Indication for light sneutrinos and gauginos from precision electroweak data

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Abstract: The present standard model fit of precision data has a low confidence level, and is characterized by a few inconsistencies. We look for supersymmetric effects that could improve the agreement among the electroweak precision measurements and with the direct lower bound on the Higgs mass. We find that this is the case particularly if the 3.6 σ discrepancy between $\sin^2\theta_{\text{eff}}$ from leptonic and hadronic asymmetries is finally settled more on the side of the leptonic ones. After the inclusion of all experimental constraints, our analysis selects light sneutrinos, with masses in the range 55 – 80 GeV, and charged sleptons with masses just above their experimental limit, possibly with additional effects from light gauginos. The phenomenological implications of this scenario are discussed.

Keywords: Standard Model, Supersymmetric Standard Model, LEP HERA and HERAsPhysics.
1. Introduction

The results of the electroweak precision tests as well as of the searches for the Higgs boson and for new particles performed at LEP and SLC have now been presented in a close to final form. Taken together with the measurements of $m_t$, $m_W$ and the searches for new physics at the Tevatron, and with some other data from low energy experiments, they form a very stringent set of precise constraints to compare with the Standard Model (SM) or with any of its conceivable extensions. When confronted with these results, on the whole the SM performs rather well, so that it is fair to say that no clear indication for new physics emerges from the data. However, if we look at the results in detail, there are a number of features that are either not satisfactory or could indicate the presence of small new physics effects. We will describe in quantitative terms the experimental results and their consistency among themselves and with the SM in the next section. Here we anticipate a qualitative discussion.

One problem is that the two most precise measurements of $\sin^2 \theta_{\text{eff}}$ from $A_{LR}$ and $A_{FB}^b$ differ by 3.5 $\sigma$. More in general, there appears to be a discrepancy between $\sin^2 \theta_{\text{eff}}$ measured from leptonic asymmetries and from hadronic asymmetries. The result from $A_{LR}$ is actually in good agreement with the leptonic asymmetries measured at LEP, while all hadronic asymmetries are better compatible with the result of $A_{FB}^b$. It is well known that this discrepancy is not likely to be explained by some new physics effect in the $b\bar{b}Z$ vertex. In fact $A_{FB}^b$ is the product of lepton- and $b$-asymmetry factors: $A_{FB}^b \propto A_L A_b$, where $A_f = 2g_A^f g_V^f / (g_A^2 + g_V^2)$. The sensitivity
of $A_{FB}^b$ to $A_b$ is limited, because the $A_\ell$ factor is small, so that, in order to reproduce the measured discrepancy, the new effect should induce a large change of the $b$ couplings with respect to the SM. But then this effect should be clearly visible in the direct measurement of $A_b$ performed at SLD using the LR polarized $b$ asymmetry, even within the moderate precision of this result, and it should also appear in the accurate measurement of $R_b \propto g^b_A + g^b_V$. Neither $A_b$ nor $R_b$ show deviations of the expected size. One concludes that most probably the observed discrepancy is due to a large statistical fluctuation and/or to an experimental problem. Indeed, the measurement of $A_{FB}^b$ not only requires $b$ identification, but also distinguishing $b$ from $\bar{b}$, and therefore the systematics involved are different than in the measurement of $R_b$. At any rate, the disagreement between $A_{FB}^b$ and $A_{LR}$ implies that the ambiguity in the measured value of $\sin^2 \theta_{\text{eff}}$ is larger than the nominal error obtained from averaging all the existing determinations.

Another point of focus is the relation between the fitted Higgs mass and the lower limit on this mass from direct searches, $m_H > 113$ GeV, as it was recently stressed in ref. \cite{2}. The central value of the fitted mass is systematically below the limit. In particular, given the experimental value of the top mass, the measured results for $m_W$ (with perfect agreement between LEP and the Tevatron) and $\sin^2 \theta_{\text{eff}}$ measured from leptonic asymmetries, taken together with the results on the $Z_0$ partial widths, push the central value of $m_H$ very much down. In fact, if one arbitrarily excludes $\sin^2 \theta_{\text{eff}}$ measured from the hadronic asymmetries, the fitted value of $m_H$ becomes only marginally consistent with the direct limit, to a level that depends on the adopted value and the error for $\alpha_{\text{QED}}(m_Z)$. Consistency is reinstated if the results from hadronic asymmetries are also included, because they drive the fitted $m_H$ value towards somewhat larger values.

