Non radial motions and the large scale structure of the Universe

A. Del Popolo
Dipartimento di Matematica, Università Statale di Bergamo, Piazza Rosate, 2 - I 24129 Bergamo, Italy

M. Gambera
Istituto di Astronomia dell’Università di Catania, Viale A.Doria, 6 - I 95125 Catania, Italy

Abstract. We study the effect of non-radial motions on the mass function, the VDF and on the shape of clusters of galaxies using the model introduced in Del Popolo & Gambera (1998a,b; 1999). The mass function of clusters, obtained using the quoted model, is compared with the statistical data by Bahcall & Cen (1992) and Girardi et al. (1998), while the VDF is compared with the CfA data by Zabludoff et al. (1993) for local clusters. In both cases the model predictions are in good agreement with the observational data showing once more how non-radial motions can reduce many of the discrepancies between CDM model predictions and observational data. Besides we study the effect of non-radial motions on the intrinsic shape of clusters of galaxies showing that non-radial motions produce clusters less elongated with respect to CDM model.

1. Introduction

At its appearance the CDM model contributed to obtain a better understanding of the origin and evolution of the large scale structure in the Universe (White et al. 1987). The principal assumptions of the standard CDM (SCDM) model (Liddle & Lyth 1993) are:

- a flat Universe dominated by weakly interacting elementary particles having low velocity dispersion at early times. The barionic content is determined by the standard big bang nucleosynthesis model;
- critical matter density;
- expansion rate given by $h = 0.5$;
- a scale invariant and adiabatic spectrum with a spectral index, $n \equiv 1$;
- the condition required by observations, that the fluctuations in galaxy distribution, $(\delta \rho/\rho)_g$, are larger than the fluctuations in the mass distribution, $(\delta \rho/\rho)_\rho$ by a factor $b > 1$. 

1
If this last assumption is not introduced, the pairwise velocity dispersion is larger than that deduced from observations and the galaxy correlation function is steeper than that observed (Davis et al. 1985). After the great success of the model in the 80’s, a closer inspection of the model has shown a series of deficiencies, namely:

- the strong clustering of rich clusters of galaxies, $\xi_{cc}(r) \simeq (r/25h^{-1}\text{Mpc})^{-2}$, far in excess of CDM predictions (Bahcall & Soneira 1983);

- the overproduction of clusters abundance. Clusters abundance is a useful test for models of galaxy formation. This is connected to three relevant parameters: the mass function, the VDF and the temperature function. Using N-body simulations, Jing et al. (1994) studied the mass function of rich clusters at $z = 0$ for the CDM model. They found that the CDM model with the COBE normalization produces a temperature function of clusters higher than that given by the observations by Edge et al. (1990) and by Henry & Arnaud (1991);

- the conflict between the normalization of the spectrum of the perturbation which is required by different types of observations;

- the incorrect scale dependence of the galaxy correlation function, $\xi(r)$, on scales $10 \div 100\ h^{-1}\text{Mpc}$, having $\xi(r)$ too little power on the large scales compared to the power on smaller scales (Maddox et al. 1990; Saunders et al. 1991; Lahav et al. 1989; Peacock 1991; Peacock & Nicholson 1991).

Several alternative models have been proposed in order to solve the quoted problems (Peebles 1984; Valdarnini & Bonometto 1985; Bond et al. 1988; Holtzman 1989; Efstathiou et al. 1990; Turner 1991; Schaefer 1991; Schaefer & Shafi 1993; Holtzman & Primack 1993; Bower et al. 1993). Most of them propose in some way a modification of the primeval spectrum of perturbations. In two previous papers (Del Popolo & Gambera 1998a; 1999) we showed how, starting from a CDM spectrum and taking into account non-radial motions, at least the problem of the clustering of clusters of galaxies and the problem of the X-ray temperature can be considerably reduced.

Here, we study the effect of non-radial motions on the mass function, the VDF and on the shapes of galaxy clusters.

In Sect. 2 we shall use the same model introduced by Del Popolo & Gambera (1998a,b,1999) to compare the mass function calculated using the CDM model, taking into account non-radial motions, with the observed mass function obtained by Bahcall & Cen (1992). Then, we repeat the calculation for the VDF and compare the theoretical VDF with the CfA data by Zabludoff et al. (1993). In Sect. 4 we study the effect of non-radial motions on the ellipticity of clusters and finally in Sect. 5 we give our conclusions.

