We investigated the possibility that nonlinear gravitational effects influence the preheating era after inflation, using numerical solutions of the inhomogeneous Einstein field equations. We compared our results to perturbative calculations and to solutions of the nonlinear field equations in a rigid (unperturbed) spacetime, in order to isolate gravitational phenomena. We confirm the broad picture of preheating obtained from the nonlinear field equations in a rigid background, but find gravitational effects have a measurable impact on the dynamics. The longest modes in the simulation grow much more rapidly in the relativistic calculation than with a rigid background. We used the Weyl tensor to quantify the departure from homogeneity in the universe. We saw no evidence for the sort of gravitational collapse that leads to the formation of primordial black holes.

For nearly a decade it has been known that coherent processes at the end of inflation can drive explosive particle production [1–3] via parametric resonance, leading to an era of preheating in the post-inflationary universe. Preheating is intrinsically nonlinear and takes place far from thermal equilibrium.

A full treatment of parametric resonance must include the inhomogeneous metric induced by the inhomogeneous matter. Incorporating metric perturbations into the analysis of parametric resonance has been the subject of considerable recent attention. (See [4] and references therein.)

As a testbed for preheating, we considered the chaotic inflationary model driven by the potential $V(\phi) = \lambda \phi^4/4$ where $\phi$ is the inflaton field. The onset of resonance can be studied perturbatively, however perturbation theory cannot give a full description of resonance, since once the $\delta \phi_k$ grow large, mode-mode couplings become significant. The next level of sophistication is to include all terms which are nonlinear in the fields, but to assume that the background FLRW spacetime is unperturbed. However if the spatial gradients are non-zero, the metric cannot be perfectly homogeneous, so gravitational
FIG. 1. The growth of $C^2/R^2$ during the course of the simulation indicates the growth of inhomogeneity. It levels off at late times suggesting primordial black holes do not form.

effects on preheating are ignored when this equation is solved. Therefore one is led finally to numerically solving the full Einstein field equations.

To reduce the computational complexity of the problem, we assumed that the universe has a planar symmetry; the metric functions depend only on $\eta$ and $\zeta$, and are independent of $x$ and $y$. We used the metric

$$ds^2 = \alpha^2(\eta, \zeta) (d\eta^2 - d\zeta^2) - \beta^2(\eta, \zeta) (dx^2 + dy^2).$$

The equations of motion and details of our numerical code may be found in [4]. We began our simulations at the end of inflation, so $\phi$ is described by small fluctuations overlaid on top of the homogeneous zero mode. The fluctuations are quantum in origin, and are generated by the usual inflationary mechanism.

For the longest modes there are clear differences between the results found in a fixed FLRW background and the full relativistic calculation. Both the relativistic calculation and the nonlinear field equations in an unperturbed background predict that the longest modes of the field perturbation grow significantly. However, the relativistic results predict that this growth begins earlier and is an order of magnitude larger than that found from the field evolution in a rigid background. Note that in both cases we are seeing the growth of modes that lie outside the instantaneous Hubble horizon.

As a non-perturbative measure of inhomogeneity, we considered the ratio of $C^2$, the spatial average of $C^2 = C_{\mu\nu\gamma\lambda}C^{\mu\nu\gamma\lambda}$, to $R^2$, the spatial average of $R^2$. Because $C^2$ vanishes everywhere in an FLRW spacetime, the dimensionless quantity $C^2/R^2$ is a measure of how far spacetime departs from a perfectly FLRW universe. If the enhanced density perturbations generated during resonance led to localized gravitational collapse and the formation of primordial black holes (or “black walls” in this case), it would be obvious from $C^2/R^2$ as the curvature would diverge in a collapsing region. Since this does not happen (see Fig. (1)) we find no evidence that preheating after $\lambda \phi^4$ inflation leads to primordial black hole formation.

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