Linear and nonlinear realizations of superbranes
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The coordinate transformations which establish the direct relationship between the actions of linear and nonlinear realizations of supermembranes are proposed. It is shown that the Roček-Tseytlin constraint known in the framework of the linear realization of the theory is simply equivalent to a limit of a "pure" nonlinear realization in which the field describing the massive mode of the supermembrane puts to zero.

1. Introduction

One of the well-known examples of theory with partially broken global supersymmetry (PBGS) is the $N = 1$, $D = 4$ supermembrane which can be derived either from the nonlinear realization (NR) of global N = 2, D = 3 supersymmetry partially broken down to N = 1, D = 3 or entirely from a corresponding linear realization (LR) of this supersymmetry [1]. The first approach is more transparent. It gives the manifestly covariant description of the action in terms of vacuum Goldstone excitations associated with the generators of the spontaneously broken symmetries. The second one is more simple but less transparent due to the presence of a bit skillful nilpotency constraints suppressing the massive degree of freedom of the supermembrane [2]. In General these two approaches use different Goldstone superfields and by now there is absent a manifestly covariant procedure establishing the direct relation between them.

Here we would like to propose a new approach which establishes the one to one correspondence between the quantities of linear and nonlinear realizations of theory. The latter is based on a special kind of superembedding of a complex superspace $\mathbb{C}^{3|2}$ into a superspace $\mathbb{C}^{4|2}$ in which the third spatial coordinate is identified with a Goldstone superfield associated with a central charge. The action of the supermembrane appears as a consequence of a nilpotency constraint [2] imposed on the real part of the Goldstone superfield. Note that because of this constraint the massive mode of the supermembrane does not contribute into the action. We show that from the geometrical point of view it corresponds to a limit of a pure nonlinear realization in which the masses of all the massive modes of theory tend to infinity.

2. PBGS in complex superspace

It was shown in [1,3] that the action of the $N = 1$, $D = 4$ supermembrane can be considered as the effective action of the superfield theory with partial breaking global $N = 2$, $D = 3$ supersymmetry. This approach is based on the geometrical ideas of Refs. [4,7–10] where the standard method of the NR of supersymmetry in superspace were used. In this section we are going to reproduce the basic entities of this approach proceeding from the another prescription based on the ideas of supergravity [14,6,15,5].

2.1. Linear realization

Let us shortly consider the linear realization of the $N = 2$, $D = 3$ supersymmetry in the real superspace $\mathbb{R}^{3|4}$

\[
\begin{align*}
    x^{\alpha\beta} &= x^{\alpha\beta} - \frac{i}{4} (\epsilon^{\alpha} \tau^\beta + \xi^{\alpha} \omega^\beta + \alpha \leftrightarrow \beta), \\
    x^{\alpha'} &= \tau^\alpha + \epsilon^{\alpha'}, \\
    \omega^{\alpha'} &= \omega^\alpha + \xi^{\alpha},
\end{align*}
\]

where $\alpha$, $\beta$ are the spinor indices of the Lorentz group $SL(2,R)$. This superspace can be regarded as the real subspace of the complex superspace
\[ C^{3|2} \]

\[
x_{L}^{\alpha} = x^{\alpha 3} + \frac{1}{4}(\tau^{\alpha} \omega^{\beta} + \alpha \leftrightarrow \beta), \tag{2}
\]

\[
\theta^{\alpha} = \frac{1}{\sqrt{2}}(\tau^{\alpha} + i\omega^{\alpha}),
\]

where the original supergroup is realized as follows

\[
x_{L}^{\alpha \beta} = x_{L}^{\alpha} - \frac{i}{2}(\xi^{\alpha}_{c} \theta^{\beta} + \xi^{\beta}_{c} \theta^{\alpha}) + \frac{i}{4}(\xi^{\alpha}_{c} \xi^{\beta}_{c} + \xi^{\beta}_{c} \xi^{\alpha}_{c}),
\]

\[
\theta^{\alpha \prime} = \theta^{\alpha} + \xi^{\alpha}_{c}, \tag{3}
\]

\[
\xi^{\alpha}_{c} = \frac{1}{\sqrt{2}}(\xi^{\alpha} + i\tau^{\alpha}), \quad \xi^{\alpha}_{c} = \frac{1}{\sqrt{2}}(\xi^{\alpha} - i\tau^{\alpha}).
\]

To provide the partial breaking of this supersymmetry let us consider the model of superembedding of the original superspace \( C^{3|2} \) into an extended complex superspace \( C^{4|2} \) with an additional bosonic coordinate \( q \)

\[
q_{L} = q - \frac{1}{2}(\xi^{\alpha}_{c} \theta^{\alpha} - \xi^{\alpha}_{c} \theta^{\alpha}), \tag{4}
\]

\[
q_{L} = qL - \xi^{\alpha}_{c} \theta^{\alpha} - \frac{1}{2} \xi^{\alpha}_{c} \varepsilon_{ca}, \quad qL = q - \frac{1}{2} \theta^{\alpha} \bar{\theta}^{\alpha}. \tag{5}
\]

