In this paper, we discuss physics potential of the Very Long Base-Line (VLBL) Neutrino-Oscillation Experiments with the High Intensity Proton Accelerator (HIPA), which is planned to be built by 2006 in Tokaimura, Japan. We propose to use conventional narrow-band νμ beams (NBB) from HIPA for observing the νμ → ντ transition probability and the νμ survival probability. The pulsed NBB allows us to obtain useful information through counting experiments at a huge water-Cherenkov detector which may be placed in our neighbor countries. We study sensitivity of such an experiment to the neutrino mass hierarchy, the mass-squared differences, the mixing angles and the CP phase of the 3 × 3 lepton flavor mixing matrix (MNS matrix). The CP phase can be measured with a 100kt detector if both the mass-squared difference and Uei3 elements of the MNS matrix are sufficiently large.

1 Introduction

Very long base-line (VLBL) neutrino-oscillation experiments is one of the attractive experiments in the near future. In this paper, we discuss physics potential of the VLBL neutrino-oscillation experiments with the High Intensity Proton Accelerator (HIPA), which was approved by the Japanese government last December and will be built by 2006 in Tokaimura, Japan.

The JHF Neutrino Working Group proposes the first phase neutrino-oscillation experiments with HIPA and Super-Kamiokande, whose base-line length is 295 km. The Letter of Intent (LOI) is available on their web site.

In this paper, we discuss possible future VLBL neutrino-oscillation experiment between HIPA and Beijing. The base-line length of this experiment is about 2100 km.

In our analysis, we propose to use the conventional pulsed narrow-band νμ beams (NBB) from HIPA for observing the νμ → ντ transition probability and the νμ survival probability at a huge 100kt-level detector in Beijing. We can then obtain useful information through counting experiments e.g. by adopting a water-Cherenkov detector. We study the sensitivity of such experiment to the neutrino mass hierarchy, that is the sign of mass-squared differences, the mixing angles and the CP phase in the 3 × 3 lepton flavor mixing matrix, the MNS (Maki-Nakagawa-Sakata) matrix, and the magnitudes of the two mass-squared differences. We can determine the neutrino mass hierarchy pattern from this experiment. The CP phase can be measured if both the mass-squared difference and all three mixing angles are sufficiently large.

2 MNS matrix

In this analysis, we assume three neutrino spices. We define the MNS matrix as

\[
J^\mu_\alpha = \begin{pmatrix} & & & \end{pmatrix}(1 - \gamma_5)U^T_{\text{MNS}}(u, c, t)T + \begin{pmatrix} & & & \end{pmatrix}(1 - \gamma_5)V^\dagger_{\text{CKM}}(\nu_1, \nu_2, \nu_3)T, \tag{1}
\]

where \(u, d, c, s, t, b\) are the quark mass-eigenstates, \(e, \mu, \tau\) are the charged-lepton mass-eigenstates, and \(\nu_i \ (i = 1, 2, 3)\) is the neutrino mass-eigenstate. The MNS matrix connects the mass eigenstate \(\nu_i \ (i = 1, 2, 3)\) to the flavor eigenstate \(\nu_\alpha \ (\alpha = e, \mu, \tau)\)

\[
\nu_\alpha = \sum_{i=1}^{3} (V_{\text{MNS}})_{\alpha i} \nu_i . \tag{2}
\]

The MNS matrix can be parameterized as

\[
V_{\text{MNS}} = U_{\text{MNS}} \begin{pmatrix} & & & \end{pmatrix} = \begin{pmatrix} U_{\mu e} & U_{\mu \mu} & U_{\mu \tau} \\ U_{\tau e} & U_{\tau \mu} & U_{\tau \tau} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{pmatrix} \tag{3}
\]

If the LSND results are confirmed, we have to revise this analysis.
where the matrix $\mathcal{P}$ is the Majorana-phase matrix. If neutrinos were not Majorana particles, this phase matrix can be absorbed away by the phases of the right-handed neutrino fields. The neutrino oscillation experiments are not sensitive to this phase matrix. The matrix $U_{MNS}$ has three mixing angles and one $CP$ phase, just like the CKM matrix. We can always take these upper-right elements, $U_{e2}$, $U_{e3}$, and $U_{\mu 3}$, as the independent parameters of the MNS matrix. Both $U_{e2}$ and $U_{e3}$ have the non-negative real values and $U_{\mu 3}$ can be complex number, which are related to the three mixing angles and one $CP$ phase. The other elements are determined by the unitarity conditions.