In conclusion, if one takes all available measurements into account the $\chi^2$ of the SM fit is not good, with a probability of about 4%, partly because the measurements of $\sin^2 \theta_{\text{eff}}$ are not in good agreement among them. If, on the other hand, one only takes the results on $\sin^2 \theta_{\text{eff}}$ from the leptonic asymmetries, then the $\chi^2$ of the SM fit considerably improves, but the consistency with the direct limit on $m_H$ becomes marginal.

In this article we enlarge the discussion of the data from the SM to the Minimal Supersymmetric Standard Model (MSSM). We look for regions of the MSSM parameter space where the corrections are sufficiently large and act in the direction of improving the quality of the fit and the consistency with the direct limit on $m_H$ with respect to the SM, especially in the most unfavourable case for the SM that the results on $\sin^2 \theta_{\text{eff}}$ from the hadronic asymmetries are discarded. We will show that, if sleptons (and, to a lesser extent, charginos and neutralinos) have masses close to their present experimental limits, it is possible to considerably improve the overall picture. In particular the possible MSSM effects become sizeable if we allow the sneutrino masses to be as small as allowed by the direct limits on $m_{\tilde{\nu}}^2$ and by
those on charged slepton masses, which are related by $m_{\tilde{\nu}_\ell}^2 = m_\nu^2 + m_{W}^2 \cos 2\beta$. At moderately large values of $\tan\beta$ (i.e. for $| \cos 2\beta \sim 1$), light sneutrinos with masses as low as 55 GeV are not excluded by present limits, while charged sleptons must be heavier than 96 GeV. These low values of the sneutrino mass can still be compatible with the neutralino being the lightest supersymmetric particle. This region of parameter space was not emphasized in some past analyses [3,4,5]. We recall that $\tan\beta \gtrsim 2 - 3$ is required by LEP, and large $\tan\beta$ and light sleptons are indicated by the possible deviation observed by the recent Brookhaven result [6] on the muon $g - 2$, if this discrepancy is to be explained by a MSSM effect. We find it interesting that, by taking seriously the small hints that appear in the present data, one can pinpoint a region of the MSSM which match the data better than the SM, and is likely to be within reach of the present run of the Tevatron and, of course, of the LHC.

For this analysis in the MSSM we use the technique of the epsilon parameters $\epsilon_1$, $\epsilon_2$, $\epsilon_3$ and $\epsilon_b$, introduced in ref. [7]. The variations of $\epsilon_1$, $\epsilon_2$ and $\epsilon_3$ due to new physics contributions are proportional to the shifts in the $T$, $U$, and $S$ parameters [8], respectively, if one keeps only oblique contributions (i.e. terms arising from vacuum polarization diagrams), expanded up to the first power in the external momentum squared. But in the MSSM not all important contributions are of this kind [5]. We recall that the starting point of the epsilon analysis is the unambiguous definition of the $\epsilon_i$ in terms of four basic observables that were chosen to be $\sin^2 \theta_{\text{eff}}$ from $A_{FB}^\mu$, $\Gamma_\mu$, $m_W$ and $R_b$. Given the experimental values of these quantities, the corresponding experimental values of the $\epsilon_i$ follow, independent of $m_t$ and $m_H$, with an error that, in addition to the propagation of the experimental errors, also includes the effect of the present ambiguities in $\alpha_s(m_Z)$ and $\alpha_{\text{QED}}(m_Z)$.

If one assumes lepton universality, which is well supported by the data within the present accuracy, then the combined results on $\sin^2 \theta_{\text{eff}}$ from all leptonic asymmetries can be adopted together with the combined leptonic partial width $\Gamma_\ell$. At this level the epsilon analysis is model-independent within the stated lepton universality assumption. As a further step we can observe that by including the information on the hadronic widths arising from $\Gamma_Z$, $\sigma_b$, $R_\ell$, the central values of the $\epsilon_i$ are not much changed (with respect to the error size) and the errors are slightly decreased. Thus one may decide of including or not including these data in the determination of the $\epsilon_i$, without affecting the results.

