Charmonium - hadron cross sections are of crucial importance in the context of quark-gluon plasma physics [1]. Small $J/\psi$ - hadron dissociation cross sections may favor an interpretation of the recent Pb + Pb data in terms of the production of a new phase of matter. Part of these interactions happens in the early stages of the nucleus - nucleus collisions and therefore at high energies ($\sqrt{s} \simeq 10 - 20$ GeV) and one may try to apply perturbative QCD. However, even in this regime, nonperturbative effects may be important [2]. Interestingly, estimates using quite different methods give results clustering around the value of 3. Interestingly, this effect may be important [2].

On the other hand, a significant part of the charmonium - hadron dissociation cross sections obtained with models based on meson exchange, but are close to those obtained with quark exchange models.


Charmonium - hadron cross sections are of crucial importance in the context of quark-gluon plasma physics [1]. Small $J/\psi$ - hadron dissociation cross sections may favor an interpretation of the recent Pb + Pb data in terms of the production of a new phase of matter. Part of these interactions happens in the early stages of the nucleus - nucleus collisions and therefore at high energies ($\sqrt{s} \simeq 10 - 20$ GeV) and one may try to apply perturbative QCD. However, even in this regime, nonperturbative effects may be important [2]. Interestingly, estimates using quite different methods give results clustering around the value of 3 - 5 mb in this energy range.

On the other hand, a significant part of the charmonium - hadron interactions occurs when other light particles have already been produced, forming a “fireball”. Interactions inside this fireball happen at much lower energies ($\sqrt{s} \leq 5$ GeV) and one has to apply nonperturbative methods.

One possible nonperturbative reaction mechanism is meson exchange, which can be studied by means of effective Lagrangians, constrained by flavor and chiral symmetries as well as by gauge invariance. This approach was first introduced in ref. [3] and further developed by other groups [4–7]. Another reaction mechanism is quark interchange driven by Born-order matrix elements of the standard nonrelativistic quark model [8,9]. In this approach, once the masses and sizes of the mesons are fixed, there are no free parameters left.

The results of the calculations for the charmonium-pion cross sections based on these two approaches can differ by two orders of magnitude in the relevant energy range. The situation clearly calls for different types of calculations that are constrained by other, independent pieces of phenomenology. In this work we use the QCD sum rules (QCDSR) technique [10,11] to study the $J/\psi - \pi$ dissociation. The QCDSR technique allows to compute hadronic quantities like masses, coupling constants and form factors in terms of quark and gluon properties and universal matrix elements which represent the properties of the QCD vacuum. In view of our relatively poor understanding of $J/\psi$ reactions in nuclear matter and considering the large discrepancies between different model estimates, we believe that our work adds to a better understanding of this important topic.

We consider all four channels $J/\psi \pi \rightarrow \bar{D}D^*, \bar{D}D$ and $D^*D^*$. Let us start with the the four-point function for the process $J/\psi \pi \rightarrow \bar{D}D^*$:

\[
\Pi_{\mu\nu} = \int \frac{d^4 x}{4\pi} \frac{d^4 y}{4\pi} \frac{d^4 z}{4\pi} e^{-ip_1 \cdot x} e^{-ip_2 \cdot 0} e^{ip_3 \cdot y} e^{ip_4 \cdot z} \langle 0 | \{ j_\rho(x) j_{\rho'}^\dagger(y) j_{\rho''}^\dagger(0) J_{D}(z) \} | 0 \rangle ,
\]

with the currents given by $j_\rho = \bar{u} d \gamma_5 u$, $j_{\rho'} = \bar{u} d c$, and $J_D = \bar{c} \gamma_5 d$ [11], where $c$, $u$ and $d$ are the charm, up and down quark fields respectively, and $p_1, p_2, p_3$ and $p_4$ are the four-momenta of the mesons $\pi$, $J/\psi$, $D^*$ and $D$ respectively, with $p_1 + p_2 = p_3 + p_4$.