Eq. (4) together with (1) describes the transformations of \( N = 2, D = 3 \) supersymmetry with a central charge \( Z \) and the following generators of supertranslations

\[
\bar{S}_{\alpha} = \frac{\partial}{\partial \theta^{\alpha}} - \frac{i}{2} \tau^{\beta} \frac{\partial}{\partial \theta^{\alpha \beta}} - \frac{1}{2} \omega_{\alpha} Z \equiv S_{\alpha} - \frac{1}{2} \omega_{\alpha} Z,
\]

\[
\bar{Q}_{\alpha} = \frac{\partial}{\partial \omega^{\alpha}} - \frac{i}{2} \tau^{\beta} \frac{\partial}{\partial \omega^{\alpha \beta}} + \frac{1}{2} \tau_{\alpha} Z \equiv Q_{\alpha} + \frac{1}{2} \tau_{\alpha} Z,
\]

\[
Z \equiv i \frac{\partial}{\partial q}.
\]

To avoid the presence of the redundant coordinate \( q \) in the theory we impose the following superembedding condition

\[
q_{L} = i \xi^{-1} \phi(x_{L}, \theta), \tag{6}
\]

It deserves mentioning that in accordance with the definition (6) the superfield \( \phi(x_{L}, \theta) \) transforms inhomogeneously with respect to the transformations (4)

\[
\phi(x'_{L}, \theta') = \phi(x_{L}, \theta) + i \xi \xi^{\alpha}_{c} \theta^{\alpha} + \frac{i}{2} \xi^{\alpha}_{c} \varepsilon_{ca}. \tag{7}
\]

Therefore (7) can be assumed to be a Goldstone superfield of \( N = 2, D = 3 \) supersymmetry. To justify this assumption it is instructive to notice that the real and imaginary parts of the superfield \( \phi(x_{L}, \theta) \equiv \Re \phi_{L}(x, \tau, \omega) + i \Im \phi_{L}(x, \tau, \omega) \) satisfy the chirality conditions

\[
D^{(\tau)} \Re \phi_{L} = D^{(\omega)} \Im \phi_{L}, \quad D^{(\omega)} \Re \phi_{L} = -D^{(\tau)} \Im \phi_{L}, \tag{8}
\]

\[
D^{(\tau)} \phi_{L} = \frac{\partial}{\partial \tau^{\alpha}} + \frac{i}{2} \tau^{\beta} \frac{\partial}{\partial \omega^{\alpha \beta}}, \quad D^{(\omega)} \phi_{L} = \frac{\partial}{\partial \omega^{\alpha}} + \frac{i}{2} \omega^{\beta} \frac{\partial}{\partial \tau^{\alpha \beta}}.
\]

This means that if we introduce the shifted superfield

\[
\phi_{L}(x, \tau, \omega) = \phi_{L}(x, \tau, \omega) - \frac{i}{4}(\tau^{2} + \omega^{2}), \tag{9}
\]

which transforms as the real variable \( q \)

\[
\phi'_{L}(x', \tau', \omega') = \phi_{L}(x, \tau, \omega) - \frac{\xi}{2}(\xi^{\alpha}_{c} \omega_{\alpha} - \xi^{\alpha}_{c} \tau_{\alpha}) \tag{10}
\]

and on which the central charge generator \( Z \) is realized as

\[
Z \phi_{L} = -\xi, \quad Z \bar{\phi}_{L} = \xi, \tag{11}
\]

we reveal that its real and imaginary parts would satisfy the same chirality conditions as the original superfield does but with the modified spinor derivatives of the extended supersymmetry

\[
D^{(\tau)} = D^{(\tau)} + \frac{1}{2} \omega_{\alpha} Z, \quad D^{(\omega)} = D^{(\omega)} - \frac{1}{2} \tau_{\alpha} Z. \tag{12}
\]

Thus the superfield \( \phi_{L} \) is also chiral (but in contrast to \( \phi_{L} \)) with respect to the extended supersymmetry. Another very important feature of the superfield \( \phi_{L} \) is that its real part possesses the homogeneous transformation law under the both of supersymmetries. The careful analysis shows that \( \phi_{L} \) indeed insures the partial breaking of \( N = 2, D = 3 \) supersymmetry and one of its spinor \( N = 1 \) components describes the Goldstone excitations of the \( 4D \) supermembrane associated with \( Q \)-supertranslations. Let us write down the Grassmann decomposition of the original chiral superfield (6)

\[
\Re \phi_{L} = A + i \omega^{\alpha} A_{\alpha} - \frac{i}{4} \omega^{2} F, \tag{13}
\]

\[
\Im \phi_{L} = B + i \omega^{\alpha} \Sigma_{\alpha} - \frac{i}{4} \omega^{2} G.
\]
Substituting (13) into the chirality conditions (8) and solving them one gets

\[ \text{Re} \phi_L = A - \omega^\alpha D^{(r)}_\alpha B + \frac{1}{4} \omega^2 D^{(r)\alpha} D^{(r)}_\alpha A, \]
\[ \text{Im} \phi_L = B + \omega^\alpha D^{(r)}_\alpha A + \frac{1}{4} \omega^2 D^{(r)\alpha} D^{(r)}_\alpha B. \]

