Small Numbers From Tunnelling Between Brane Throats

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Generic classes of string compactifications include “brane throats” emanating from the compact dimensions and separated by effective potential barriers raised by the background gravitational fields. The interaction of observers inside different throats occurs via tunnelling and is consequently weak. This provides a new mechanism for generating small numbers in Nature. We apply it to the hierarchy problem, where supersymmetry breaking near the unification scale causes TeV sparticle masses inside the standard model throat. We also design naturally long-lived cold dark matter which decays within a Hubble time to the approximate conformal matter of a long throat. This may soften structure formation at galactic scales and raises the possibility that much of the dark matter of the universe is conformal matter. Finally, the tunnelling rate shows that the coupling between throats,
mediated by bulk modes, is stronger than a naive application of holography suggests.

April 2001
1. Introduction

The enormous differences in scales that appear in Nature present a formidable challenge for any unified theory of forces. Grand unification addresses this problem by postulating an energy desert separating the gravitational and the electroweak scale [1]. The supersymmetric version of this picture [2], the supersymmetric standard model (SSM), has had a quantitative success: the unification prediction of the value of the weak mixing angle [2], subsequently confirmed by the LEP and SLC experiments. While this picture is attractive, it leaves many fundamental questions unanswered. There are 125 unexplained parameters, many of them mysteriously small; these include the masses of the three generations of particles and the cosmological constant. String theory provides a natural framework for addressing these questions. Many scenarios for string phenomenology involve localized gauge fields. Perhaps the simplest is the minimal Hořava-Witten theory [3,4]; other models use “D-brane” defects on which gauge dynamics occurs [5]. A striking possibility emerging from these ingredients is a new explanation the weakness of gravity [6]. These ideas are providing new avenues for exploring physics beyond the Standard Model, and novel mechanisms for explaining small numbers [7,8,9].

Hořava-Witten theory and the perturbative $E_8 \times E_8$ heterotic string [10] have been well studied in calculable, weakly-coupled regimes. In this note we will study string phenomenology in a different calculable regime, which can arise when there are many branes transverse to the compactification manifold $M$. The tension of the branes curves the space around them. The backreaction is proportional to the sum of brane tensions, and therefore to the total number of branes in some region of space. Hence solitary branes have little effect and their neighborhood is nearly flat. Such “dilute gases” of branes are commonly studied in e.g. perturbative string orientifold constructions. In other regimes of couplings where a (super)gravity description is valid, large stacks of branes in the compactification manifold $M$ significantly alter the metric on $M$. The regions of space where the branes reside may be viewed as gravitational funnels, or throats. Examples in this regime arise in F-theory compactifications on elliptic Calabi-Yau fourfolds [11]. From the 4d point of view the geometry is “warped” – the scale factor of the 4d metric depends on the distance down the throat.
In such models, the ensuing geometry of the compactification resembles an “octopus,” where the legs represent throats arising from stacks of branes, as depicted in Fig. 1. The (super)gravity modes in the throat and the low-energy field theory on the branes are dual to each other [12]: the degrees of freedom localized at the ends of the throats are dual to infrared (IR) excitations of the field theory, while the excitations closer to the mouth of the throat are dual to the ultraviolet (UV) degrees of freedom.

This geometry suggests a new mechanism for generating small numbers in 4d physics. The mutual couplings of the IR degrees of freedom residing in different throats are suppressed, as these modes must tunnel through the bulk to communicate. In this paper, we make this intuition precise in a 5d toy model of Fig. 1, which appears in Fig. 2. We study a pair of brane throats that are joined at a “UV brane” playing the role of the bulk of $M$. We then show that the KK modes of the 5d gravity theory localized in adjacent throats must tunnel to communicate with each other; this effect can generate small numbers. We study applications to SUSY breaking and to astrophysical dark matter. For simplicity and to facilitate a holographic interpretation we will take AdS metrics in the throats of our toy model. The effects we study would persist with much more generic warped metrics (including those with only power-law warping). This note is a summary of [13], where detailed derivations appear.
2. Tunnelling, Glueball Decay, and Dark Matter