The phenomenological side of the correlation function, $\Pi_{\mu\nu}$, is obtained by the consideration of $J/\psi$, $\pi$, $D$ and $D^*$ state contribution to the matrix element in Eq. (1):

\[
\Pi_{\mu\nu}^{\text{phen}} = -\frac{m_{\pi}^2 f_{\pi}^2}{m_u + m_d} \frac{m_{D} f_{D}}{m_c} \frac{m_{\psi} f_{\psi}}{m_\pi} \mathcal{M}_{\alpha\beta} \frac{\mathcal{C}_\alpha \mathcal{C}_\beta}{(p_3^2 - m_\pi^2)(p_4^2 - m_D^2)} \times \frac{g_{\mu\nu} - p_{1\mu} p_{2\nu}/m_{\pi}^2}{p_1^2 - m_{\pi}^2} \frac{g_{\mu\nu} - p_{3\mu} p_{3\nu}/m_D^2}{p_3^2 - m_D^2} + \text{h. r.}
\]

where h. r. means higher resonances and the hadronic amplitude for the process $J/\psi \pi \rightarrow \bar{D}D^*$ is given by

\[
\mathcal{M} = \mathcal{M}_{\mu\nu}(p_1, p_2, p_3, p_4) \epsilon_\mu^* \epsilon_\nu^* .
\]

We note that one has $1/p_1^2$ pole in Eq. (2) in the limit of a vanishing pion mass. Following Reinders, Rubinstein, and Yazaki [11], and others [12–14], we can write a sum rule at $p_1^2 = 0$ and single out the leading terms in the operator product expansion (OPE) of Eq. (1) that match the $1/p_1^2$ term. The perturbative diagram does not contribute with $1/p_1^2$ and, up to dimension four, only the diagrams proportional to the quark condensate, shown in Fig. 1, contribute. After collecting the $1/p_1^2$ terms on the theoretical side and taking the limit $p_1 \mu \rightarrow 0$ in the residue of the pion pole, one obtains for the contribution of these two graphs

\[
\Pi_{\mu\nu}^{<\bar{q}q>} = -\frac{2m_c \langle \bar{q}q \rangle}{p_1^2} \frac{p_{1\nu}(p_{1\mu} + p_{2\mu} - 2p_{3\mu}) - p_{1\mu} p_{2\nu}}{(p_3^2 - m_\pi^2)(p_4^2 - m_D^2)} .
\]
Contracting the hadronic amplitude with the numerators of \( J/\psi \) and \( D^* \) propagators in Eq. (2) and comparing with Eq. (4), the structure defining \( M_{\mu \nu} \) in Eq. (3) is easily identified. Therefore, defining

\[
M_{\mu \nu} = \Lambda_{DD^*}(p_1\mu_1p_\nu - p_1\mu_2p_\nu - 2p_1\nu p_3\mu),
\]

we can write a sum rule for \( \Delta \rho_{DD^*} \) in any of the three structures appearing in Eq. (5). To improve the matching between the phenomenological and theoretical sides we follow the usual procedure and make a single Borel transformation to all the external momenta (except \( p_1^2 \)) taken to be equal: \( -p_2^2 = -p_3^2 = -p_4^2 = P^2 \rightarrow M^2 \).

The problem of doing a single Borel transformation is the fact that terms associated with the pole-continuum transitions are not suppressed [15]. In Ref. [15] it was explicitly shown that the pole-continuum transitions have explicitly shown that the pole-continuum transition has been found good stability for the two-point sum rules \([13–15]\). Thus, neglecting terms associated with the pole-continuum transitions, we use in our approach \( \Lambda \) is just a number and \( \Lambda \) will use thresholds, and to radiative corrections. Therefore, we introduce a parameter \( \Lambda \) in the phenomenological and theoretical sides, the parameter \( \Lambda_{DD^*} \) to account for possible nondiagonal transitions.

