(c' = 8, -) \\
\{4\} & : \bar{C} (\bar{c} = -5, +), \quad \bar{C}' (\bar{c}' = 4, -) \\
10 & : H(h = -2, +), \quad H'(h' = 4, -) \\
1 & : Z(z = -3, -), \quad \bar{Z}(\bar{z} = -3, -), \quad S(s = 5, +)
\end{align*}
\]

Table 1, The indices represent the anomalous $U(1)_A$ charge and parity.

Here we have written typical values of the anomalous $U(1)_A$ charges. Among these fields, $A$, $C$, $\bar{C}$, $Z$ and $\bar{Z}$ are expected to obtain non-vanishing VEVs around the GUT scale. Here for simplicity we assume that the fields with positive $U(1)_A$ charges have vanishing VEV, although we can give more rigorous argument for this.

Since the fields with non-vanishing VEVs have negative charges, only the $F$-flatness conditions of fields with positive charge must be counted for determination of their VEVs. (Generally $c$ or $\bar{c}$ can be positive (although we are now taking $c = -1$ and $\bar{c} = -5$), because it is sufficient for keeping “SUSY zero” that the charge $c + \bar{c}$ of the gauge invariant operator $\bar{C}C$ becomes non-positive. Even in this case, the following argument does not change so much.) Moreover we have only to take account of the terms in the superpotential which contain only the fields with non-positive. Even in this case, the following argument does not change so much.) Moreover we have only to take account of the terms in the superpotential which contain only one field with positive charge. This is because the terms with more positive charge fields do not contribute to the $F$-flatness conditions, since the positive fields are assumed to have zero VEV. Therefore, in general, the superpotential required by determination of the VEVs can be written as

\[
W = W_{H'} + W_{A'} + W_S + W_{C'} + W_{\bar{C}'}.
\]

Here $W_X$ denotes the terms linear in $X$ field which has positive anomalous $U(1)_A$ charge. Notice, however, that terms including two fields with positive charge like $\lambda H'H'$ have contribution to the mass terms but not for the determination of the VEVs. $W_{A'}$ is for producing the DW form for the VEV of $A$, $\langle A \rangle = i\tau_2 \times \text{diag}(v, v, v, 0, 0)$, which is proportional to the generator $B - L$. Such a VEV of $A$ gives a super heavy mass to the color triplets in $H$ and $H'$ through $W_H' = HAH'$ term, while keeping the weak doublets massless. This means that the $F$-flatness condition of $H'$ forces vanishing VEV of the colored Higgs in $H$, but not the VEV of the doublet Higgs in $H$. The mass term $\lambda^{2H'}(H')^2$ gives the mass $\sim \lambda^{2H'}$ to the extra doublet in $H'$. Therefore it is realized that only one pair of doublet Higgs in $H$ becomes massless.

Let us discuss the determination of the VEVs successively. For determination of the VEVs, it is sufficient to take account of the superpotential terms which includes only the fields with VEVs except one $A'$. If $-3a \leq a' < -5a$, the superpotential $W_{A'}$ is in general written as

\[
W_{A'} = \lambda^{a+a'} \alpha A'A + \lambda^{a'+3a}(\beta (A'A)_1 (A^2)_1 + \gamma (A'A)_{54} (A^2)_{54}),
\]

where the suffixes 1 and 54 indicate the representation of the composite operators under the $SO(10)$ gauge symmetry and $\alpha$, $\beta$ and $\gamma$ are parameters of order one. Here we take $a + a' + c + \bar{c} < 0$ to forbid the term $\bar{C}A'AC$ which destabilizes the DW form of the VEV $\langle A \rangle$. If we take $\langle A \rangle =$
\(i\pi_2 \times \text{diag}(x_1, x_2, x_3, x_4, x_5)\). F-flatness of \(A'\) field requires \(x_i(\alpha \lambda^{-2a} + 2(\beta - \gamma)(\sum_j x_j^2) + \gamma x_i^2) = 0\), which gives only two solutions \(x_i^2 = 0\). Here \(N = 1 \sim 5\) is the number of \(x_i \neq 0\) solutions. The DW form is obtained when \(N = 3\). Notice that the higher terms \(A'A^{2L+1}(L > 1)\) are forbidden by SUSY zero. If they are allowed, the number of the possible VEVs other than the DW form becomes larger, so it becomes less natural to obtain the DW form. This is an critical point of this mechanism and the anomalous must have the same direction because of the \(D\) flatness condition of \(S\), which plays an essential role to forbid the undesired terms. And it is also interesting that the scale of the VEV is automatically determined by the anomalous \(U(1)_A\) charge of \(A\) as noted in the previous section.