We take into account existing constraints for the MNS matrix and the mass-squared differences as follows. From the atmospheric-neutrino oscillation measurements

$$\sin^2 2\theta_{\text{ATM}} = 1.0, \delta m_{\text{ATM}}^2 = 3.5 \times 10^{-3} \text{(eV)}^2. \quad (4)$$

From solar-neutrino deficit observations

$$\sin^2 2\theta_{\text{SOL}} = 0.8, \delta m_{\text{SOL}}^2 = 5.15 \times 10^{-5} \text{(eV)}^2, \quad (5)$$

large-mixing-angle MSW$^9$ solution (LMA)

$$\sin^2 2\theta_{\text{SOL}} = 7 \times 10^{-3}, \delta m_{\text{SOL}}^2 = 5 \times 10^{-6} \text{(eV)}^2, \quad (6)$$

small-mixing-angle MSW$^9$ solution (SMA)

$$\sin^2 2\theta_{\text{SOL}} = 0.9, \delta m_{\text{SOL}}^2 = 7 \times 10^{-11} \text{(eV)}^2. \quad (7)$$

From the CHOOZ reactor experiments$^{11}$,

$$\sin^2 2\theta_{\text{CHOOZ}} < 0.1 \text{ when } \delta m_{\text{CHOOZ}}^2 > 10^{-3} \text{(eV)}^2. \quad (8)$$

The four independent parameters of the MNS matrix are related to the above observables and $CP$ phase :

$$2|U_{e3}|^2 = 1 - \sqrt{1 - \sin^2 2\theta_{\text{CHOOZ}}}, \quad (9)$$

$$2|U_{\mu 3}|^2 = 1 - \sqrt{1 - \sin^2 2\theta_{\text{ATM}}}, \quad (10)$$

$$2|U_{e2}|^2 = 1 - |U_{e3}|^2 - \sqrt{(1 - |U_{e3}|^2)^2 - \sin^2 2\theta_{\text{SOL}}}, \quad (11)$$

$$\arg(U_{\mu 3}) = -\delta_{\text{MNS}}. \quad (12)$$

The first three equations are obtained for the observed mass-squared differences

$$\delta m_{\text{SOL}}^2 = |\delta m_{12}|^2 \ll |\delta m_{13}|^2 = \delta m_{\text{ATM}}^2, \quad (13)$$

where $\delta m_{ij}^2 \equiv m_i^2 - m_j^2$.

### 3 Probability and mass hierarchies

The Hamiltonian in the matter is written as

$$\mathcal{H}_{\alpha\beta} = \frac{1}{2E_e} \left( \delta m_{12}^2 U_{e1}^* U_{\nu_2} + \delta m_{13}^2 U_{e2}^* U_{\nu_3} + \delta \text{m}_{ijkl} \delta_{\beta\gamma} \right), \quad (14)$$

where $\alpha$ and $\beta$ are flavor indices $(e, \mu, \tau)$ and $A$ measures the matter effect,

$$A = 2\sqrt{2} G_F Y_e \rho E_e, \quad (15)$$

$$= 7.56 \times 10^{-5} \left( \frac{\rho}{\text{g/cm}^2} \right) \left( \frac{E_e}{\text{GeV}} \right).$$

We assume that the matter density of the earth’s crust is constant at $\rho = 3$. We can then diagonalize the Hamiltonian, eq.(14) as

$$\mathcal{H} = \frac{1}{2E_e} \hat{U} \left( \begin{array}{ccc} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{array} \right) \hat{U}^\dagger, \quad (16)$$

where $\hat{U}$ is the MNS matrix in the matter. We introduce the oscillation phase parameters in the matter as

$$\hat{\Delta}_{ij} = \frac{\lambda_i - \lambda_j}{2E_e} L/h c. \quad (17)$$

By using eq.(16) and eq.(17), the oscillation probability in the matter is obtained as