The Tunnelling Calculation

To get the model depicted in Fig. 2, we choose a 5d metric:

$$ds^2 = \frac{L^2}{(|z| + L)^2} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2), \quad -l_1 \leq z \leq l_2 .$$  \hfill (2.1)

Here $x^\mu$ are the coordinates of our 4d world, and $z$ is the coordinate down the throat. $l_1 = L e^{R_1/L}$ and $l_2 = L e^{R_2/L}$, and $R_{1,2}$ are the proper distances from the UV brane to the left/right IR branes. To analyze the KK spectrum, define [8]

$$h_{\mu\nu}(x, z) = \sqrt{\frac{L}{|z| + L}} e^{ip \cdot x} \psi_{\mu\nu}(z)$$  \hfill (2.2)

where $h_{\mu\nu}$ is the 5d graviton. The transverse, traceless modes of $h_{\mu\nu}$ satisfy

$$\partial_z^2 \psi + (m^2 - \frac{15}{4(|z| + L)^2}) \psi = 0, \quad -l_1 \leq z \leq l_2$$  \hfill (2.3)

with appropriate boundary conditions at the branes, where $p^2 = m^2$ is the 4d KK mass of the mode. This is an effective Schrödinger equation with a potential barrier arising from
the warped metric (2.1). The 4d and 5d Planck masses scale out this equation. We expect the low-lying modes in the left/right throat to have masses $\sim 1/l_{1,2}$, so they must tunnel to communicate.

Qualitatively similar barriers arise for any minimally coupled modes in non-AdS backgrounds. The only differences are the explicit relationship between the proper bulk distances and the conformal distances $z$, and between the parameters of the potential and the bulk scales. Hence our analysis should carry over to those cases.

We would like to analyze the tunnelling amplitude, for which the WKB approximation will suffice. The potential in the Schrödinger equation (2.3) is

$$V(z) = \frac{15}{4(|z| + L)^2}.$$  \hspace{1cm} (2.4)

The turning points for classical motion of a mode of energy $m^2$ occur at $|z| + L = \pm \sqrt{\frac{15}{2}} \frac{1}{m}$. Evaluating $\int \sqrt{V - E}$ between the turning points as required, and neglecting the $m^2$ in $\sqrt{V - E}$, one finds the WKB suppression factor $e^{-S} \sim (mL)^{\sqrt{15}}$.

Thus the tunnelling probability for a left mode of mass $m$ going to the right is

$$P \sim (mL)^{\sqrt{15}}.$$ \hspace{1cm} (2.5)

The flux of a left mode at the barrier goes like the mass $m \sim \frac{1}{l_1}$, so the tunnelling rate is:

$$\Gamma \sim m(mL)^{\sqrt{15}}.$$ \hspace{1cm} (2.6)

This can be expressed in terms of $R_1$ using $m \sim \frac{1}{l_1} = \frac{1}{L} e^{-\frac{2\pi}{l_1}}$. A more precise calculation, discussed in [13], yields

$$\Gamma \sim m(mL)^4,$$ \hspace{1cm} (2.7)

so the approximation (2.6) is reasonable.

**Holographic Interpretation**

The AdS/CFT correspondence [12] states that each AdS throat can be viewed as a large $N$ gauge theory at strong 't Hooft coupling. The IR branes provide a schematic representation of confinement or a mass gap. However, when applying this correspondence to our background, the usual UV/IR relationship becomes more complicated: there are light degrees of freedom localized in each throat, as well as light “closed string modes” localized in the bulk of $M$. The light KK modes in each throat can be thought of as “glueballs” of the dual large $N$ gauge theories. Therefore, eq. (2.6) or (2.7) is the rate
at which the heavier glueballs of one strongly coupled gauge theory decay to the lighter glueballs of the other.