Next we argue the \(F\)-flatness condition of \(S\), which determines the scale of the VEV \(\langle \bar{C}C \rangle\). \(W_S\), which is linear in \(S\) field, is given by

\[
W_S = \lambda^{c+\bar{c}}S((\bar{C}C) + \lambda^{-(c+\bar{c})} + \sum_k \lambda^{-(c+\bar{c})+2ka}A^{2k}),
\]

if \(s \geq -(c + \bar{c})\). Then the \(F\)-flatness condition of \(S\) forces \(\langle \bar{C}C \rangle \sim \lambda^{-(c+\bar{c})}\) and \(D\)-flatness condition requires \(|\langle C \rangle| = |\langle \bar{C} \rangle| \sim \lambda^{-(c+\bar{c})/2}\). The scale of the VEV is determined only by the charges of \(C\) and \(\bar{C}\) again. If we take \(c + \bar{c} = -6\), then we obtain the VEVs of the field \(\bar{C}\) and \(C\) as \(\lambda^3\), which are different from the expected values \(\lambda^{-c}\) or \(\lambda^{-\bar{c}}\) if \(c \neq \bar{c}\). Notice that a composite operator with positive anomalous \(U(1)_A\) charge larger than \(-(c + \bar{c}) - 1\) may play the same role as the singlet \(S\) if any. (In the above example, we have no candidate.)

Finally let us discuss the \(F\)-flatness of \(C'\) and \(\bar{C'}\), which realize the alignment of the VEVs \(\langle C \rangle\) and \(\langle \bar{C} \rangle\) and give masses to the PNG fields. This simple mechanism is proposed by Barr and Raby [11]. We can easily assign the anomalous \(U(1)_A\) charges which allow the following superpotential

\[
W_{C'} = \bar{C}(\lambda^{\bar{c}+c}A + \lambda^{\bar{c}+c}Z)C',
\]

\[
W_{\bar{C'} = \bar{C}'}(\lambda^{\bar{c}+c}A + \lambda^{\bar{c}+c}Z)C'\).
\]

The \(F\)-flatness conditions \(F_{C'} = F_{\bar{C'}} = 0\) give \((\lambda^{a-z}A + Z)C = \bar{C}(\lambda^{a-z}A + \bar{Z}) = 0\). Recall that the VEV of \(A\) is proportional to the \(B - L\) generator \(Q_{B-L}\) as \(<A> = \frac{3}{2}\nu Q_{B-L}\). And \(C, 16\), is decomposed into \((3, 2, 1)_{1/3}, (3, 1, 2)_{-1/3}, (1, 2, 1)_{-1}\) and \((1, 2, 1)_{1}\) under \(SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}\). Since \(\langle \bar{C}C \rangle \neq 0\), not all components in the spinor \(C\) vanish. Then \(Z\) is fixed to be \(Z \sim -\frac{3}{2}\nu Q_{B-L}\), where \(Q_{B-L}\) is \(B - L\) charge of the component field in \(C\) which has non-vanishing VEV. It is interesting that no other component fields can have non-vanishing VEV because of the \(F\)-flatness conditions. If \((1, 2, 1)_{1}\) field obtains the VEV (therefore, \(\langle Z \rangle \sim -\frac{3}{2}\nu\)), then the gauge group \(SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}\) is broken to the standard gauge group. Once the direction of the VEV \(\langle C \rangle\) is determined, the VEV \(\langle \bar{C} \rangle\) must have the same direction because of the \(D\)-flatness condition. Therefore, \(\langle \bar{Z} \rangle \sim -\frac{3}{2}\nu\). Thus, all VEVs have now been fixed.

