Quantum entanglement, first noted by Einstein-Podolsky-Rosen (EPR) [1] and Schrödinger [2], is one of the essential features of quantum mechanics. Its famous embodiment $\Phi^{\pm} = \frac{1}{\sqrt{2}}((111) \pm |000\rangle)$, $\Psi^{\pm} = \frac{1}{\sqrt{2}}((10\rangle \pm |01\rangle)$ was shown by Bell [3] to have stronger correlations than allowed by any local hidden variable theory. For the multi-particle entanglement states, there are many properties more peculiar than the two-party ones. For example, the Greenberger-Horne-Zeilinger (GHZ) state [4,5] $\Phi^{ABC} = \frac{1}{\sqrt{3}}(|111\rangle + |000\rangle)$, a canonical three-particle entanglement state exhibits the contradiction between local hidden variable theories and quantum mechanics even for nonstatistical predictions, as opposed to the statistical ones for the EPR states. Many papers have discussed the multipartite entanglement and its applications [6,7]. In the paper [8], the authors proved that there exists another kind of peculiar genuine tripartite entanglement W states $W = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$, which is inequivalent to the GHZ states in the sense that they cannot be converted to each other even under stochastic local operations and classical communication (SLOCC); that is, through LOCC but without imposing that it has to be achieved with certainty [9]. The GHZ state is maximally in unit probability an EPR state shared between any two of the three parties. Another relevant feature is that when any one of the three qubits is traced out, the remaining two are in separable - and therefore unentangled - state. Thus, the entanglement properties of the GHZ state are very fragile under particle losses. Oppositely, the entanglement of the W state has the highest degree of endurance against loss of one of the three qubits which is argued as an important property in any situation where one of the three parties decide not to cooperate with the other two [8]. For the generalized form $W_n = \frac{1}{\sqrt{n}}(n-1, 1)$, where $|n-1, 1\rangle$ denotes the totally symmetric state including $n-1$ zeros and 1 ones, the concurrence (which is related to the formation entanglement) of any reduced density operators $\rho_{k,u}$, $C_{k,u}(\rho_{k,u}) = 2/n$, which indicates the maximal entanglement achievable for any reduced two parties of system in any pure state [8,11]. The states which can been converted to each other under SLOCC belong to the same class, then there are at least two inequivalent classes of multiparticle entanglement states: the GHZ state class and the W state class [8].

Recently, it has been realized that quantum resources can be useful in information processing where quantum entanglement plays a key role in many such applications like quantum teleportation [12], computer [13], cryptography [14]. And it has been shown that multipartite states have some advantages over the two-particle Bell states in their application to cloning [15,16], teleportation [17], and dense coding [18]. Then the preparation and manipulation of the entanglement states becomes a critical technique for these quantum information processing. Many schemes such as these employing the Jaynes-Cummings model in the cavity quantum electrodynamics (QED) [19], ion trap [20], NMR [21] have been proposed. In experiment, two particles entangled states have been realized in both cavity QED [22] and ion traps [23]. But in most of the previous schemes for quantum information processing in cavity QED and ion traps, the cavity and ion motion act as memories. Thus the decoherence of the cavity field becomes one of the main obstacles for the implementation of quantum information in the cavity field, while in the ion traps is the difficulty to achieve the joint ground state of the ion motion and the heating of the ions. In paper [24], Sørensen and Mølmer have proposed schemes for realizing quantum computation in the ion traps via virtual vibrational excitations. They [25] have also proposed a scheme for the generation of multi-particle states in the GHZ state class in ion traps without the requirement of the full control of the ion motion, which has been accomplished in experiment [26]. In the cavity QED, Zheng and Guo [27] have proposed a novel scheme for two-atom entanglement and quantum information processing.

\[\text{PACS number(s): 03.65.Ud, 03.67.Lx, 42.50.Dv}\]

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whose experiment implementation has also been reported by Osnahgi et al. very quickly [28].

In this paper, we present firstly a scheme for the generation of the multi-particle entanglement states of both the GHZ state class and the W state class in cavity QED. The scheme does not require the transfer of quantum information between the atoms and the cavity. As the cavity is only virtually excited, the requirement on the quality of the cavities is greatly loosened and the efficient decoherence time of it is greatly prolonged.

Firstly, we consider the model of two identical two-level atoms simultaneously interacting with a single-mode cavity field. The interaction Hamiltonian in the interaction picture is

$$H_i = g \sum_{j=1}^{n} (e^{-i\delta t} a^+ s_j^- + e^{i\delta t} a s_j^+), \quad (1)$$

where $s_j^+ = |1\rangle_{jj} \langle 0|$ and $s_j^- = |0\rangle_{jj} \langle 1|$, with $|1\rangle_j$ and $|0\rangle_j (j = 1, 2, ..., n)$ being the excited and ground states of the $j$th atom, $a^+$ and $a^-$ are, respectively, the creation and annihilation operators for the cavity mode, $g$ is the atom-cavity coupling strength, and $\delta$ is the detuning between the atomic transition frequency $\omega_0$ and cavity frequency $\omega$.