A priori, one might have expected our model to be dual to a system of two gauge theories coupled to each other via 4d gravity (naively extending the suggestions in [14]). The term in the interaction Lagrangian inducing glueball decay would be the dimension 8 operator
\[ \mathcal{L}_{\text{int}} \sim \frac{1}{M_4^4} T^{(L)}_{\mu\nu} T^{\mu\nu(R)}, \] (2.8)
where \( T^{(L,R)} \) are the stress tensors of the left and right dual gauge theories. Such an interaction would yield a decay rate
\[ \Gamma \sim \frac{m^9}{M_4^8}. \] (2.9)

Instead, Eq. (2.7) is consistent with a coupling of excitations in the two throats via a dimension 6 operator, at the scale \( M_{UV} = 1/L \). Thus the 4d holographic description of our background geometry consists of two gauge theories coupled by KK modes, at scales set by the compactification geometry – in this case, \( 1/L \). This lower scale and the lower dimension of the operator mean that the inter-throat coupling in these backgrounds can be much larger than that induced by the coupling to 4d gravity. This is an example of the more general fact that the effective field theories arising from string compactifications can have several relevant scales set by the compactification geometry, beyond just \( M_4 \) which depends only on the overall volume of \( M \).

**CFT Dark Matter**

It is fascinating to contemplate the possibility that the dark matter which constitutes about 90% of the mass of the universe is described by a CFT, and that we are immersed inside an ocean of scale-free matter.\(^1\) In its simplest form this idea is in conflict with observation: CFT matter would have a relativistic equation of state, acting as hot dark matter (HDM); but the large scale structure of the universe suggests that non-relativistic, cold dark matter (CDM) dominates the dynamics of the universe since \( t \sim 10^4 \) years. A way to bypass this difficulty is to postulate that the universe has, until recently, been dominated by an unstable CDM particle which decays into CFT matter, with a lifetime of order of the age of the universe. The two-throat model of Fig. 2 provides such a scenario.

\(^1\) This possibility has been entertained independently by many physicists, including T. Banks, M. Dine, and J. Maldacena.
Suppose the SM is localized on the left IR brane, and that the right throat is dual to a real CFT (i.e. $l_2 \to \infty$). Now introduce a bulk particle, the “bulky,” which is distinct from the graviton and which has a bulk parity symmetry under which it changes sign. This symmetry protects it from decay to SM fields. One can show [13] along the lines of [15] that the relic abundance of bulkies today would be

$$\frac{\rho}{\rho_c} \sim 16\pi^2 \times 10^{-2}(M_5L)^2 \frac{m^2}{T_{eV}^2}.$$  \hspace{1cm} (2.10)

So if $LM_5 \sim 10$ and $m \sim 100 GeV$, then the left bulkies would close the Universe.

The left bulkies will decay into their much lighter right cousins with a rate given by Eq. (2.7). A lifetime of the age of the Universe requires $L^{-1} \sim 10^{14} GeV$ and $M_5 \sim 10^{15} GeV$. These scales are of the order of the unification scale and should arise naturally in model building.

In this scenario, the dark matter is slowly decaying into CFT degrees of freedom in our epoch. This can have important observational effects: it can lead to a softening of the dark matter density profile within our galactic halo, by spreading it into extragalactic space, as shown in simulations of decaying CDM performed by Cen [16]. This may help account for the absence of the excess small scale structure predicted in the canonical CDM scenario [16,17,18]. A variation of this scenario is to add a right brane with characteristic scale of less than the galactic halo size, $\sim 400 kpc$. This may confine most of the approximately conformal matter within the halo.

3. Tunnelling Mediated Supersymmetry Breaking

Low-energy SUSY is one of the most attractive scenarios for physics beyond the Standard Model, because it stabilizes scalar masses at the SUSY breaking scale [2]. We must still explain the origin of the low SUSY breaking scale and the 125 physical parameters in the MSSM [19]. There is a variety of SUSY breaking mechanisms such as gravity [20,21,22], gauge [23,24], anomaly [25] and gaugino [26] mediation in hidden sector scenarios. Tunnelling effects between brane throats provide a new mechanism for generating a small SUSY breaking scale. We will present some basic results of this approach here, leaving the detailed exposition for [13].

Our scenario is as follows. Both IR branes are close to the UV brane. SUSY is broken on the left IR brane via a soft, R-symmetry breaking, Majorana-like mass term for bulk
fermions. This induces SUSY breaking mass splittings on the right IR brane, where the SSM resides. A hierarchy is generated because SUSY breaking at a high scale on the left IR brane induces small SUSY breaking mass splittings in the SSM, due to tunnelling suppression. We choose the distance between the right IR brane and the UV brane, as well as the bulk parameters $M_5$ and $L$, to be near the GUT scale $M_{GUT} \sim 10^{16}$ GeV. This ensures that the cutoff on the right SSM brane is $M_{GUT}$; consequently supersymmetric gauge coupling unification [2,27] can be preserved.