Different is the case of including the results from the hadronic asymmetries in the combined value of $\sin^2 \theta_{\text{eff}}$. In this case, obviously, the determination of $\epsilon_i$ is sizeably affected and one remains with the alternative between an experimental problem or a bizarre effect of some new physics in the $b$ coupling (not present in the MSSM). But if we remain within the first stage of purely leptonic measurements plus $m_W$ and $R_b$, the $\epsilon_i$ analysis is quite general and, in particular, is independent of an assumption of oblique correction dominance.
The comparison with the SM can be repeated in the context of the $\epsilon_i$. The predicted theoretical values of the $\epsilon_i$ in the SM depend on $m_H$ and $m_t$, while they are practically independent of $\alpha_s(m_Z)$ and $\alpha_{QED}(m_Z)$. If we only take the leptonic measurements of $\sin^2 \theta_{\text{eff}}$, for $m_H = 113$ GeV and $m_t = 174.3$ GeV one finds that the experimental value of $\epsilon_1$ agrees within the error with the prediction, while both $\epsilon_2$ and $\epsilon_3$ are below the theoretical expectation by about 1 $\sigma$. We recall that $m_W$ is related to $\epsilon_2$ and the fact that the experimental value is below the prediction for this quantity corresponds to the statement that $m_W$ would prefer a value of $m_H$ much smaller than $m_H = 113$ GeV. Similarly the smallness of the fitted value of $\epsilon_3$ with respect to the prediction has to do with the marked preference for a light $m_H$ of $\sin^2 \theta_{\text{eff}}$ from all leptonic asymmetries. The agreement between fitted value and prediction for $\epsilon_1$, which, contrary to $\epsilon_2$ and $\epsilon_3$, contains a quadratic dependence on $m_t$, reflects the fact that the fitted value of $m_t$ is in agreement with the measured value. The other variable that depends quadratically on $m_t$ is $\epsilon_b$. The agreement of the fitted and predicted values of $\epsilon_b$ reflects the corresponding present normality of the results for $R_b$.

2. The data and their comparison with the standard model

We start by summarising the different existing determinations of $\sin^2 \theta_{\text{eff}}$ and their mutual consistency. The two most precise measurements from $A_{LR}$ by SLD and $A_{FB}^b$ by LEP lead to

$$\sin^2 \theta_{\text{eff}} = 0.23098 \pm 0.00026 \quad (A_{LR})$$

$$\sin^2 \theta_{\text{eff}} = 0.23240 \pm 0.00031 \quad (A_{FB}^b).$$

As already mentioned these two measurements differ by 3.5$\sigma$. If we take $\sin^2 \theta_{\text{eff}}$ from the combined LEP/SLD leptonic or hadronic asymmetries we have

$$\sin^2 \theta_{\text{eff}} = 0.23114 \pm 0.00020 \quad \text{(all leptonic asymmetries)}$$

$$\sin^2 \theta_{\text{eff}} = 0.23240 \pm 0.00029 \quad \text{(all hadronic asymmetries)}.$$  

The resulting discrepancy is at 3.6 $\sigma$, thus at about the same level. By combining all the above measurements one obtains

$$\sin^2 \theta_{\text{eff}} = 0.23156 \pm 0.00017 \quad \text{(all asymmetries)}.$$  

We see that the dispersion between the results from leptonic and hadronic asymmetries is much larger than the nominal error in the combination.

The experimental values of the most important electroweak observables which are used in our analysis are collected in table 1.
A quantity which plays a very important role in the interpretation of the electroweak precision tests is the value of $\alpha_{QED}(m_Z)$, the QED coupling at the scale $m_Z$ or, equivalently, $\Delta \alpha_h$, the hadronic contribution to the shift $\Delta \alpha$, with $\alpha_{QED}(m_Z) = \alpha/(1 - \Delta \alpha)$. We adopt here as our main reference values those recently derived in ref. [9]:

$$\Delta \alpha_h = 0.02761 \pm 0.00036,$$

$$\alpha_{QED}^{-1}(m_Z) = 128.936 \pm 0.049 \quad (\text{BP01}). \quad (2.6)$$

A larger set of recent determinations of $\Delta \alpha_h$ will also be used for comparison (see table 2).