This leads to the following solution for the \( N = 1 \) components of the shifted superfield

\[ \text{Re} \phi_\xi = A - \frac{i}{4} \xi^2 - \omega^\alpha D^{(r)}_\alpha B + \frac{1}{4} \omega^2 ( -i \xi + D^{(r)\alpha} D^{(r)}_\alpha A ), \]
\[ \text{Im} \phi_\xi = B + \omega^\alpha D^{(r)}_\alpha A + \frac{1}{4} \omega^2 D^{(r)\alpha} D^{(r)}_\alpha B. \]

It is not hard to verify that the superfield \( \Lambda_\alpha = i D^{(r)}_\alpha B \) transforms inhomogeneously under the second supersymmetry \( \delta = -\xi^{\alpha} Q_\alpha \)

\[ \delta A = \xi \xi + \frac{1}{2} \xi_\alpha D^{(r)\beta} D^{(r)}_\beta A \xi - \frac{1}{2} \xi^\beta \partial_\alpha \beta A_\xi, \]
\[ A_\xi = A - \frac{i}{4} \xi^2, \]

because of the transformation law of the Goldstone superfield \( B \) associated with the central charge

\[ \delta B = -i \xi - \xi^\alpha D^{(r)}_\alpha A_\xi. \]

Thus we conclude that the superfields \( A_\alpha \) and \( B \) actually represent the Goldstone excitations of a theory with partially broken \( N = 2, D = 3 \) supersymmetry.

Now we can directly follow the prescriptions of Ref.[2] and impose the covariant constraint

\[ (\text{Re} \phi_\xi)^2 = 0, \]

which allows one to eliminate the massive mode of supermembrane \( A \) in terms of the Goldstone superfield \( \Lambda_\alpha \)

\[ A_\xi \equiv A - \frac{i}{4} \xi^2 = \frac{-i \Lambda^2}{\xi + \sqrt{\xi^2 + (D^{(r)})^2 \Lambda^2}}. \]

This equation coincides with that obtained in [1]. The difference is in the method of its derivation.

In our approach it was obtained in the frame of the linear realization of supersymmetry after imposing the nilpotency constraint (18), while in [1] the starting point was the nonlinear realization and the constraint (18) was not exploited at all. The reason of this discrepancy is that in the NR method of PBGS the massive mode of the supermembrane and the corresponding Goldstone superfield are transformed independently with respect to both supersymmetries. Moreover there always exists the possibility of avoiding the massive mode of the supermembrane by putting to zero the corresponding \( N = 1 \) superfield without violating the covariant properties of the \( N = 2 \) chiral superfield composed of this mode and the related Goldstone superfield. Further we shall prove that both of these approaches are canonically equivalent to each other. More precisely, it will be shown that there exists the change of the coordinates involved which transforms the superfield action in the LR method of the \( 4D \) supermembrane

\[ S = \int d^3 x d^2 \tau A_\xi (x, \tau) \]
\[ \equiv -\frac{i}{4} \int d^3 x d^2 \tau d^2 \omega^2 \text{Re} \phi_\xi, \]

into the related action of the NR method and vice versa if the constraint (18) is replaced by the equivalent manifestly covariant constraint of the “pure” NR formulation

2.2. Unusual form of the nonlinear realization

We have already seen that the chiral superspace \( \mathbb{C}^{3|2} \) is the most suitable basis for describing models with partial breaking of \( N = 2, D = 3 \) supersymmetry. It seems therefore naturally to introduce the Goldstone superfields of the underlying nonlinear realization \( \psi_\alpha (\tilde{x}, \tilde{\tau}) \) and \( Q_\alpha (\tilde{x}, \tilde{\tau}) \) directly in this basis. This is a very essential point of our approach because following the chirality principle we are forced to begin with transformations of a little bit unusual form

\[ \tilde{x}_L^{\alpha \beta} = \tilde{x}_L^{\alpha \beta} - \frac{i}{4} (\xi^\alpha \tilde{\theta}^\beta + \xi^\beta \psi^\alpha (\tilde{x}_L, \tilde{\theta}) + \alpha \leftrightarrow \beta), \]
\[ \psi^\alpha (\tilde{x}_L', \tilde{\theta}') = \psi^\alpha (\tilde{x}_L, \tilde{\theta}) + \xi^\alpha. \]

\[ \tilde{\theta}^\alpha = \tilde{\theta}^\alpha + \epsilon^\alpha, \]
Note that instead original form [1] of the nonlinear realization, which is compatible with the transformations of the coordinates in real superspace (1), this one is based on the nonlinear transformations of the Goldstone superfields $\bar{\psi}_\alpha(\bar{x}, \bar{\tau})$ and $Q(\bar{x}, \bar{\tau})$ in the complex superspace $\mathbb{C}^{5|2} = \{\bar{x}, \bar{\tau}\}$. Below we will see that although this form of the nonlinear realization looks quite unusual at first glance it is equivalent to the ordinary one and transforms into it upon the canonical redefinition of coordinates. But let us first consider some unconventional properties of this nonlinear realization. It is quite easy to check that the superfunctions