In the case $\delta \gg g$, there is no energy exchange between the atomic system and the cavity. The effective Hamiltonian obtained by adiabatically eliminating the atomic coherence is given by

$$H = \lambda \sum_{i,j=1}^{n} (s_i^- s_i^+ a^+ a - s_i^+ s_i^- a^+ a), \quad (2)$$

where $\lambda = g^2 / \delta$. This can been viewed as a generalized Jaynes-Cummings model Hamiltonian describing a cavity mode interacting with $n$ atoms. When $n = 1$, the Hamiltonian is

$$H = \lambda (|1\rangle \langle 1| a a^+ - |0\rangle \langle 0| a^+ a), \quad (3)$$

which represents the far-off-resonant case of the Jaynes-Cummings model [29]. When $n = 2$, the Hamiltonian is

$$H = \lambda \sum_{j=1,2} (|1\rangle_{jj} \langle 1| a a^+ - |0\rangle_{jj} \langle 0| a^+ a) + (s_1^+ s_2^- + s_2^+ s_1^-), \quad (4)$$

which has been shown to be useful in the generation of two-atom maximally entangled states and the realization of quantum controlled-not gates and quantum teleportation with dispersive cavity QED [27]. The procedure in this scheme, essentially insensitive to thermal fields and to photon decay, opens promising perspectives for complex entanglement manipulations [28].

Now we consider the case of multi-atom. Assume that the cavity field is initially in the vacuum state, the Hamiltonian reduces to

$$H = \lambda \sum_{j=1}^{n} |1\rangle_{jj} \langle 1| + \sum_{i,j=1,i\neq j}^{n} s_j^+ s_i^- \quad (5)$$

It is obvious that there is no quantum information transfer between the atoms and cavity. For the case of $n = 3$, the Hamiltonian can be written as

$$H = \lambda \sum_{j=1,2,3} |1\rangle_{jj} \langle 1| + (s_1^+ s_2^- + s_1^- s_2^+ + s_1^+ s_3^- + s_1^- s_3^+ + s_2^+ s_3^- + s_2^- s_3^+). \quad (6)$$

The first term describe the Stark shifts in the vacuum cavity, and the rest terms describe the dipole coupling between any of the two atoms induced by the cavity mode. Assume the atoms are initially in the state $|001\rangle$, then the state evolution of the system can be represented by

$$W_3(t) = \frac{e^{-i3\lambda t} + 2}{3} |001\rangle + \frac{e^{-i3\lambda t} - 1}{3} (|010\rangle + |100\rangle). \quad (7)$$

With the choice of $\lambda t = \frac{2\pi}{3}$, we obtain the W states [8]

$$W_3 = \frac{1}{\sqrt{3}} (e^{i\frac{2\pi}{3}} |001\rangle + |010\rangle + |100\rangle), \quad (8)$$

where the common phase factor $e^{-i\frac{2\pi}{3}}$ has been discarded.
Generally if initially the first \(n-1\) atoms are in the state \(|0\rangle\) and the last atom is in \(|1\rangle\), the evolution of the state goes as follows:

\[
W_n(t) = \frac{e^{-in\lambda t} + n - 1}{n} |0\rangle_{1,2,\ldots,n-1} |1\rangle_n + \frac{e^{-in\lambda t} - 1}{n} |n-2, 1\rangle_{1,2,\ldots,n-1} |0\rangle_n,
\]

where \(|n-2, 1\rangle_{1,2,\ldots,n-1}\) denotes the symmetric \(n-1\) particles states involving \(n-2\) zeros and 1 ones. With the different choice of evolution time, one can get various \(n\)-particle state of the W state class. This result can be understood from the properties of the Hamiltonian: the parity bit of the state is unchanged in the evolution process governed by this Hamiltonian, then the population becomes distributed on all the state with the same parity bit which forms the states of W state class. Obviously, only in the case of \(n \leq 4\) can \(|e^{-in\lambda t + n-1} - e^{-in\lambda t - 1}|\) equal \(|e^{-in\lambda t - 1}|\) which represents the maximally entangled W state. But if we measure the \(n\)th atoms at sometime \(t\) and get \(|0\rangle_n\), then the rest \(n-1\) atoms becomes in the state

\[
W_{n-1} = \frac{1}{\sqrt{n-1}} |n-2, 1\rangle.
\]

In this way, we can get the \((n-1)\)-particle maximal entangled W states with the probability of \(|\frac{e^{-in\lambda t}}{n} - 1|^2\) which gets its maximal value for the case of \(t = \frac{\pi}{n}\) and approximately inversely proportionate to the atom number \(n\).