We consider now different SM fits to the observables $m_t$, $m_W$, $\Gamma_t$, $R_b$, $\alpha_s(m_Z)$, $\alpha_{QED}$, with different assumptions on the input value of $\sin^2 \theta_{\text{eff}}$. These fits are based on up-to-date theoretical calculations [10]. We start by considering $\sin^2 \theta_{\text{eff}}$ from all leptonic asymmetries, eq. (2.3), and $\sin^2 \theta_{\text{eff}}$ from all hadronic asymmetries, eq. (2.4), as two distinct inputs in the same fit. In this case, we find $\chi^2/\text{d.o.f.} = 18.4/4$, corresponding to C.L.=0.001. When a more complete analysis is performed, including all 20 observables in the global fit, the situation is still not satisfying, although less dramatic: ref. [4] reports $\chi^2/\text{d.o.f.} = 26/15$, with C.L.=0.04. If we now exclude $\sin^2 \theta_{\text{eff}}$ from all hadronic asymmetries, the quality of the fit of our seven observables significantly improves, giving $\chi^2/\text{d.o.f.} = 2.5/3$, C.L.=0.48, while the fit to all observables except $A^b_{FB}$ gives [4] $\chi^2/\text{d.o.f.} = 15.8/14$, C.L.=0.33. Finally, if we instead exclude $\sin^2 \theta_{\text{eff}}$ from all leptonic asymmetries, we find $\chi^2/\text{d.o.f.} = 15.3/3$, C.L.=0.0016. Thus it appears that the leptonic value of $\sin^2 \theta_{\text{eff}}$ leads to the best overall consistency in terms of C.L.

We now consider the corresponding fitted values of the Higgs mass, and the 95% C.L. upper limits. In the first case studied above, namely when $\sin^2 \theta_{\text{eff}}$ from both hadronic and leptonic asymmetries are included, with $\Delta \alpha_h^{\text{BP01}}$ given in eq. (2.6), we obtain a central value for the Higgs mass of $m_H = 100 \text{ GeV}$, with a 95% C.L. limit $m_H < 212 \text{ GeV}$. These values are indeed in complete agreement with the SM fit results presented by the LEP Electroweak Group [4], based on the complete set

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Data (March 2001)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_Z$ (GeV)</td>
<td>91.1875(21)</td>
</tr>
<tr>
<td>$\Gamma_Z$ (GeV)</td>
<td>2.4952(23)</td>
</tr>
<tr>
<td>$\sigma_h$ (nb)</td>
<td>41.540(37)</td>
</tr>
<tr>
<td>$R_t$</td>
<td>20.767(25)</td>
</tr>
<tr>
<td>$R_b$</td>
<td>0.21664(68)</td>
</tr>
<tr>
<td>$\Gamma_\ell$ (MeV)</td>
<td>83.984(86)</td>
</tr>
<tr>
<td>$A_{FB}$</td>
<td>0.01714(95)</td>
</tr>
<tr>
<td>$A_\ell$</td>
<td>0.1439(41)</td>
</tr>
<tr>
<td>$A_e$</td>
<td>0.1498(48)</td>
</tr>
<tr>
<td>$A^b_{FB}$</td>
<td>0.0982(17)</td>
</tr>
<tr>
<td>$A^c_{FB}$</td>
<td>0.0689(35)</td>
</tr>
<tr>
<td>$A_b$ (SLD direct)</td>
<td>0.921(20)</td>
</tr>
<tr>
<td>$\sin^2 \theta_{\text{eff}}$ (all lept. asym.)</td>
<td>0.23114(20)</td>
</tr>
<tr>
<td>$\sin^2 \theta_{\text{eff}}$ (all hadr. asym.)</td>
<td>0.23240(29)</td>
</tr>
<tr>
<td>$m_W$ (GeV) (LEP2+pp)</td>
<td>80.448(34)</td>
</tr>
<tr>
<td>$m_t$ (GeV)</td>
<td>174.3(5.1)</td>
</tr>
<tr>
<td>$\alpha_s(m_Z)$</td>
<td>0.119(3)</td>
</tr>
</tbody>
</table>

Table 1: Observables included in our global fit.
Table 2: Different determinations of $\Delta \alpha_h$ and their influence on the fitted Higgs mass.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>$\Delta \alpha_h$</th>
<th>$m_H$ (GeV)</th>
<th>95% C.L. limit (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BP01</td>
<td>0.02761(36)</td>
<td>42</td>
<td>109</td>
</tr>
<tr>
<td>J01</td>
<td>0.027896(395)</td>
<td>34</td>
<td>91</td>
</tr>
<tr>
<td>Jeucl01</td>
<td>0.027730(209)</td>
<td>40</td>
<td>98</td>
</tr>
<tr>
<td>MOR00</td>
<td>0.02738(20)</td>
<td>52</td>
<td>124</td>
</tr>
<tr>
<td>DH98</td>
<td>0.02763(16)</td>
<td>42</td>
<td>104</td>
</tr>
<tr>
<td>KS98</td>
<td>0.02775(17)</td>
<td>38</td>
<td>96</td>
</tr>
<tr>
<td>EJ95</td>
<td>0.02804(65)</td>
<td>28</td>
<td>90</td>
</tr>
</tbody>
</table>