$$Y^\alpha_L(\bar{x}_L, \bar{\theta}) = \bar{x}_L^\alpha + \frac{1}{4} \left( \tilde{\theta}^\alpha \bar{\psi}_\beta(\bar{x}_L, \bar{\theta}) + \alpha \leftrightarrow \beta \right),$$

$$Y^\alpha_L(\bar{x}_L, \bar{\theta}) = \frac{1}{\sqrt{2}}(\tilde{\theta}^\alpha + i\psi^\alpha(\bar{x}_L, \bar{\theta})).$$

are transformed under (21) as the coordinates of the superspace $\mathbb{C}^{5|2}$ in eq. (3). Therefore we can suggest the transformations

$$x^{\alpha\beta}_L = Y^{\alpha\beta}_L(\bar{x}_L, \bar{\theta}), \quad \theta^\alpha = Y^\alpha_L(\bar{x}_L, \bar{\theta}),$$

(23)

which establish the map $\mathbb{C}^{5|2} \rightarrow \mathbb{C}^{5|2}$. Our next step resembles the geometrical approach to supergravity by Ogievetsky and Sokatchev [5]. We consider the embedding of a real superspace $\mathbb{R}^{3|4}$ into the complex one $\mathbb{C}^{5|2}$. This embedding as we know is described by the relation (2). If we take into account the coordinate transformations (22) and (23) we find a new nontrivial axial-vector object

$$\tilde{z}^{\alpha\beta}_L = \tilde{x}^{\alpha\beta}_L + \tilde{H}^{\alpha\beta}(\tilde{x}, \tilde{\tau}, \tilde{\omega}), \quad \tilde{\theta}^\alpha = \tilde{\tau}^\alpha + i\tilde{\omega}^\alpha$$

(24)

which provides a suitable map $\mathbb{R}^{3|4} \rightarrow \mathbb{R}^{3|4}$. Note that in accordance with (21) the real spinor variable $\tilde{\omega}^\alpha$ entering eqs. (24) as the imaginary part of the complex spinor $\tilde{\theta}^\alpha$ does not transform at all with respect to both supersymmetries. This fact reflects the idea of spontaneous breaking of global supersymmetry in its pure geometrical form. The explicit form of the superfield $\tilde{H}^{\alpha\beta}(\tilde{x}, \tilde{\tau}, \tilde{\omega})$ as well as the expressions of the “old” real variables $\{x, \tau, \omega\}$ through the “new” ones $\{\tilde{x}, \tilde{\tau}, \tilde{\omega}\}$ can be read out straightforwardly from the Eqs. (2), (22) and (23). Here we shall only present the final results of these calculations

$$\tilde{H}^{\alpha\beta}(\tilde{x}, \tilde{\tau}, \tilde{\omega}) = (i\tilde{\omega}^\rho T^\rho_{\mu\nu} - \frac{1}{2} \tilde{\omega}^2 D^\mu \psi^\sigma)T^{-1\alpha\beta}_{\mu\nu},$$

(25)

$$\tilde{x}^{\alpha\beta} = \tilde{x}^{\alpha\beta}_L + \frac{i}{4} (\tilde{\omega}^\alpha \psi^\beta + \tilde{\tau}^\alpha (\tilde{\omega}^\rho D^\rho \psi^\beta + \frac{\tilde{\omega}^2}{2} D^\mu \psi^\sigma D^\rho_{\sigma\rho} \psi^\beta) + \alpha \leftrightarrow \beta),$$

(26)

$$\tilde{\tau}^\alpha = \tilde{\tau}^\alpha - \tilde{\omega}^\rho D^\rho \psi^\alpha - \frac{\tilde{\omega}^2}{2} D^\mu \psi^\sigma D^\rho_{\sigma\rho} \psi^\alpha,$$

(27)

$$\tilde{\omega}^\alpha = \tilde{\omega}^\alpha + \tilde{\psi}^\alpha(\tilde{x}, \tilde{\tau}, \tilde{\omega}),$$

(28)

$$\tilde{\bar{\psi}}^\alpha(\tilde{x}, \tilde{\tau}, \tilde{\omega}) = \psi^\alpha(\tilde{x}, \tilde{\tau}) + \tilde{\omega}^2 \tilde{D}^2 \psi^\alpha(\tilde{x}, \tilde{\tau}),$$

(29)

where

$$D^\rho = \partial^\rho + T^\rho_{\mu\nu} D^\mu,$$

(30)

$$\tilde{D}_{\mu\nu} = T^{-1\alpha\beta}_{\mu\nu} \tilde{D}^{\alpha\beta},$$

(31)

$$T^\alpha_{\mu\nu} = \frac{i}{4} (\tilde{\omega}^\alpha \tilde{\psi}_\mu + \psi^\alpha \tilde{\psi}_\mu + \alpha \leftrightarrow \beta),$$