Next we examine the mass spectrum. Since for the mass terms, we must take account of not only the above terms but also the terms which contains two fields with vanishing VEVs.

Considering the additional mass term \(\lambda^{2h'}H'H'\), we write the mass matrix of the Higgs fields \(H\) and \(H'\) which are decomposed 5 and 5 of \(SU(5)\) as

\[
\begin{pmatrix}
(5_H, 5_{H'}) & \left( \begin{array}{cc}
0 & \lambda^{h+h'+a} <A>\\
\lambda^{h+h'+a} <A> & \lambda^{2h'}
\end{array} \right) \left( \begin{array}{c}
\tilde{5}_H \\
\tilde{5}_{H'}
\end{array} \right)
\end{pmatrix}.
\]
The colored Higgs obtain their masses of order $\lambda^{h+h'+a} \langle A \rangle \sim \lambda^{h+h'}$. Since in general $\lambda^{h+h'} > \lambda^{2h'}$, the proton decay is naturally suppressed. The effective colored Higgs mass is estimated as $(\lambda^{h+h'})^2/\lambda^{2h'} = \lambda^{2h}$ which is larger than the Planck scale because $h < 0$. One pair of the doublet Higgs is massless, while an additional pair of the doublet Higgs obtains mass of order $\lambda^{2h'}$, which is $\sim \lambda^8 \sim 5 \times 10^{12}$ GeV in the typical $U(1)_A$ assignment in Table 1. DW mechanism works well though we have to examine the effect of the rather light additional Higgs.

Next let us examine the mass matrices for the representations $I = Q, U^c$ and $E^c$, which are contained in the 10 of $SU(5)$. As well as the superpotential previously discussed, the additional terms $\lambda^{2a'} A' A'$, $\lambda^{c+\bar{c}'} C'C'$, $\lambda^{d+\bar{d}'} \bar{C} A'C'$ and $\lambda^{d+\bar{d}'} \bar{C} A'C'$ must be taken into account. The mass matrices are written as 4 $\times$ 4 matrices

\[
\begin{pmatrix}
I_A, I_A', I_C, I_C'
\end{pmatrix}
\begin{pmatrix}
0 & \lambda^{a+a'}\alpha_f & 0 & \frac{\lambda^{c+\bar{c}'+a}}{\sqrt{2}} \langle C \rangle \\
\lambda^{a+a'}\alpha_f & \lambda^{2a'} & 0 & \frac{\lambda^{c+\bar{c}'+a}}{\sqrt{2}} \langle C \rangle \\
0 & 0 & \lambda^{c+\bar{c}'+a} \beta_{1U} & \lambda^{c+\bar{c}'} \\
\frac{\lambda^{c+\bar{c}'+a}}{\sqrt{2}} \langle C \rangle & \frac{\lambda^{c+\bar{c}'+a}}{\sqrt{2}} \langle C \rangle & \lambda^{c+\bar{c}'+a} \beta_{1U} & \lambda^{c+\bar{c}'}
\end{pmatrix}
\begin{pmatrix}
I_A \\
I_A' \\
I_C \\
I_C'
\end{pmatrix},
\tag{4.7}
\]