We will outline the calculation of the SSM gaugino masses in [13]. The action for the bulk fermions is:

$$S_F = \int d^4x dz \left\{ X^\dagger i\sigma^\mu \partial_\mu X + Z^\dagger i\bar{\sigma}^\mu \partial_\mu Z + \frac{1}{2} X^\dagger \partial^z Z - \frac{1}{2} Z^\dagger \partial^z X - M_F a(X^\dagger Z + Z^\dagger X) + \delta(z + l_1) \left( q_1 [Z^T i\sigma^2 Z - Z^\dagger i\sigma^2 Z^*] - q_2 [X^T i\sigma^2 X - X^\dagger i\sigma^2 X^*] \right) \right\}. \quad (3.1)$$

Here $a^2$ is the warp factor in (2.1). $X$ and $Z$ are bulk spinors, which we have rescaled by powers of $a$ to have canonical kinetic terms. $M_F$ is a bulk Dirac mass for the fermion and its superpartner. The SUSY breaking is encoded in the dimensionless parameters $q_1$ and $q_2$, both less than unity, which split the bulk fermions from their superpartners. Defining:

$$\Sigma = \left( \begin{array}{c} Z \\ X \end{array} \right), \quad (3.2)$$

the equations of motion for $\Sigma$ have solutions of the form:

$$\Sigma_L = \sqrt{w} \left( \begin{array}{c} A_L J_{\nu+1/2}(w) + B_L J_{-\nu-1/2}(w) \\ A_L J_{\nu-1/2}(w) - B_L J_{\nu+1/2}(w) \end{array} \right) \quad z < 0$$

$$\Sigma_R = \sqrt{w} \left( \begin{array}{c} A_R J_{\nu-1/2}(w) - B_R J_{\nu+1/2}(w) \\ A_R J_{\nu+1/2}(w) + B_R J_{\nu-1/2}(w) \end{array} \right) \quad z > 0 \quad (3.3)$$

where $w = m(|z| + L)$ and $\nu = M_F L$. Similar fermion spectroscopy in warped geometries has been analyzed in [28,29].

We take $l_1 \geq l_2$ for ease of computation [13], and set $q_1 = 0$. The boundary conditions on the branes remove the fermion zero mode from the spectrum. At low energies, the states (3.3) break up into left-localized states, for which $A_R, B_R, B_L \sim (mL)^{2\nu} A_L$, and right-localized states, for which $A_L, B_L, B_R \sim (mL)^{2\nu} A_R$. The masses and mass splittings of the left-localized states are:

$$m_L \sim [\left( \frac{\nu - 1}{2} \right) \pi + n\pi] \frac{1}{l_1} \quad \delta m_L = \pm O(1) \frac{q_2}{l_1}, \quad (3.4)$$

8
while for the right-localized states:

$$m_R \sim \left[ \frac{(\nu - 1)\pi}{2} + n\pi \right] \frac{1}{l_2} \quad \delta m_R = \pm \mathcal{O}(1) \frac{q_2}{l_2} (mL)^{4\nu} . \quad (3.5)$$

The mass splittings on the right IR brane arise from loops of the bulk modes which couple to the SSM. The couplings in the 4d effective action are determined by their coupling in five dimensions, given by the appropriate (fractional) power of $1/M_5$, and by giving canonical normalization to the kinetic term of the 4d fields. The latter requires rescaling by powers of the warp factor $a$, and by the wavefunction of bulk modes evaluated at the IR brane. Thus the Yukawa couplings of left and right localized bulk fermions to fields on the right IR brane are

$$g^{L\ R\ 4D} = \mathcal{O}(1) \sqrt{\frac{M_5 L}{l_1}} (mL)^{2\nu} \quad g^{R\ R\ 4D} = \mathcal{O}(1) \sqrt{\frac{M_5 L}{l_2 + (mL)^{4\nu} l_1}} \quad (3.6)$$

respectively. The coupling of left-localized modes to the right IR brane is suppressed by a barrier penetration factor $(mL)^{2\nu}$ relative to that of the right-localized modes.