(32)

$$T^{-1\alpha\beta}_{\mu\nu} = \frac{i}{4} (\tilde{\psi}^\alpha \tilde{\psi}_\mu + \tilde{\psi}^\alpha \tilde{\psi}_\mu + \alpha \leftrightarrow \beta),$$

(33)

are the vielbein and covariant derivatives of the $N = 2, D = 3$ NR obtained in [1]. Thus we see that the intrinsic geometry of the $N = 2, D = 3$ PBGS theory is closely related to the axial-vector prepotential $\tilde{H}^{\alpha\beta}(\tilde{x}, \tilde{\tau}, \tilde{\omega})$ which involves all the basic ingredients of the corresponding NR method. It deserves mentioning that this example is a generalization of the geometrical approach to supergravity proposed in [6] for the models with completely broken local supersymmetries. It was shown that in these cases the transition to the complex superspace also gives the nontrivial axial-vector prepotential which contains in its flat limit the covariant objects of the Volkov-Akulov
nonlinear realization [4]. It is naturally to assume, that eq.(25) describes the flat limit of the prepotential related to \( N = 2, D = 3 \) supergravity with partially broken local supersymmetry. We are not ready to consider this problem here to full extent as it raise questions which go beyond the framework of this report, and will proceed with the discussion of the superbrane theory with rigid worldvolume supersymmetry.

2.3. Connection with the usual form

Let us note that the restriction of the map \( C^{3|2} \Rightarrow \tilde{C}^{3|2} \) onto the real superspace (23), (24) leads to the following unconventional form of the NR

\[
\hat{x}^{\alpha \beta} = \tilde{x}^{\alpha \beta} - \frac{i}{4} [\epsilon^{\alpha \beta \gamma} \psi^\gamma + \xi^\alpha \psi^\beta (\tilde{x}, \tilde{\tau}, \tilde{\omega}) + \alpha \leftrightarrow \beta],
\]

\[
\hat{\tau}^\alpha = \tilde{\tau}^\alpha + \epsilon^\alpha, \quad \hat{\omega}^\alpha = \tilde{\omega}^\alpha,
\]

(33)

This form, however, is replaced by the usual one

\[
\hat{x}^{\alpha \beta} = \tilde{x}^{\alpha \beta} - \frac{i}{4} [\epsilon^{\alpha \beta \gamma} \psi^\gamma (\tilde{x}, \tilde{\tau}) + \alpha \leftrightarrow \beta],
\]

\[
\hat{\tau}^\alpha = \tilde{\tau}^\alpha + \epsilon^\alpha, \quad \hat{\omega}^\alpha = \tilde{\omega}^\alpha,
\]

(34)

\[
\psi^\alpha (\tilde{x}', \tilde{\tau}', \tilde{\omega}') = \tilde{\psi}^\alpha (\tilde{x}, \tilde{\tau}, \tilde{\omega}) + \xi^\alpha.
\]

upon a redefinition of the coordinates

\[
\tilde{z} = \{ \hat{x}, \hat{\tau}, \hat{\omega} \} \Rightarrow \hat{z} = \{ \tilde{x}, \tilde{\tau}, \tilde{\omega} \}
\]

(35)

To get the explicit form of the transformations (35) one should change the variables as follows

\[
x^{\alpha \beta} = \hat{x}^{\alpha \beta} + \frac{i}{4} [\omega^\alpha \psi^\beta (\hat{x}, \hat{\tau}) + \alpha \leftrightarrow \beta],
\]

\[
\tau^\alpha = \hat{\tau}^\alpha, \quad \omega^\alpha = \hat{\omega}^\alpha + \psi^\alpha (\hat{x}, \hat{\tau}),
\]

which provides the usual form of the nonlinear realization (34) \(^3\), and then inverting the transformations (36)

\[
\hat{x}^{\alpha \beta} = x^{\alpha \beta} - \frac{i}{4} (\omega^\alpha \psi^\beta + \alpha \leftrightarrow \beta),
\]

(37)

\(^3\)One can check that when applied to the r.h.s. of the Eqs. (36) the transformations (34) imply the ordinary transformation laws of the variables \( \{ x, \tau, \omega \} \).

\[
\hat{\omega}^\alpha = \omega^\alpha - \psi^\alpha + \frac{i}{2} \omega^\mu \psi^\nu \partial_{\mu \nu} \psi^\alpha
\]

\[- \frac{1}{8} \omega^2 \varepsilon^{\mu \rho \sigma} (\partial_{\rho \sigma} \psi^\nu \partial_{\mu \nu} \psi^\alpha + \frac{1}{2} \psi^\nu \partial_{\mu \nu} \partial_{\rho \sigma} \psi^\alpha),
\]

\[
\hat{\tau}^\alpha = \tau^\alpha
\]

by substituting into (36) the superfunctions (26), (28), (29) instead of the variables \( z = \{ x, \tau, \omega \} \).

3. The limit of pure nonlinear realization

Now we are in a position to consider in more detail the condition (18). We have already mentioned that this requirement can be treated as the condition of transition to the pure version of the NR. What does it mean from the physical point of view? Here we are going to get the transparent answer to this question using the standard technic based on the map of the variables in superspace \([14,6]\) which establishes the connection between linear and nonlinear realizations.