where $\alpha_f$ vanishes for $I = Q$ and $U^c$ because these are Nambu-Goldstone modes, but $\alpha_{E^c} \neq 0$. On the other hand, $\beta_1 = \frac{1}{2}(B-L)f - 1$, namely $\beta_0 = -1$, $\beta_{1E^c} = -2$ and $\beta_{2E^c} = 0$. Thus for each $I$ the 4 $\times$ 4 matrix has one vanishing eigenvalue, which corresponds to the Nambu-Goldstone mode eaten by the Higgs mechanism. The mass spectrum of the remaining three modes is $(\lambda^{c+\bar{c}'+v}, \lambda^{c+\bar{c}'+v}, \lambda^{2a'})$ for the color-triplet modes $Q$ and $U^c$, and $(\lambda^{a+a'}, \lambda^{a+a'}, \lambda^{c+\bar{c'}})$ or $(\lambda^{a+a'} \langle C \rangle, \lambda^{c+\bar{c}'+a} \langle C \rangle, \lambda^{2a'})$ for the color-singlet modes $E^c$. (These are dependent on the anomalous $U(1)_A$ charges.) If we take typical anomalous $U(1)_A$ charges as shown in the previous Table, then the light modes are $Q$, $U^c$, $E^c$ and their hermitian conjugate fields, which are contained in a pair of 10 and $\bar{10}$ of $SU(5)$ with the mass of order $\lambda^{12} \sim 10^{10}$ GeV. Though in principle the mass of the color-triplet fields and that of the color-singlet field are determined independently, it is interesting that all the fields in a single multiplet 10 of $SU(5)$ become light together. This fact makes us expect that the success of the gauge coupling unification may not be drastically changed with these light modes.

If we simply omit the rows and columns of $A$ and $A'$ in Eq. (4.7), then we obtain 2 $\times$ 2 mass matrices, which are for the representations $D^c$ and $L$ and their conjugates. Since $\beta_D = -1/2$ and $\beta_L = -3/2$, the color triplets obtain the masses of $\lambda^{c+\bar{c}'}/2$ and $\lambda^{c+\bar{c}'}/2$ while the weak doublets get masses of $3\lambda^{c+\bar{c}'}/2$ and $3\lambda^{c+\bar{c}'}/2$.

The adjoint fields $A$ and $A'$ contain two $(8, 1)_0$ and two $(1, 3)_0$ of the standard gauge group, which obtain mass of $\lambda^{a+a'}$. Moreover, they contain two pairs of $(3, 2)_{-5/6} + h.c.$, one of which is eaten by Higgs mechanism, but another pair has rather light mass of $\lambda^{2a'}$, which may break the success of the coincidence of the running gauge couplings.

Once we determine the anomalous $U(1)_A$ charges, the mass spectrum of all fields is determined, so we can calculate the Weinberg angle. However, since the estimation is strongly dependent on the assignment of the anomalous $U(1)_A$ charges as shown in the above argument and on the details of the DT splitting sector and the matter sector, here we no more discuss it.

There are several terms which must be forbidden for the stability of the DW mechanism. For examples, $H^2$, $HZH'$ and $HZH'$ induce a large mass to the doublet Higgs, and the term $\bar{C} A' AC$ would destabilize the DW form of $\langle A \rangle$. We can easily forbid these terms using SUSY zero, for examples, if we take $h < 0$, then $H^2$ is forbidden and if we take $\bar{c} + c < a + a'$, then $\bar{C} A' AC$ is forbidden. (It is interesting that the negative $U(1)_A$ charge $h$, which is required for the DT splitting, enhances the left-handed neutrino masses as discussed in section 2.)
Once these dangerous terms are forbidden by SUSY zero, higher dimensional terms which are also dangerous, for examples, \( \bar{C}A'A^3C \) or \( \bar{C}AC\bar{C}AC \), are automatically forbidden, since only gauge invariant operators with negative charge can have non vanishing VEVs. This is also an attractive point of our scenario. Actually, the symmetry discussed in Ref.\[7\] does not forbid the dangerous term \( (\bar{C}AC)^2 \), which destabilizes the DW form of \( \langle A \rangle \).

In this section, we have proposed an attractive DT splitting mechanism in which the anomalous \( U(1)_A \) gauge symmetry plays critical roles and the VEVs and mass spectrum are automatically determined by the anomalous \( U(1)_A \) charges.

In the next section, we examine the simplest model with this DT splitting mechanism, which gives realistic mass matrices of quarks and leptons.

5 The simplest model

In this section, we examine the simplest model to demonstrate how to determine everything from the anomalous \( U(1)_A \) charges.