Furthermore, using this generalized Jaynes-Cummings model, we can also prepare the states in the GHZ class. Assume four atoms are initially in the state \(|0011\rangle\), the evolution under the four-atom Hamiltonian is

\[
|\phi\rangle = \frac{1}{6}(e^{-i6\lambda t} + 3e^{-i3\lambda t} + 2)|0011\rangle + \frac{1}{6}(e^{-i6\lambda t} - 3e^{-i3\lambda t} + 2)|1100\rangle \\
+ \frac{1}{6}(e^{-i6\lambda t} - 1)(|1001\rangle + |0101\rangle + |1010\rangle + |0110\rangle).
\]

Also with the choice of \(\lambda t = \frac{\pi}{3}\), we obtain a state belonging to the GHZ state class

\[
|\phi\rangle = \frac{e^{-i\phi}}{2}(|0011\rangle + i\sqrt{3}|1100\rangle).
\]

Noticeably, although any \(n\)-particle W state can be generated straightforwardly in the present scheme, the \(m\)-particle GHZ state where \(m \geq 5\) cannot be prepared directly this way. But it has been well known that entangled states involving higher numbers of particles can be generated from entangled states involving lower numbers of particles by employing the same procedure as entanglement swapping [30]. The basic ingredients are a Bell state measuring device and lower numbers of particles entanglement states. Now it has been proved that there are at least two inequivalent classes multi-particle entanglement states which can not been converted to each other under SLOCC [8]. Then the lower numbers of particles entanglement states for the preparation of a higher numbers of particles state of GHZ state class or W state class must be GHZ state and W state respectively [31]. Here we have shown that both classes of states can be generated in the present scheme. Bell states measurement can also be realized in this generalized Jaynes-Cummings model of the \(n = 2\) case [27]. Then any multi-particle state of either the W state class or the GHZ state class can be prepared in this scheme of the QED cavity.

The discussion on the experimental matters is similar to the paper [27]. The two atoms experiment to prepare EPR pair using the present model of \(n = 2\) case has been realized recently [28]. As there is a probability of 0.78, 0.19, 0.025 respectively to have 0, 1, 2 atoms in one atom pulse and events in which only one atom is detected in two pulses are recorded, then in approximately 25% of these events, there are in fact two atoms in one of the pulses, one of them escaping detection. In addition to the probabilities \(P(e_1, g_2), P(g_1, e_2)\), there are also some spurious channels probabilities \(P(e_1, e_2), P(g_1, g_2)\) caused by possible three atoms collision. All these probabilities could be calculated using the present multi-atom model in detail:

\[
\begin{align*}
P(e_1, e_2) &= P(g_1, g_2) = 0.028(1 - \cos(3\lambda t)), \\
P(e_1, g_2) &= 0.514 + 0.375 \cos(2\lambda t) + 0.111 \cos(3\lambda t), \\
P(g_1, e_2) &= 0.430 - 0.375 \cos(2\lambda t) - 0.055 \cos(3\lambda t),
\end{align*}
\]

where the two atom pulses are assumed initially in excited and ground state respectively and the state discriminating errors are omitted. The result is shown in Figure 1. The experiment of [28] have shown the existence of \(P(e_1, e_2)\) and \(P(g_1, g_2)\), further experiment should reveal the oscillation of these probabilities with the interacting time.
This generalized Jaynes-Cummings model requires the atoms be sent through the cavity simultaneously, otherwise there will be an error. But the influence of time difference is not as severe as expected. Even assume the third atom in the excited state enters the cavity 10\% later than the other two ground state atoms ( the time difference between these two atoms is nonsignificant ) in the generation of the W state. Then the three atoms are finally prepared in the state \( W_3(0.90t_0) \). In the case that the third atom leaves the cavity 10\% earlier than the other two atoms, the three atoms final state becomes

\[
W'_3(0.90t_0) = \frac{e^{-i\lambda t} + 2}{3} |001\rangle + \frac{1}{3} e^{-i0.1\lambda t_0} (|010\rangle + |100\rangle)
\]  

(14)

If we still choosing \( \lambda t_0 = \frac{2\pi}{N} \) we have

\[
\begin{align*}
|\langle W'_3(0.90t_0)|W_3(t_0)\rangle|^2 &\simeq 0.99, \\
|\langle W'_3(0.90t_0)|W'_3(t_0)\rangle|^2 &\simeq 0.99.
\end{align*}
\]  

(15)

The operation is only slightly affected.

In conclusion, we have present a generalized Jaynes-Cummings model involving a single-mode cavity field and \( n \) identical two-level atoms. One of its applications for the preparations of the multi-particle states is analyzed. In addition to the GHZ state, the W states can also be generated in this scheme. The further analysis for the experiment of the model of \( n = 2 \) case is also presented by considering the possible three-atom collision. The most distinct advantage of this model is that cavity initially is in vacuum state and no quantum information transfer is required. Thus the requirement on the quality factor of the cavity is greatly loosened and then implementation is foreseeable.

This work was supported by the National Natural Science Foundation of China.

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Figure Captions:
Figure 1: The graph of the measurement probabilities $P(e_1, g_2), P(g_1, e_2), P(e_1, e_2)$ and $P(g_1, g_2)$ versus $\lambda t$. The solid line denotes the probability $P(e_1, g_2)$, the dashed line denotes $P(g_1, e_2)$, and the dotted line denotes $P(e_1, e_2)$ and $P(g_1, g_2)$.