For consistency we use in our analysis the QCDRSR expressions for the decay constants of the \( J/\psi \), \( D^* \) and \( D \) mesons up to dimension four in lowest order of \( \alpha_s \):

\[
f_D = \frac{3m_c^2}{8\pi^2m_D^3} \int_{m_D^2}^{u_D} du \left( \frac{u - m_c^2}{u} \right) e^{(m_D^2 - u)/M^2_D},
\]

\[
-f_{D^*} = \frac{1}{8\pi^2m_{D^*}^2} \int_{m_{D^*}^2}^{u_{D^*}} ds \left( \frac{(s - m_c^2)^2}{s} \right) \left( 2 + \frac{m_c^2}{s} \right)
\times e^{(m_{D^*}^2 - s)/M^2_{D^*}},
\]

\[
f_{\psi} = \frac{1}{4\pi^2} \int_{m_{\psi}^2}^{u_{\psi}} dr \left( \frac{s + m_{\psi}^2}{s} \right) \sqrt{s - 4m_{\psi}^2} e^{(m_{\psi}^2 - r)/M^2_{\psi}},
\]

where \( M^2_{D^*} \) represents the Borel mass in the two-point function. We have also omitted the numerically insignificant contribution of the gluon condensate.

The parameter values used in all calculations are \( m_u = m_d = 14 \text{ MeV} \), \( m_c = 1.5 \text{ GeV} \), \( m_s = 140 \text{ MeV} \), \( m_D = 1.87 \text{ GeV} \), \( m_{D^*} = 2.01 \text{ GeV} \), \( m_\psi = 3.097 \text{ GeV} \), \( f_D = 131.5 \text{ MeV} \). \( (\bar{q}q) = -0.23^3 \text{ GeV}^4 \). We parametrize the continuum thresholds as \( u_M = \left( m_M + \Delta_u \right)^2 \). The values of \( u_M \) are, in general, extracted from the two-point function sum rules for \( f_D \) and \( f_{D^*} \) and \( f_\psi \). Using the Borel region \( 3 \leq M_{D^*}^2 \leq 6 \text{ GeV}^2 \) for the \( D^* \) and \( D \) mesons and 6 \( \leq M_{\psi}^2 \leq 10 \text{ GeV}^2 \) for the \( J/\psi \), we found good stability for \( f_D \), \( f_{D^*} \), and \( f_\psi \) with \( \Delta_u \leq 0.6 \text{ GeV} \). We obtained \( f_D = 155 \pm 5 \text{ MeV} \), \( f_{D^*} = 195 \pm 5 \text{ MeV} \) and \( f_\psi = 225 \pm 10 \text{ MeV} \), which are acceptable values for these decay constants [16]. However, instead of using numerical values for these decay constants we are going to use directly the sum rules in Eqs. (7), (8) and (9) when evaluating \( M \). Ref. [17] it was found that relating the Borel parameters in the two- and three-point functions through \( M^2 = 2M_{D^*}^2 \), is a crucial ingredient for the incorporation of heavy quark symmetries, and leads to a considerable reduction of the sensitivity to input parameters, such as the continuum thresholds, and to radiative corrections. Therefore, we will use \( M^2 = 2M_{D^*}^2 \) to relate the Borel parameters and will work in the Borel range \( 8 \leq M^2 \leq 16 \text{ GeV}^2 \). We recall that this region corresponds to \( 4 \leq M_{\psi}^2 \leq 8 \text{ GeV}^2 \), in which we have obtained good stability for the two-point sum rules of Eqs. (7), (8) and (9). This region also covers the range of the average values of the masses of the \( D \), \( D^* \) and \( J/\psi \) mesons.