In addition to the Higgs sector in Table.1, we introduce only three 16 representations \( \Psi_i \) with the anomalous \( U(1)_A \) charges \( (\psi_1 = n + 3, \psi_2 = n + 2, \psi_3 = n) \) and one 10 field \( T \) with charge \( t \) as the matter content. These matter fields are assigned odd R-parity, while those of the Higgs sector even R-parity. Such an assignment of R-parity guarantees that the argument on VEVs in the previous section does not change if these matter fields have vanishing VEVs. We can argue the allowed region of the anomalous \( U(1)_A \) charges to obtain desired terms and while forbidding dangerous terms. Though it is a straightforward argument, here we do not discuss it. Instead, we give a set of anomalous \( U(1)_A \) charges by which all conditions are satisfied and a novel neutrino mass matrix is obtained: \( n = 3, t = 4, h = -6, h' = 8, c = -4, \bar{c} = -1, c' = 4, \bar{c}' = 7, s = 5 \). Then the mass term of \( 5 \) and \( \bar{5} \) of \( SU(5) \) is written as

\[
5_T(\lambda^6 \langle C \rangle, \lambda^5 \langle C \rangle, \lambda^3 \langle C \rangle, \lambda^8)
\]

\[
\begin{pmatrix}
\bar{5}_{\psi_1} \\
\bar{5}_{\psi_2} \\
\bar{5}_{\psi_3} \\
\bar{5}_T
\end{pmatrix}
\]

(5.1)

Since \( \langle \bar{C} \rangle = \langle C \rangle \sim \lambda^{5/2} \) because \( c + \bar{c} = -5 \), massive mode \( \bar{5}_M \), the partner of \( 5_T \), is given by

\[
\bar{5}_M \sim \bar{5}_{\psi_3} + \lambda^{5/2} \bar{5}_T.
\]

(5.2)

Therefore three massless modes \( (\bar{5}_1, \bar{5}_2, \bar{5}_3) \) are written as \( (\bar{5}_{\psi_1}, \bar{5}_T + \lambda^{5/2} \bar{5}_{\psi_3}, \bar{5}_{\psi_2}) \). Dirac mass matrices for quarks and leptons can be obtained from the interaction

\[
\lambda \psi_i + \psi_j + b \psi_i \psi_j H.
\]

(5.3)

Mass matrices for up quark sector and for down quark sector are

\[
M_u = \begin{pmatrix}
\lambda^6 & \lambda^5 & \lambda^3 \\
\lambda^5 & \lambda^4 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix} \langle H_u \rangle, \quad M_d = \lambda^2 \begin{pmatrix}
\lambda^4 & \lambda^{7/2} & \lambda^3 \\
\lambda^3 & \lambda^{5/2} & \lambda^2 \\
\lambda^1 & \lambda^1 & 1
\end{pmatrix} \langle H_d \rangle.
\]

(5.4)

Notice that the Yukawa couplings for \( \bar{5}_2 \sim \bar{5}_T + \lambda^{5/2} \bar{5}_{\psi_3} \) are obtained only through the Yukawa couplings for the component \( \bar{5}_{\psi_3} \), because we have no Yukawa couplings for \( T \). We can estimate
the CKM matrix from these quark matrices as

\[ U_{CKM} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \] (5.5)

which is consistent with the experimental value if we take \( \lambda \sim 0.2 \). Since the ratio of the Yukawa couplings of top and bottom quarks is \( \lambda^2 \), small \( \tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle \sim O(1) \) is predicted by these mass matrices. Yukawa matrix for charged lepton sector is the same as the transpose of \( M_d \) at this stage except an overall factor \( \eta \) induced by the renormalization group effect. Mass matrix for the Dirac mass of neutrinos is given by

\[ M_D = \lambda^2 \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda \\ \lambda^{7/2} & \lambda^{5/2} & \lambda^{1/2} \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \langle H_u \rangle \eta. \] (5.6)

The right-handed neutrino masses come from the interaction

\[ \lambda^{\psi_i + \psi_j + 2e} \bar{\Psi}_i \bar{\Psi}_j \bar{C} \bar{C} \] (5.7)
as

\[ M_R = \lambda^{\psi_i + \psi_j + 2e} \langle \bar{C} \rangle^2 = \lambda^9 \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}. \] (5.8)