of observables: $m_H = 98$ GeV and $m_H < 212$ GeV. Neglecting the fact that the dispersion of the various measurements corresponds to a very poor $\chi^2$, there is no significant contradiction with the direct limit on $m_H$. However, it is well known and was recently emphasized in ref. [2] that, if instead we use $\sin^2 \theta_{\text{eff}}$ measured from leptonic asymmetries only, see eq. (2.3), which leads to the best value of $\chi^2$/d.o.f., then the fitted value of $m_H$ markedly drops and the consistency with the direct limit becomes marginal. In fact, in this case, all other inputs being the same, we find $m_H = 42$ GeV and $m_H < 109$ GeV. In table 2 we report the corresponding results for some other determinations of $\Delta \alpha_h$. We see that, while there is some sensitivity to this choice, the conclusion that the compatibility of the fitted value of $m_H$ with the direct limit becomes marginal is quite stable. Similarly, we believe that uncalculated higher order effects cannot have a serious impact, as they can be estimated [17] to shift the 95% C.L. up by at most 10-15 GeV.

It must however be recalled that the level of compatibility is sensitive to the top mass, and is increased if $m_t$ is moved up within its error bar: for a shift up by 1σ we find, using $\Delta \alpha_h^{\text{BP01}}$, $m_H = 58$ GeV and $m_H < 156$ GeV.

We now consider the epsilon analysis. As already mentioned, the predicted values of the epsilon variables in the SM depend on $m_t$ and $m_H$, while they are practically insensitive to small variations of $\alpha_s(m_Z)$ and $\alpha_{\text{QED}}(m_Z)$. We report here the values of $\epsilon_i$ for $m_H = 113$ GeV and $m_t = 174.3 - 5.1, 174.3, 174.3 + 5.1$ GeV, respectively:

$$
\begin{align*}
\epsilon_1 &= 5.1 \times 10^{-3}, \quad 5.6 \times 10^{-3}, \quad 6.0 \times 10^{-3} \\
\epsilon_2 &= -7.2 \times 10^{-3}, \quad 7.4 \times 10^{-3}, \quad 7.6 \times 10^{-3} \\
\epsilon_3 &= 5.4 \times 10^{-3}, \quad 5.4 \times 10^{-3}, \quad 5.3 \times 10^{-3} \\
\epsilon_b &= -6.2 \times 10^{-3}, \quad -6.6 \times 10^{-3}, \quad -7.1 \times 10^{-3}.
\end{align*}
$$

We first consider the observables $\sin^2 \theta_{\text{eff}}$ measured from leptonic asymmetries, see eq. (2.3), $\Gamma_\ell$, $m_W$, and $R_b$. From these observables we obtain the following values of the $\epsilon_i$:

$$
\epsilon_1 = (5.1 \pm 1.0) \times 10^{-3}
$$
\[ \epsilon_2 = (-9.0 \pm 1.2) \times 10^{-3} \]
\[ \epsilon_3 = (4.2 \pm 1.0) \times 10^{-3} \]
\[ \epsilon_b = (-4.2 \pm 1.8) \times 10^{-3}. \] (2.8)

(in our fits, the value of \( \alpha_s \) was kept fixed). The errors also include the effect of the quoted errors on \( \alpha_s(m_Z) \) and \( \alpha_{QED}(m_Z) \). At this stage we have only assumed lepton universality and the derivation of the \( \epsilon_i \) is otherwise completely model independent. For example, no assumption of oblique corrections dominance is to be made. It is interesting to observe that if we add to the previous set of observables the information on the hadronic widths arising from \( \Gamma_Z, \sigma_h, R_\ell \) we obtain for the \( \epsilon_i \)
\[ \epsilon_1 = (5.0 \pm 1.0) \times 10^{-3} \]
\[ \epsilon_2 = (-9.1 \pm 1.2) \times 10^{-3} \]
\[ \epsilon_3 = (4.2 \pm 1.0) \times 10^{-3} \]
\[ \epsilon_b = (-5.7 \pm 1.6) \times 10^{-3}. \] (2.9)

The central values are only changed by a small amount (in comparison with the error size) with respect to the previous fit. We interpret this result by concluding that the hadronic \( Z_0 \) widths are perfectly compatible with the leptonic widths. Thus, if there are new physics corrections in the widths, these must be mostly of universal type like from vacuum polarization diagrams. A posteriori we can add this information in the epsilon analysis which allows to slightly reduce the errors on the individual \( \epsilon_i \).