In Fig. 2 we show, for \( \Delta_u = 0.6 \text{ GeV} \), the QCD sum rule results for \( \Lambda_{DD^*} + A_{DD^*}M^2 \) as a function of \( M^2 \) (dots). We see that they follow a straight line in the Borel region \( 8 \leq M^2 \leq 16 \text{ GeV}^2 \). The value of the amplitude \( \Lambda \) is obtained by the extrapolation of the line to \( M^2 = 0 \) [13–15]. Fitting the QCD sum rule results to a straight line we get

\[
\Lambda_{DD^*} \simeq 17.71 \text{ GeV}^{-2}.
\]

As expected, in our approach \( \Lambda \) is just a number and all dependence of \( M_{\mu \nu} \) (Eq. (5)) on particle momenta is contained in the Dirac structure. This is a consequence of our low energy approximation.

Next, we consider the process \( J/\psi \pi \rightarrow D D (J/\psi \pi \rightarrow D^* D^*) \). In this case we have to change the current \( j_{\psi}^D \) (\( j_{D^*} \)) in Eq. (1) by \( \bar{u}i\gamma_5c(\bar{c}\gamma_aD) \). The phenomenological side is obtained as

\[
\Pi_{\mu}^{phen} = -\frac{m_{\psi}f_{\psi}}{m_u + m_d} \left( m_{\psi}^2f_{\psi} - g_{\mu} + p_{2\mu}p_{2\mu} \right)
\times \frac{m_{\psi}f_{\psi}M_{D^*}^2}{(p_{D^*}^2 - m_{D^*}^2)(p_{D^*}^2 - m_{D^*}^2)} + h. r. ,
\]

for \( J/\psi \pi \rightarrow D D \), where the hadronic amplitude is defined by \( M = M_{\mu}(p_1,p_2,p_3,p_4) \epsilon_2^\mu \). In the same way, for \( J/\psi \pi \rightarrow D^* D^* \) we get
the denominator of Eqs. (11) and (12), and doing a single
have neglected the structure
$\epsilon$
contains a term
$p$
states contributions. Since these contributions are con-

Similarly to the case $J/\psi \pi \to D \bar{D}^*$, in the OPE side
the only diagrams, up to dimension four, contributing
with the corresponding hadronic amplitude defined by
hadronic amplitudes:

$$M_{\mu
\nu} = \Lambda_{DD} \epsilon_{\mu\alpha\beta\sigma} p_1^\alpha p_3^\beta p_4^\sigma, \quad M_{\mu
\nu} = \Lambda_{D^*D^*} \epsilon_{\mu\nu}.$$ (16)

It is important to notice that in writing Eq. (16) we have neglected the structure $\epsilon_{\alpha\mu\nu\beta} p_1^\alpha$ in $M_{\mu\nu}$. This is because, as can be seen from Eq. (14), this structure contains a term $p_3 p_4$ that can be rewritten in terms of $p_3^2 - m_c^2$ and $p_4^2 - m_c^2$ and, therefore, will contribute with a single pole which contains information about pole-excited states contributions. Since these contributions are considered in the phenomenological side as a parameter, we do not need to include them explicitly in the OPE side.

We can write a sum rule for $\Lambda_{DD}$ in the structure $\epsilon_{\mu\alpha\beta\sigma} p_1^\alpha p_3^\beta p_4^\sigma$, and a sum rule for $\Lambda_{D^*D^*}$ in any of the structures appearing in Eq. (15). Thus, neglecting $m_c^2$ in the denominator of Eqs. (11) and (12), and doing a single Borel transform in $-p_3^2 = -p_4^2 = P^2$, we get

$$f_M(M^2) = \frac{m_u + m_d}{m_D^2 - m_c^2} \left( \sum_{\mu} \frac{m_\mu}{m_D^2 m_\psi f_\psi} \right) \times 2 \langle \bar{q}q \rangle e^{-m_c^2/M^2}.$$ (17)

where the subscript $M$ stands for the $D$ or $D^*$ mesons, with $C_D = \frac{m_c^2}{m_D^2}$, $C_{D^*} = 1$ and

$$f_M(M^2) = \frac{e^{-m_c^2/M^2} - e^{-m_\mu^2/M^2} - e^{-m_c^2/M^2}}{m_c^2 - m_\mu^2}.$$ (18)