Therefore we can estimate the neutrino mass matrix:

\[ M_\nu = M_D M_R^{-1} M_D^T = \lambda^{-5} \begin{pmatrix} \lambda^2 & \lambda^{3/2} & \lambda \\ \lambda^{3/2} & \lambda & \lambda^{1/2} \\ \lambda & \lambda^{1/2} & 1 \end{pmatrix} \langle H_u \rangle^2 \eta^2. \] (5.9)

Notice that the overall factor \( \lambda^{-5} \) has negative power which can be induced by the effects discussed in section 2 and 3. From these mass matrices in the lepton sector the MNS matrix is obtained as

\[ U_{MNS} = \begin{pmatrix} 1 & \lambda^{1/2} & \lambda \\ \lambda^{1/2} & 1 & \lambda^{1/2} \\ \lambda & \lambda^{1/2} & 1 \end{pmatrix}. \] (5.10)

This gives bi-maximal mixing angles for neutrino sector, since \( \lambda^{1/2} \sim 0.5 \). A prediction is \( m_{\nu_\mu} / m_{\nu_\tau} \sim \lambda \), which is consistent with the experimental data: \( 1.6 \times 10^{-3} \text{(eV)}^2 \leq \Delta m_{\text{atm}}^2 \leq 4 \times 10^{-3} \text{(eV)}^2 \) and \( 2 \times 10^{-5} \text{(eV)}^2 \leq \Delta m_{\text{solar}}^2 \leq 1 \times 10^{-4} \text{(eV)}^2 \). \( \nu_e \sim \lambda \) is also an interesting prediction from this matrix, though CHOOZ gives severe upper limit \( V_{e3} \leq 0.15 \) [26]. The neutrino mass is given by \( m_{\nu_\tau} \sim \lambda^{-5} \langle H_u \rangle^2 \eta^2 / M_P \sim m_{\nu_\tau} / \lambda \sim m_{\nu_\mu} / \lambda^2 \). If we take \( \langle H_u \rangle \eta = 100 \) GeV, \( M_P \sim 10^{18} \) GeV and \( \lambda = 0.2 \), then we get \( m_{\nu_\tau} \sim 3 \times 10^{-2} \) eV, \( m_{\nu_\mu} \sim 6 \times 10^{-3} \) eV and \( m_{\nu_e} \sim 1 \times 10^{-3} \) eV. It is surprising that such a rough estimation gives good values for explaining the experimental values from the atmospheric neutrino and large mixing angle (LMA) MSW solution for solar neutrino problem [25]. This LMA solution for the solar neutrino problem gives the best fitting to the present experimental data [23].

In addition to the Eq.(5.3), interactions

\[ \lambda^{\psi_i + \psi_j + 2a + h} \bar{\Psi}_i A^2 \bar{\Psi}_j H \] (5.11)
also contribute to the Yukawa couplings. Here $A$ is squared since it has odd parity. Since $A$ is proportional to the generator of $B - L$, the contribution to lepton Yukawa coupling is nine times larger than that to quark Yukawa coupling, which can change the unrealistic prediction $m_\mu = m_s$ at the GUT scale. Since the prediction $m_s/m_d \sim \lambda^{5/2}$ at GUT scale is wonderful, the enhancement factor $2 \sim 3$ of $m_\mu$ can improve the situation. Note that the additional terms contribute mainly on lepton sector. If we adopt $a = -2$, the additional matrices are

$$
\frac{\Delta M_u}{\langle H_u \rangle} = \frac{v^2}{4} \begin{pmatrix}
\lambda^2 & \lambda & 0 \\
\lambda & 1 & 0 \\
0 & 0 & 0
\end{pmatrix},
\frac{\Delta M_d}{\langle H_d \rangle} = \frac{v^2}{4} \begin{pmatrix}
\lambda^2 & \lambda & 0 \\
0 & 0 & 0 \\
\lambda & 1 & 0
\end{pmatrix},
(5.12)
$$

$$
\frac{\Delta M_e}{\langle H_d \rangle} = \frac{9v^2}{4} \begin{pmatrix}
\lambda^2 & 0 & \lambda \\
\lambda & 0 & 1 \\
0 & 0 & 0
\end{pmatrix}.
(5.13)
$$