3.1. Quasilinear scalar supermultiplet

Let us, as in the LR method (see Eqs.(1) and (10)) supply the nonlinear realizations (34) with the Goldstone superfield associated with the central charge

\[
\hat{\phi}_\xi (\tilde{x}', \tilde{\tau}', \tilde{\omega}') = \hat{\phi}_\xi (\hat{x}, \hat{\tau}, \hat{\omega}) + \xi^\alpha (\hat{x}, \hat{\tau}, \hat{\omega})
\]

(41)

From this equation it is quite easy to see that the superfield \( \hat{\phi}_\xi (\hat{x}, \hat{\tau}, \hat{\omega}) - (\xi/2) \tilde{\omega}_\alpha \) has the same transformation law as the LR superfield \( \phi_\xi (x, \tau, \omega) \) and therefore can be identified with the former

\[
\phi_\xi (x, \tau, \omega) = \hat{\phi}_\xi (\hat{x}, \hat{\tau}, \hat{\omega}) - \frac{\xi}{2} \tilde{\omega}_\alpha.
\]

(39)

Thus, after performing the change of the variables (36) in the LR superfields (9) one gets

\[
\phi_L (x, \tau, \omega) = \hat{\phi}_L (\hat{x}, \hat{\tau}, \hat{\omega}) + \frac{\xi}{2} \tilde{\omega}_\alpha (\hat{x}, \hat{\tau})
\]

\[- \frac{\xi}{2} \tilde{\tau}_\alpha \tilde{\omega}_\alpha + \frac{\xi}{4} \tilde{\omega}^2,
\]

(40)

here the chiral superfield of the NR is introduced

\[
\hat{\phi}_L (\hat{x}, \hat{\tau}, \hat{\omega}) = \hat{\phi}_\xi (\hat{x}, \hat{\tau}, \hat{\omega}) + \frac{\xi}{4} (\tilde{\tau}^2 + \psi^2 (\hat{x}, \hat{\tau})).
\]

(41)
Eqs. (40) and (41) together with (36) establish the required interrelations between the superfields of linear and nonlinear realizations describing the 4D supermembrane. Recall, however, that when deriving these relations we introduced an additional superfield \( \psi_\alpha(\hat{x}, \tau) \), which is a new independent superfield unless it is canonically related to the analogous Goldstone superfield of the LR method \( \Lambda_\alpha(x, \tau) \). To avoid the doubling of the Goldstone degrees of freedom in our approach let us study the transformation laws of the real and imaginary parts of the superfield \( \hat{\phi}_\xi \). As it follows from the eq. (39) its real part does not transform at all because of the relation

\[
\text{Re} \hat{\phi}_\xi(x, \tau, \omega) = \text{Re} \hat{\phi}_\xi(\hat{x}, \tau, \hat{\omega}) \tag{42}
\]

and the transformations (10). Hence, all the \( N = 1 \)-components in the \( \hat{\omega} \)-decomposition

\[
\text{Re} \hat{\phi}_\xi = \hat{A} - \frac{i \xi}{4} (\tau^2 + \psi^2) + i \hat{\omega}^\alpha \dot{\Lambda}_\alpha - \frac{i}{4} \hat{\omega}^2 \hat{F}, \tag{43}
\]

which follows from the Eq.(41) and the corresponding decompositions of chiral superfield

\[
\text{Re} \hat{\phi}_L = \hat{A} + i \hat{\omega}^\alpha \dot{\Lambda}_\alpha - \frac{i}{4} \hat{\omega}^2 \hat{F}, \tag{44}
\]

\[
\text{Im} \hat{\phi}_L = \hat{B} + i \hat{\omega}^\alpha \dot{\Sigma}_\alpha - \frac{i}{4} \hat{\omega}^2 \hat{G},
\]

are transformed independently of each other. This allows one to put any of them equal to zero without the violation of the covariant properties of the theory. For instance, one can check that the constraint

\[
\dot{\Lambda}_\alpha = 0 \tag{45}
\]

makes equal the number of the Goldstone degrees of freedom in the two sides of the eq. (39). To see this let us substitute the eq. (40) into the chirality condition (8). Using the transformations of the vector and spinor covariant derivatives

\[
D^{(\tau)}_\alpha = \partial^{(\tau)}_\alpha + \frac{i}{4} (\hat{\omega}^\rho \partial_\rho \psi^\sigma + \rho \leftrightarrow \sigma) \partial_{\rho \sigma} - \partial_\alpha \psi \partial_\rho \hat{\omega}^\rho, \tag{46}
\]

\[
\partial_{\alpha \beta} = X_{\alpha \beta}^{-1}(\partial_{\rho \sigma} - \partial_{\rho} \psi \partial_{\sigma} \hat{\omega}),
\]

\[
X_{\alpha \beta}^{\rho \sigma} = \delta_{\alpha \beta}^{\rho \sigma} + \frac{i}{4} (\hat{\omega}^\rho \partial_\rho \psi^\sigma + \rho \leftrightarrow \sigma),
\]