### 2. The mass function and the velocity dispersion function

One of the most important constraints that a model for large-scale structure must overcome is that of predicting the correct number density of clusters of galaxies. This constraint is crucial for several reasons (Peebles 1993). The
Non radial motions and the large scale structure of the Universe

abundance of clusters of galaxies, together with the mass distributions in galaxy halos and in rich clusters of galaxies, the peculiar motions of galaxies, the spatial structure of the microwave background radiation is one of the most readily accessible observables which probes the mass distribution directly. The most accurate way of assessing the cluster abundance is via numerical simulations. However, there is an excellent analytic alternative, Press & Schechter’s theory (Press & Schechter 1974; Bond et al. 1991). Press-Schechter’s theory states that the fraction of mass in gravitationally bound systems larger than a mass, $M$, is given by the fraction of space in which the linearly evolved density field, smoothed on the mass scale $M$, exceeds a threshold $\delta_c$:

$$F(> M) = \frac{1}{2} \text{erfc} \left( \frac{\delta_c}{\sqrt{2} \sigma(R_f, z)} \right)$$

where $R_f$ is the comoving linear scale associated with $M$. Press-Schechter’s result predicts that only half of the mass of the Universe ends up in virialized objects but in particular cases this problem can be solved (Peacock & Heavens 1990; Cole 1991; Blanchard et al. 1992).

The mass variance present in Eq. (1) can be obtained once a spectrum, $P(k)$, is fixed:

$$\sigma^2(M) = \frac{1}{2\pi^2} \int_0^\infty dk k^2 P(k) W^2(kR)$$

where $W(kR)$ is a top-hat smoothing function:

$$W(kR) = \frac{3}{(kR)^3} (\sin kR - kR \cos kR)$$

and the power spectrum $P(k) \propto k^n T^2(k)$ is fixed giving the transfer function $T(k)$ (here, we adopt that by Klypin et al. 1993), where $k$ is the wave-number measured in units of Mpc$^{-1}$. This spectrum is valid for $k < 30$Mpc$^{-1}$ and $z < 25$. The accuracy of the spectrum is 5%. It is more accurate than Holtzman’s (1989) spectrum, used by Jing et al. (1994) and Bartlett & Silk (1993) to calculate the mass function and the X-ray temperature function of clusters, respectively. The spectrum is lower by 20% on cluster mass scales than Holtzman’s (1989). The spectrum was normalized to the COBE quadrupole $Q_2 = 17\mu K$, corresponding to $\sigma_8 = 0.66$. As shown by Bartlett & Silk (1993) the X-ray distribution function, obtained using a standard CDM spectrum, over-produces the clusters abundances data obtained by Henry & Arnaud (1991) and by Edge et al. (1990). This has lead some authors (White et al. 1993) to cite the cluster abundance as one of the strongest pieces of evidence against the standard CDM model when the model is normalized so as to reproduce the microwave background anisotropies as seen by the COBE satellite (Bennett et al. 1996).

The discrepancy can be reduced taking into account the non-radial motions that originate when a cluster reaches the non-linear regime as follows. A fundamental role in Press-Schechter’s theory is played by the value of $\delta_c$. Using a top-hat window function $\delta_c = 1.7 \pm 0.1$ while for a Gaussian window the threshold is significantly lower. In a non-spherical context the situation is more complicated. Considering the collapse along all the three axes the threshold is higher, whereas the collapse along the first axis (pancake formation) or the first two
axes (filament formation) corresponds to a lower threshold (Monaco 1995). The threshold, $\delta_c$, does not depend on the background cosmology. As shown by Del Popolo & Gambera (1998a; 1999), if non-radial motions are taken into account, the threshold $\delta_c$ is not constant but is function of mass, $M$ (Del Popolo & Gambera 1998a; 1999):

$$\delta_c(\nu) = \delta_{co} \left[ 1 + \int_{r_i}^{\tau_a} \frac{r_L^2 \cdot dr}{GM^3 r^3} \right]$$

where $\delta_{co} = 1.68$ is the critical threshold for a spherical model, $r_i$ is the initial radius, $r_{\tau_a}$ is the turn-around radius and $L$ the angular momentum. In terms of the Hubble constant, $H_0$, the density parameter at current epoch, $\Omega_0$, the expansion parameter $a$ and the mean fractional density excess inside a shell of a given radius, $\delta$, Eq. (4) can be written as (Del Popolo & Gambera 1998a; 1999):

$$\delta_c(\nu) = \delta_{co} \left[ 1 + \frac{8G^2}{\Omega_0^3 H_0^6 r_i^{10/3} (1 + \delta)^3} \int_{a_{\min}}^{a_{\max}} \frac{L^2 \cdot da}{a^3} \right]$$

where $a$ is the expansion parameter, and $a_{\min}$ its value corresponding to $r_i$. The mass dependence of the threshold parameter, $\delta_c(\nu)$, was obtained in the same way sketched in Del Popolo & Gambera (1999). The result of the calculation is shown in Fig. 1. Here, the mass function of clusters, derived using a CDM model without taking into account non-radial motions (dashed line) and taking account non-radial motions (solid line) compared with Bahcall & Cen (1992) data (full dots) and with that of Girardi et al. (1998) (open squares).
Non radial motions and the large scale structure of the Universe