It is interesting that this modification essentially changes the eigenvalues of only the first and second generation, so it is natural to expect that the realistic mass pattern can be obtained by this modification. This is one of the largest motivations to assign $a = -2$. Notice that this charge assignment determines also the scale of $\langle A \rangle \sim \lambda^2$. It is suggestive that the fact that the $SO(10)$ breaking scale is a bit smaller than the Planck scale is correlated with the discrepancy between the naive prediction of the ratio $m_\mu/m_s$ from the unification and the experimental value. It is also interesting that the SUSY zero plays an essential role again. When $z, \bar{z} \geq -4$, the terms $\lambda^{\psi_i + \psi_j + a + z + h} Z \Psi_i A \Psi_j H + \lambda^{\psi_i + \psi_j + 2z + h} Z^2 \Psi_i \Psi_j H$ also contribute to the fermion mass matrices, though only to the first generation.

Proton decay mediated by the colored Higgs is strongly suppressed in this model. As mentioned in the previous section, the effective mass of the colored Higgs is of order $\lambda^2h \sim \lambda^{-16}$, which is much larger than the Planck scale. Proton decay is also induced by the non-renormalizable term

$$
\lambda^{\psi_i + \psi_j + \psi_k + \psi_l} \Psi_i \Psi_j \Psi_k \Psi_l,
(5.14)
$$

which is also strongly suppressed.

Unfortunately in this model we obtain the extra light doublet Higgs with mass of order $\lambda^{2h} \sim \lambda^{16}$ and the extra fields $(3, 2)_{-5/6} + h.c.$ with mass of order $\lambda^{2d'} \sim \lambda^{12}$, which may break the success of gauge coupling unification. Actually rough estimation shows that the meeting point of the gauge couplings of $SU(3)_C$ and $SU(2)_L$ is too low to keep the proton stability even if we take any $U(1)_A$ charge assignment. However, since the mass spectrum is strongly dependent on the details of DT splitting models and matter sector, we expect that in a certain charge assignment of a certain DT splitting model and matter sector, the coupling unification is recovered. In other words, the requirement of coupling unification gives a strong constraint on these models.

6 Discussions

In this paper, we have examined a DT splitting model and emphasized that the anomalous $U(1)_A$ gauge symmetry plays essential roles to realize DT splitting by DW mechanism. This is a general statement. Actually we can make various types of DT splitting models in which the anomalous $U(1)_A$ gauge symmetry plays the same roles as discussed in this paper. For examples, if we exchange the parity between $C'$ and $\bar{C}$, the term $CC'H$ can be allowed. After obtaining the VEV of $C$ field, the massless doublet Higgs becomes linear combination of $\bar{5}_H$ and
of $SU(5)$, which may give richer structure to quark and lepton matrices, though a dangerous term $CA'C$ must be forbidden by SUSY zero. We can introduce additional Higgs pair, $F: 16$ and $\bar{F}: \bar{16}$, to obtain the massless doublet Higgs which is linear combination of $\bar{5}_H$ and $\bar{5}_F$. Yet another way to modify DT splitting model is to introduce an additional adjoint field $45 A_{+}(a_{+}, +)$ with $a_{+} < 0$. Then the mass spectrum of light modes are quite different from the model discussed in this paper because of the term $A'A_{+}A$. Moreover, we have assumed that the anomalous $U(1)_A$ charges are integer, but in principle, we can adopt rational numbers as in Ref.[8]. We have not examined well all these modified DT splitting models. We will examine various possibilities in future. The condition for gauge coupling unification must be an useful guide to select these models.