\[
D^{(\omega)}_\alpha = \partial_{\alpha}^{(\omega)} + \frac{i}{2} \hat{\omega}^\beta \partial_{\alpha \beta}, \quad \partial_{\alpha}^{(\omega)} = \frac{\partial}{\partial \omega^\alpha}
\]

where \( D_\alpha \) and \( D_{\alpha \beta} \) are defined in (31), (32), and taking into account eqs. (44) one can solve the constraints (8) in terms of fields of the NR method. The final form of the solutions is rather complicated but it is significantly simplified when the restriction (45) together with the covariant constraint

\[
\hat{A}_\xi \equiv \hat{A} - \frac{i \xi}{4} (\tau^2 + \psi^2) = 0, \tag{47}
\]

are imposed. In this case we arrive at the following form of the quasilinear chiral superfield of the NR

\[
\text{Re} \hat{\phi}_L = \frac{i \xi}{4} (\tau^2 + \psi^2) + \frac{i \xi}{2} \hat{\omega}^2 \frac{1}{1 + \frac{1}{2} D_\alpha \psi \hat{D}_\alpha \psi}, \tag{48}
\]

\[
\text{Im} \hat{\phi}_L = \frac{i \xi}{4} \hat{\omega}^2 \frac{\hat{D}_\gamma \psi \hat{D}_\gamma \psi}{1 + \frac{1}{2} D_\alpha \psi \hat{D}_\alpha \psi}.
\]

Note that in eqs. (48) superfields \( \psi_\alpha \) and \( \hat{B} \) are not independent but express through the scalar Goldstone superfield \( Q \)

\[
\psi_\alpha = i \xi^{-1} D_\alpha Q, \quad \hat{B} = Q + \frac{i \xi}{2} \hat{\epsilon}^\beta \psi_\beta,
\]

associated with the central charge in the NR [1]. This superfield is canonically equivalent to the superfield \( B \) of LR owing to the Eqs.(36) and (41). For instance putting the first of them onto the surface \( x^{\alpha \beta} = \hat{x}^{\alpha \beta}, \omega^\alpha = \psi^\alpha \) one obtains

\[
\hat{A} = A + i \psi \dot{\Lambda}_\alpha + \frac{1}{4} \psi^2 D^{(\tau)}_\alpha D^{(\tau)}_\beta A, \tag{50}
\]

where \( \hat{A} \) and \( A \) are defined in (47) and (19). One can checks that the equation (50) has the following well-known solution [1,3]

\[
\psi_\alpha = \frac{\Lambda^\alpha}{\xi + \frac{1}{2} D^{(\tau)} \hat{D}_\alpha \Lambda}, \tag{51}
\]

\[
M \equiv \frac{\Lambda^2}{\xi + \sqrt{\xi^2 + 4 D^{(\tau)} \hat{D}_\alpha \Lambda^2}}.
\]

Substituting this solution into the corresponding equation for the second scalar superfield

\[
\hat{B} = B + \psi_\alpha D^{(\tau)}_\alpha A + \frac{1}{4} \psi^2 D^{(\tau)}_\alpha D^{(\tau)}_\beta B \tag{52}
\]
we get one more relation
\[ Q = B - \frac{3}{4} \frac{M D^{(\tau)}_{\alpha} D^{(\tau)}_{\beta} B}{\xi + \sqrt{\xi^2 + D^{\alpha} D^{\beta}}}, \tag{53} \]
which proves the equivalence of linear and nonlinear parameterizations of theory.

In this connection one should note that after imposing the constraint (47) all the components of the superfield \( \phi_L \) are expressed through the scalar Goldstone superfield of the central charge only. This superfield (which we call quasilinear superfield) is associated with the limit of the pure nonlinear realization. From the physical point of view the latter describes the models in which the mass of the massive mode of supermembrane tends to infinity. Thus we see that eq. (47) describes the constraint which removes the contribution of the corresponding degree of freedom in this limit in its manifestly covariant and physically transparent form. The remarkable feature of this constraint is that it together with (45) automatically reproduces the nilpotency constraint à la Roček and Tseytlin (18).

3.2. Quasilinear vector supermultiplet
In the case of a \( D^2 \)-brane the situation becomes substantially more complicated because of another linear supermultiplet which involves a corresponding worldvolume Goldstone superfield [1,13].