with \( \Omega_0 = 1, h = 1/2 \) normalized to \( Q_{\text{COBE}} = 17 \mu \text{K} \) and taking into account non-radial motions (solid line), is compared with the statistical data by Bahcall & Cen (1992) (full dots) and with that of Girardi et al. (1998) (open squares) and with a pure CDM model with \( \Omega_0 = 1, h = 1/2 \) (dashed line). Bahcall & Cen (1992) estimated the cluster mass function using optical data (richness, velocities, luminosity function of galaxies in clusters) as well as X-ray data (temperature distribution function of clusters). Groups poorer than Abell clusters have also been included thus extending the mass function to lower masses than the richer Abell clusters. Girardi et al. (1998) data are obtained from a sample of 152 nearby (\( z \leq 0.15 \)) Abell-ACO clusters. As shown, the CDM model that does not take account of the non-radial motions over-produces the clusters abundance. The introduction of non-radial motions (solid line) reduces remarkably the abundance of clusters with the result that the model predictions are in good agreement with the observational data. This result confirms what found in Del Popolo & Gambera (1999) showing how a mass dependent threshold, \( \delta_c(M) \) (dependence caused by the developing of non-radial motions) can solve several of SCDM discrepancies with observations.

The VDF is defined in a similar way to the mass function, namely it is the number of objects per unit volume with velocity dispersion larger than \( \sigma_v \). Since the velocity dispersion \( \sigma_v \) can be observed directly, VDF provides a good test of theoretical models. Observed \( \sigma_v \) comes from the measurement of galaxy redshift. The VDF can be calculated starting from the mass function:

\[
n(\sigma_v) = n(M) \frac{dM}{d\sigma_v}
\]  

(6)

The cumulative VDF can be obtained integrating Eq. (6):

\[
n(> \sigma_v) = \int_{\sigma_v}^{\infty} n(\sigma_v')d\sigma_v'
\]

(7)

In order to use Eq. (6) to calculate the VDF we need a relation between the velocity dispersion, \( \sigma_v \), and mass, \( M \). To determine the relation between \( \sigma_v \) and \( M \) we use both the relation for the typical virial temperature and the result by Thomas & Couchman (1992) and that by Evrard (1989, 1990, 1997) found in N-body simulations. We find that the necessary relation between \( \sigma_v \) and \( M \) is given by:

\[
\sigma_v = 824 \text{km/s} \left( \frac{hM}{10^{15} M_\odot} \right)^{1/3}
\]

(8)

(Evrard 1989; Lilje 1990). In Fig. 2 we compare the VDF obtained from a CDM model taking account of non-radial motions (solid line) with the CfA data by Zabludoff et al. (1993) (full dots) based on their survey of \( R \geq 1 \) Abell clusters within \( z \leq 0.05 \) and with the data by Mazure et al. (1996) (full triangles) and Fadda et al. (1996) (open squares). Mazure et al. (1996) data are obtained from a volume-limited sample of 128 \( R_{\text{ACO}} \geq 1 \) clusters while that of Fadda et al. (1996) are obtained from a sample of 172 nearby galaxy clusters (\( z \leq 0.15 \)). We also plot the VDF obtained from a CDM model without non-radial motions (dashed line). The SCDM model predicts more clusters than the CfA observation.
Figure 2. Cumulative VDF calculated using a CDM model without taking into account non-radial motions (dashed line) and taking into account non-radial motions (solid line) compared with Zabludoff et al. (1993) data (full dots) and with those by Mazure et al. (1996) (full triangles) for $R \geq 1$ clusters and Fadda et al. (1996) (open squares) for $R \geq -1$ clusters. The theoretical curves are obtained using a $\sigma_v$-$M$ relation with zero scatter.

except at $\sigma_v \simeq 1100$ km/s. As reported by Jing & Fang (1994) the SCDM model can be rejected at a very high confidence level ($> 6\sigma$). The reduction of the normalization reduces the formation of clusters, thus resolving the problem of too many clusters, but leads to a deficit at $\sigma_v \simeq 1100$ km/s. When non-radial motions are taken into account (solid line) we obtain a good agreement between theoretical predictions and observations. Both CDM and CDM with non-radial motions predict more clusters of low velocity dispersion ($\sigma_v \leq 300$ km/s) than the observation. This discrepancy is not significant because the data at $\sigma_v \leq 300$ km/s could be seriously underestimated (Zabludoff et al. 1993; Fadda et al. 1996; Mazure et al. 1996).