In principle, we may adopt an anomaly free $U(1)_A$ gauge symmetry and the F-I $D$-term instead of the anomalous $U(1)_A$ gauge symmetry, though it seems to be difficult to find a consistent $U(1)_A$ charge assignment. Moreover, since we have no reason to take the scale of the F-I $D$-term less than the Planck scale, we think it more natural to adopt the anomalous $U(1)_A$ gauge symmetry.

Finally we discuss SUSY breaking. Since we should assign the anomalous $U(1)_A$ charges dependent on the flavor to produce the hierarchy of Yukawa couplings, generally the non-degenerate scalar fermion masses are induced through the anomalous $U(1)_A D$-term. The large SUSY breaking scale can avoid the flavor changing neutral current (FCNC) problem [27, 28], but in our scenario it does not work because the anomalous $U(1)_A$ charge of the Higgs $H$ is inevitably negative to forbid the Higgs mass term in tree level. Therefore the anomalous $U(1)_A D$-term contribution, which is dependent on the flavor, must be dominated by other flavor independent contribution to sfermion mass terms. In principle, it is possible, for example, that the $F$-term of the dilation field can dominate the dangerous $D$-term contribution. Actually, Arkani-Hamed, Dine and Martin [29] pointed out that the $F$-term contribution of the dilaton field can be larger than the anomalous $U(1)_A D$-term contribution, depending on how the dilaton is stabilized, even in the case that the anomalous $U(1)_A$ gauge symmetry triggers SUSY breaking[30, 31]. It is interesting that lepton flavor violation process can be seen in the scenario [32]. Since the FCNC process gives severe constraints on the ratio of $D$-term contribution and the flavor independent contribution, it is valuable to examine the condition for the constraints to become weaker. If the first generation of $\bar{5}$ of $SU(5)$ has the same charge as the second generation, the constraint becomes much weaker [32]. The condition for the model discussed in this paper is $\psi_1 = t$. If we assign the anomalous $U(1)_A$ charge as $n = 5, t = 8, a = -2, a' = 6, h = -10, h' = 12, c = -2, \bar{c} = -3, c' = 6, \bar{c}' = 5, z = \bar{z} = -3, s = 5$, the above situation is realized, though the mass of an additional pair of Higgs becomes $\lambda^{24}$. Even if the above effect is negligible, the lepton flavor violation process may be seen through the renormalization effect of the left-handed slepton masses [33].

In the following papers [34], it is shown that the DT splitting mechanism can be non-trivially incorporated in $E_6$ unification. It is interesting that the mass matrices with bi-maximal mixing discussed in this paper appear again in the $E_6$ unified model. Moreover, the above condition $\psi_1 = t$, which make the constraints from the FCNC process weaker, is automatically satisfied.\footnote{In our case it is difficult that the anomalous $U(1)_A$ gauge symmetry triggers SUSY breaking, since we have a lot of fields with negative charge except the field $\Phi$.}
7 Conclusion

In this paper, we have pointed out that, in order to realize correct size of the neutrino mass for the atmospheric neutrino anomaly by the anomalous $U(1)_A$ gauge symmetry, it is natural to introduce the Higgs field with negative $U(1)_A$ charge and a gauge group under which the right-handed neutrino transforms non-trivially, for examples, $SO(10)$ or $E_6$ or another $U(1)$. Next we proposed an $SO(10)$ unified model in which DT splitting is naturally realized by DW mechanism. The anomalous $U(1)_A$ gauge symmetry plays essential roles in the DT splitting. Using the mechanism, we examine the simplest model in which realistic mass matrices of quarks and leptons including neutrino can be determined by the anomalous $U(1)_A$ charges. This model predicts bi-maximal mixing angle in neutrino sector, small $\tan\beta$ and $V_{e3} \sim \lambda$. Proton stability is naturally realized. It is interesting that once we fix the anomalous $U(1)_A$ charges for all fields, the orders of every parameters and scales are determined except SUSY breaking.

It is very suggestive that the anomalous $U(1)_A$ gauge symmetry motivated by the superstring theory plays critical roles in solving the two biggest problems in GUT, fermion mass hierarchy problem and doublet-triplet splitting problem. It may be the first evidence for the string theory from the phenomenological point of view.

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