The effective SUSY breaking scale in the SSM is set by the gaugino mass. To generate such a mass, we must include a massive $M \sim 1/L$ adjoint scalar on the right IR brane, coupling to the gaugino and the bulk fermion (as the bulk modes are gauge singlets). We will choose $M \sim M_{GUT}$ to preserve gauge coupling unification. Summing over the one-loop contributions from the bulk KK modes, and then performing the four-dimensional loop momentum integrals [30], gives a gaugino mass of [13]:

$$m_g \sim \mathcal{O}(1) \left( \frac{q_2}{M_5 L} \right)^2 \frac{1}{M l_1} \frac{1}{l_2 + (L/l_2)^{4\nu} l_1} \left( \frac{L}{l_2} \right)^{4\nu} . \quad (3.7)$$

The SSM cutoff is set by the conformal distance of the SSM to the UV brane $l_2^{-1} \sim 10^{15} \text{ GeV}$. Squark masses are generated by radiative corrections including gaugino loops. They start out close to zero in the UV, and rise via the RG flow in the IR. As a result, $m_{sq} \sim \text{TeV}$, and is comparable to the gaugino mass, as in no-scale models [31]. Our model predicts a gravitino with mass:

$$m_{3/2} = \mathcal{O}(1) \frac{q_2}{M_4^{2\nu} l_1} . \quad (3.8)$$

This mode is lighter than other gravitino KK modes, whose masses are $\sim l_1^{-1}, l_2^{-1}$, because it is protected by SUSY.
Tunnelling suppression produces large mass hierarchies without much effort. For example, take $M_5 \sim 10^{16} GeV$, $L \sim 5/M_5$, $l_2 \sim 5L$, $M \sim 1/L$ and $q_2 \leq \mathcal{O}(1)$. The tunnelling suppression coefficient $\nu$ and the SUSY breaking scale $l_1^{-1}$ must be chosen so that $m_g \sim TeV$. For $\nu = 1$, i.e. with little tunnelling suppression, the required SUSY breaking scale is low, $l_1^{-1} \sim 10^{10} GeV$. This scale implies a micron-range gravitino mass $m_{3/2} \sim eV$. If $\nu = 3$, the SUSY breaking scale should be $l_1^{-1} \sim 3 \times 10^{13} GeV$, closer to the unification scale. The induced gravitino mass is $m_{3/2} \sim 270 GeV$.

There are also model-independent gravity [23,20] and anomaly [25] mediated contributions to the sparticle masses that are bounded by $m_{3/2}$. They are subdominant to the tunneling mediated contributions as long as $l_1 > l_1^2 (\frac{L}{T})^{2\nu+1/2} \frac{1}{M_4 l_2}$ [13].

The hierarchies we produce do not originate from the AdS scaling as in [8]. In our case the cutoff on the SSM brane is $M_{GUT}$. Furthermore, our effect would persist with slight modifications given any warp factor which raises a barrier between different throats.

Finally, tunnelling suppression may be used to explain other small numbers such as neutrino masses and super-weakly coupled particles [13].

4. Conclusion

In this note we have studied string phenomenology in a new calculable regime which can arise when there are many branes transverse to the compactification manifold. Such compactifications generate brane throats which provide a new mechanism for producing small numbers in Nature, by utilizing the tunnelling suppressed couplings of IR sectors separated by a potential barrier.

Acknowledgements

It is a pleasure to thank N. Arkani-Hamed, S. Giddings, J. Maldacena, A. Peet, M. Peskin, J. Polchinski, A. Pomarol, M. Schmaltz, S. Shenker, P. Steinhardt, A. Strominger, L. Susskind, S. Thomas, H. Tye and H. Verlinde for interesting discussions. This work was supported in part by the NSF grants PHY-99-07949 and PHY-9870115 and the DOE under contract DE-AC03-76SF00515. We thank the ITP at Santa Barbara for hospitality. A.L. thanks the CGTP at Duke University for hospitality and support from NSF grant DMS-0074072. S.K. was supported in part by a Packard Fellowship and a Sloan Fellowship, and E.S. was supported in part by a Sloan Fellowship and a DOE OJI grant.

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