In Fig. 2 we also show, for $\Delta_u = 0.6$ GeV, the QCD
sum rule results for $\Lambda_{DD} + A_{DD} M^2$ (diamonds) and $\Lambda_{D^*D^*} + A_{D^*D^*} M^2$ (triangles) as a function of $M^2$ from
where we see that, in the Borel region $8 \leq M^2 \leq 16$ GeV$^2$, they all follow a straight line. As explained before, the value of the amplitudes $\Lambda_{DD}$ and $\Lambda_{D^*D^*}$ are obtained by the extrapolation of the line to $M^2 = 0$. We get:

$$\Lambda_{DD} \simeq 12.25\text{ GeV}^{-1}, \quad \Lambda_{D^*D^*} \simeq 11.39\text{ GeV}^{-3}.$$ (19)

Having the QCD sum rule results for the amplitude of the three processes $J/\psi \pi \to D \bar{D}^*$, $D D$, $D^* D^*$, given in Eqs. (5) and (16) with $\Lambda$ given in Eqs. (10) and (19) we can evaluate the differential cross section.

Using our QCD sum rule result in Eqs. (5), (16), (10) and (19) we show, in Fig. 3, the cross section for the $J/\psi \pi$ dissociation. It is important to keep in mind that, since our sum rule was derived in the limit $p_1 \to 0$, we cannot extend our results to large values of $\sqrt{s}$. Also, since the perturbative contribution is absent in our calculation, we were not able to properly disentangle the continuum contribution and our cross section may include contributions from higher states. Whereas they are certainly not important in the case of the pion, they may give some contribution to the heavy currents. Therefore our $J/\psi \pi$ cross section may implicitly include (at least partially) the process $\psi' \pi$. For this reason, our numbers might be regarded as upper bounds.

Our first conclusion is that our results show that, for values of $\sqrt{s}$ far from the $J/\psi \pi \to D^* D^*$ threshold, $\sigma_{J/\psi \pi \to D^* D^*} \geq \sigma_{J/\psi \pi \to D D} \geq \sigma_{J/\psi \pi \to D^* D^*}$, in agreement with the model calculations presented in [4] but in disagreement with the results obtained with the nonrelativistic quark model of [9], which show that the state $D^* D$ has a larger production cross section than $D^* D^*$. Furthermore, our curves indicate that the cross section grows monotonically with the c.m.s. energy but not as fast, near the thresholds, as it does in the calculations in Refs. [4–7]. Again, this behavior is in opposition to [9], where a peak just after the threshold followed by continuous decrease in the cross section was found.

At higher energies, due to our low energy approximation, our approach gradually looses validity. In the fiducial region, close to threshold, $4.1 \leq \sqrt{s} \leq 4.3$ GeV, we find $2.5 \leq \sigma \leq 4.0$ mb and these values are much smaller than those obtained with the effective Lagrangians without form factors in the hadronic vertices, but agree in order of magnitude with the quark model calculations of [9].
Finally, we should mention that we have been studying the dissociation processes of the J/ψ in vacuum and the quantities relevant for QGP physics are in medium cross sections. In our approach the main effect introduced by the medium is the modification of the condensates, which is thought to be very mild. Our results depend only on the quark condensate and since it decreases with the nuclear density, we expect a further reduction in our cross section in a dense nuclear environment.

In conclusion, we have used the QCD sum rule approach to evaluate the hadronic amplitude of the J/ψ π dissociation. From the hadronic amplitude we have evaluated the J/ψ π → charmed mesons dissociation cross section, and have obtained $2.5 \leq \sigma \leq 4.0 \text{ mb}$ at $4.1 \leq \sqrt{s} \leq 4.3 \text{ GeV}$. In view of the uncertainties discussed above these numbers should be taken as upper limits.

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