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \left\{ \text{Re} \left[ \bar{U}_{\nu_\alpha} \bar{U}_{\nu_3}^* \bar{U}_{\nu_2} \right] \sin^2 \frac{\hat{\Delta}_{12}}{2} \right\}$$

$$+ \left\{ \text{Re} \left[ \bar{U}_{\nu_2} \bar{U}_{\nu_3} \bar{U}_{\nu_3} \right] \sin^2 \frac{\hat{\Delta}_{23}}{2} \right\}$$

$$+ \left\{ \text{Re} \left[ \bar{U}_{\nu_3} \bar{U}_{\nu_1} \bar{U}_{\nu_1} \right] \sin^2 \frac{\hat{\Delta}_{31}}{2} \right\}$$

$$+ 2J \left[ \sin \hat{\Delta}_{12} + \sin \hat{\Delta}_{23} + \sin \hat{\Delta}_{31} \right], \quad (18)$$

where

$$J = \text{Im} \left[ \bar{U}_{\nu_1} \bar{U}_{\nu_3} \bar{U}_{\nu_2} \right] \quad (19)$$

for $(\alpha, \beta) = (e, \mu), (\mu, \tau), \text{ or } (\tau, e)$, the Jarlskog parameter$^{12}$ of the lepton sector in the matter.

There are four types of mass hierarchies, as shown at Table 1. When the MSW solutions are chosen for the solar neutrino deficit problem, only the mass hierarchies I and III are relevant. Below we show results for all four mass hierarchies because the anti-neutrino oscillation probabilities in the matter are related to those for neutrinos,

$$P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)_{I,II,III,IV} = P(\nu_\alpha \to \nu_\beta)_{IV,III,II,1} \quad (20)$$

in the limit of spherically symmetric earth. The number of anti-neutrino events for the mass hierarchy case I is obtained from that of neutrino events for the hierarchy IV, by taking account of the difference in flux and cross sections.
δm

where Mν the transition or survival probability of µ expected number of e-like and sin E peak energy, the smooth fitted curves parameterized by the neutrino number of events, N are obtained from the computer simulation and we use δm squared differences as σ ber, µ vertical axis stand for the number of e-like and µ numbers of e-like and µ only one variable, Φ(E, p)=[0, E=Eν](beam flux)ν(Time =3 ATM 2

First, we show the CP phase dependence of the expected number of e-like and µ-like events at Beijing for the NBB with Eν =4GeV and 6GeV, and for 100kyear. In this figure, we fix the mixing angles as sin2 2θSOL =0.8, sin2 2θCHOOZ =0.1, and sin2 2θATM =1.0, and the mass-squared differences as δm2SOL =1.0×10−3(eV2) and δm2ATM =3.5×10−3(eV2). In each figure, horizontal and vertical axis stand for the number of e-like and µ-like events, respectively. The solid-circle, square, open-circle, and open-square marks for δmν =0°, 90°, 180°, and 270°, respectively.

In Figure 3, we show the expected numbers of e-like and µ-like events per year at a 100kt detector on Beijing for the NBB with Eν =4GeV (left) and 6GeV (right). Predictions of the VO scenario, the SMA solution, and the LMA solution with δm2SOL =5×10−5 and 15×10−5 eV2 are shown. Five points or five circles with increasing N(e) show expectations for five values of sin2 2θCHOOZ, 0.02, 0.04, 0.06, 0.08, to 0.1. The predictions of the VO scenario are common for the mass hierarchies I and II, which are shown by the five dots with larger N(e), and also for the case III and IV, the five dots with smaller N(e). The SMA scenario predicts smaller N(µ) in cases I and IV, and larger N(µ) in cases II and III than that of the VO scenario. The LMA scenario predicts even smaller N(µ) in cases I and IV, and larger N(µ) for cases II and III, where the deviation from the VO scenario is more significant for larger δm2SOL.

6 Summary

In this paper, we study the prospects of very long baseline (VLBL) neutrino-oscillation experiments with the High Intensity Proton Accelerator (HIPA). We present results for a VLBL experiment between HIPA and Beijing, where the base-line length is about 2100 km. We propose to use conventional pulsed-narrow-band νµ beams and a huge water-Cherenkov detector of 100kt in mass. The detector should distinguish e-like, µ-like and neutral current events but it is not required to measure the neutrino-energy. We study the sensitivity of such an experiment to the signs and the magnitudes of the neutrino mass-squared differences, the mixing angles, and the CP phase of the 3 × 3 lepton flavor mixing matrix.
(MNS matrix), by using the $\nu_\mu \rightarrow \nu_e$ transition probability and the $\nu_\mu$ survival probability. We find that the neutrino mass hierarchies can be determined from this experiment within several years. The CP phase can be measured if both the mass-squared difference and all the mixing angles of the MNS matrix are sufficiently large.

References

1. JHF Neutrino Working Group, their web-site is http://neutrino.kek.jp/jhfnu.