Recall that this model is described by a real linear superfield satisfying the deformed constraints [13]
\[ (D^{(\tau)}_{\alpha} D^{(\tau)}_{\beta} - D^{(\omega)}_{\alpha} D^{(\omega)}_{\beta}) W_\xi = 2i\xi, \]
\[ D^{(\tau)}_{\alpha} D^{(\omega)}_{\beta} W_\xi = 0. \tag{54} \]
The solution of this constraints can be written in the form
\[ W_\xi = W + \frac{i\xi}{2} \omega^2, \tag{55} \]
where \( W \) is restricted by ordinary constraints with \( \xi = 0 \). To be able to derive the corresponding Born-Infeld action we should impose one more constraint
\[ W^2_\xi = 0, \tag{56} \]
which as before, allows one to avoid the massive degree of freedom. If we would like to know what does it mean from the point of view of the NR method we must perform the transformations (36) in the shifted superfield
\[ W_\xi(x, \tau, \omega) = \hat{W}_\xi(\hat{x}, \hat{\tau}, \hat{\omega}). \tag{57} \]
Now let us consider the following Ansatz of quasilinearity
\[ \hat{W}_\xi(\hat{x}, \hat{\tau}, \hat{\omega}) = -\frac{i}{4} \omega^2 \hat{F}, \tag{58} \]
which solves exactly the constraint (56). Substituting Eq. (58) into the irreducibility conditions (54) and solving it one discovers
\[ \hat{W}_\xi(\hat{x}, \hat{\tau}, \hat{\omega}) = -\frac{i\xi}{2} \omega^2 \frac{1}{1 - \frac{i}{2} D^\alpha \psi^\beta D_\alpha \psi_\beta}, \]
\[ D^\alpha \psi_\alpha = 0. \tag{59} \]
Thus we get the \( D^2 \)-brane quasilinear superfield (59). Note that it differs from the analogous superfield of the supermembrane (48) not only by the sign in the denominator of the highest component \( \hat{F} \) but also because it satisfies essentially different constraints. As in the case of supermembrane this constraint can be rewritten in terms of the Goldstone superfield of the underlying LR method
\[ D^{(\tau)}_{\alpha} \Lambda_\alpha = 0. \tag{60} \]
The latter follows immediately from eqs. (54) and (55) applied to the \( N = 1 \) decomposition of the superfield \( W \)
\[ W = A + i\omega^\alpha \Lambda_\alpha + i\omega^2 \hat{F}. \tag{61} \]
In eq. (61) the component fields \( A \) and \( \hat{F} \) are chosen to be real. As it was shown in [1] the constraint (60) ensures between the supermembrane and the \( D^2 \)-brane on the level of the corresponding superfield actions.

4. Actions
Now we would like to discuss the problem of actions. In the framework of the linear realizations the actions for the supermembrane and the
\( D = 2 \)-brane were constructed in [1]. However, the corresponding action for the superfields of the nonlinear realization is known only for the case of the supermembrane [1,3]. Having at our disposal the quasilinear superfields (48) and (59) we can get both forms of the actions starting straightforwardly from the underlying actions of the LR formulation:

\[
S_{SM} = -\frac{i}{4} \int d^3x d^2\tau d^2\omega \omega^2 \Re \phi \xi, \quad (62)
\]

\[
S_{D2} = -\frac{i}{4} \int d^3x d^2\tau d^2\omega \omega^2 W \xi.
\]

Making in (62) the change of coordinates \( z = \{x, \tau, \omega\} \Rightarrow \hat{z} = \{\hat{x}, \hat{\tau}, \hat{\omega}\} \) one gets

\[
S_{SM} = -\frac{i}{4} \int d^3\hat{x} d^2\hat{\tau} d^2\hat{\omega} \text{ sdet} \left( \frac{\partial z}{\partial \hat{z}} \right) \left( \hat{\omega} + \psi \right)^2 \Re \hat{\phi} \hat{\xi},
\]

\[
S_{D2} = -\frac{i}{4} \int d^3\hat{x} d^2\hat{\tau} d^2\hat{\omega} \text{ sdet} \left( \frac{\partial z}{\partial \hat{z}} \right) \left( \hat{\omega} + \psi \right)^2 \hat{W} \xi.
\]

Despite of its complexity though these actions look rather complicated they can be represented in a compact explicit form due to a very special structure of the quasilinear superfield (48) and (59)

\[
S_{SM} = -\frac{\xi}{2} \int d^3\hat{x} d^2\hat{\tau} \frac{\psi^2}{1 + \frac{1}{2} D^\alpha \psi^\alpha D_\alpha \psi}, \quad (63)
\]

\[
S_{D2} = -\frac{\xi}{2} \int d^3\hat{x} d^2\hat{\tau} \frac{\psi^2}{1 - \frac{1}{2} D^\alpha \psi^\alpha D_\alpha \psi}.
\]

Note that in these actions the superfields \( \psi_\alpha \) are constrained by the conditions (49) and (59), respectively.

5. Conclusion

In the present talk we have described the connection between the linear and nonlinear realizations of models with partially broken global \( N = 2 \), \( D = 3 \) supersymmetry. With the example of the 4\( D \) supermembrane and the \( D2 \)-brane we have shown that these realizations are canonically equivalent to each other off the mass shell due to the existence of the map transformations relating the corresponding Goldstone superfields. It is quite evident, however, that this approach is not restricted to these canonical models only. It can be successfully applied to any \( PBGS \) models which allow for a superspace formulation. But the most essential feature of this approach is that it opens a new possibilities of investigation of models with partially broken \textit{local} supersymmetries.

We hope to consider these problems in detail in the framework of the nonlinear realization of \( PBLS \) gauge theory in forthcoming publications.

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