3. Non-radial motions and the shape of clusters

Most clusters, like elliptical galaxies, are not spherical and their shape is not due to rotation (Rood et al. 1972; Dressler 1981). The perturbations that gave rise to the formation of clusters of galaxies are alike to have been initially aspherical (Barrow & Silk 1981; Bardeen et al. 1986) and asphericities are then amplified during gravitational collapse (Icke 1973; Barrow & Silk 1981). The elongations are probably due to a velocity anisotropy of the galaxies (Aarseth & Binney 1978) and according to Binney & Silk (1979) and to Salvador-Solé & Solanes
(1993) the elongation of clusters originates in the tidal distortion by neighboring protoclusters. In particular Salvador-Solé & Solanes (1993) found that the main distortion on a cluster is produced by the nearest neighboring cluster having more than 45 galaxies and the same model can explain the alignment between neighboring clusters (Oort 1983; Plionis 1993) and that between clusters and their first ranked galaxy (Rhee & Katgert 1987; van Kampen & Rhee 1990; West 1994).

The observational information on the distribution of clusters shapes is sometimes conflicting. Rhee et al. (1989) found that most clusters are nearly spherical with ellipticities distribution having a peak at $\epsilon \simeq 0.15$ while Plionis et al. (1991) found that clusters are more elongated with the peak of the ellipticities distribution at $\epsilon \simeq 0.5$.

To study the effect of non-radial motions on the shape of clusters we shall use a model introduced by Binney & Silk (1979). In that paper they showed that tidal interactions between protoclusters and the neighboring protostructures should yield prolate shapes (before virialization) with an axial ratio of protostructures of $\epsilon \simeq 0.5$, the typical value found in clusters. After virialization the pre-existing elongation is damped and the axial ratio leads to values of about $0.7 \pm 0.8$, that are higher with respect to observations. As observed by Salvador-Solé & Solanes (1993) this last discrepancy can be removed taking into account that tidal interaction keeps going on after virialization and that on average the damping of elongations due to violent relaxation is eliminated by its growth after virialization. Then this growth restores a value of $\epsilon$ near the one that clusters had before virialization.

According to the quoted Binney & Silk (1979) model, an initially spherical protostructure (e.g. a protocluster) of mass $M$ having at distance $r(t)$ from its centre a series of similar protostructure of mass $M'$ shall be distorted and the final ellipticity can be calculated using the following equation:

$$\epsilon \simeq \frac{3}{2} \langle \mu^2 \rangle^{1/2} G(2, x_1)$$

(9)

where $G(2, x_1)$ is defined in the quoted paper (see Eq. 13c) and $\langle \mu^2 \rangle^{1/2}$ for a CDM without non-radial motions is given by (Binney & Silk 1979) while for a CDM with non-radial motions we use the following relation:

$$\langle \mu^2 \rangle^{1/2} = \frac{1}{9 \rho_b^{1/2}} \left[ \int \frac{N(M', r) dM'}{M + M'} \right]^{1/2}$$

(10)

In Fig. 3 we show as $\epsilon$ declines with mass in agreement with the observations. Besides, we show as in a CDM model that takes into account non-radial motions (solid line) $\epsilon$ is smaller than in the simple CDM (dashed line). This show as non-radial motions reduce the elongation of clusters. For a cluster of $10^{15} M_\odot$ we get a value of $\epsilon \simeq 0.5$, if non-radial motions are excluded, while $\epsilon \simeq 0.43$ when non-radial motions are taken into account. Increasing the mass, as expected, clusters tend to become more and more spherical.
Figure 3. Ellipticity, $\epsilon$, of clusters versus mass, $M$. The dashed and solid lines represent $\epsilon$ for a CDM without and with non-radial motions, respectively.

4. Conclusions

Here, we have studied how non-radial motions change the mass function, the VDF and the shape of clusters of galaxies, using the model introduced by Del Popolo & Gambera (1998a; 1999). We compared the theoretical mass function obtained from the CDM model taking into account non-radial motions with the experimental data by Bahcall & Cen (1992) and Girardi et al. (1998). The VDF was compared with the CfA data by Zabloudoff et al. (1993), by Mazure et al. (1996) and by Fadda et al. (1996). Taking account of non-radial motions we obtained a noteworthy reduction of the discrepancies between the CDM predicted mass function, the VDF and the observations. Non-radial motions are also able to change the shape of clusters of galaxies reducing their elongations with respect to the prediction of CDM.

Acknowledgments. We are grateful to Prof. G. Moncada, E. Recami and E. Spedicato for stimulating discussions while this work was in process.

References

Non radial motions and the large scale structure of the Universe

Efstathiou G., Sutherland W.J., Maddox S.J., 1990, Nat., 348, 705
Oort J. H., 1983, ARA&A 21, 373
Plionis M., Barrow J.D., Frenk C.S., 1991, MNRAS 249, 662
Saunders W., Frenk C., Rowan-Robinson M. et al., 1991, Nat 349, 32