We can now consider the sensitivity of the \( \epsilon_i \) to the different determinations of \( \sin^2 \theta_{\text{eff}} \). We take the same set of observables as in the previous fit in eqs. (2.9), but replace \( \sin^2 \theta_{\text{eff}} \) from leptonic asymmetries with that obtained from all combined measurements, as given in eq. (2.5). The corresponding values of the \( \epsilon_i \) are given by
\[ \epsilon_1 = (5.4 \pm 1.0) \times 10^{-3} \]
\[ \epsilon_2 = (-9.7 \pm 1.2) \times 10^{-3} \]
\[ \epsilon_3 = (5.4 \pm 0.9) \times 10^{-3} \]
\[ \epsilon_b = (-5.5 \pm 1.6) \times 10^{-3}. \] (2.10)

We see that the most sensitive variable to \( \sin^2 \theta_{\text{eff}} \) is \( \epsilon_3 \) that changes by more than 1\( \sigma \) in the direction of a better agreement with the SM prediction for \( m_H = 113 \) GeV, but the value of \( \epsilon_2 \) is even further away from the SM prediction. This is in agreement with the results obtained in the direct analysis of the data in the SM.

The results of the above fits of the \( \epsilon_i \), including the error correlations among different variables, are shown in figure 3. In these figures we display the 1\( \sigma \) ellipses in the \( \epsilon_i \) plane that correspond to the fits in eqs. (2.8), (2.9) and (2.10). Note that these ellipses project \( \pm 1 \sigma \) errors on either axis. As such the probability for both \( \epsilon_i \) and \( \epsilon_j \) to fall inside the ellipse is only about 39%. The ellipses that correspond to
other significance levels can be obtained by scaling the ellipse axes by suitable well known factors. We note the following salient features. The fitted values of $\epsilon_1$ are in all cases perfectly compatible with the predicted value in the SM. This corresponds to the fact that the fitted and the measured values of $m_t$ coincide. The fitted values of $\epsilon_2$ are always below the prediction, reflecting the fact that the measured value of $m_W$ would prefer smaller $m_H$ and/or larger $m_t$. The $\epsilon_2$ deviation is larger when also the measurement of $\sin^2 \theta_{\text{eff}}$ from the hadronic asymmetries is included. The fitted values of $\epsilon_3$ are below the prediction if the value of $\sin^2 \theta_{\text{eff}}$ from leptonic asymmetries is used, while the agreement is restored if the measurement of $\sin^2 \theta_{\text{eff}}$ from the hadronic asymmetries is included.

In conclusion, the epsilon analysis reproduces the results obtained from the direct comparison of the data with the SM. The most important features are that both $m_W$ and $\sin^2 \theta_{\text{eff}}$ from leptonic asymmetries appear to favour small $m_H$ and/or large $m_t$.

In the following we will discuss the effect of supersymmetry and the choice of MSSM parameters that this trend suggests.

### 3. Supersymmetric contributions

Now we want to investigate whether low-energy supersymmetry can reconcile a Higgs mass above the direct experimental limit with a good $\chi^2$ fit of the electroweak data, in the case of $\sin^2 \theta_{\text{eff}}$ near the value obtained from leptonic asymmetries. Our approach
Figure 2: Measured values (cross) of $\epsilon_3$ and $\epsilon_2$ (left) and of $\epsilon_1$ and $\epsilon_3$ (right), with their $1\sigma$ region (solid ellipses), corresponding to case (a) of figure 1. The area inside the dashed curves represents the MSSM prediction for $m_{\tilde{e}_L}$ between 96 and 300 GeV, $m_{\tilde{\chi}^+}$ between 105 and 300 GeV, $-1000 < \mu < 1000$ GeV, $\tan \beta = 10$, $m_{\tilde{e}_R} = 1$ TeV, and $m_A = 1$ TeV.

is to discard the measurement of $A_{FB}^b$, which cannot be reproduced by conventional new physics effects, fix the Higgs mass above its present limit, and look for supersymmetric corrections that can fake a very light SM Higgs boson. As we have discussed in the previous section and as summarized in figure 1, this can be achieved if the new physics contributions to the $\epsilon$ parameters amount to shifting $\epsilon_2$ and $\epsilon_3$ down by slightly more than 1 $\sigma$, while leaving $\epsilon_1$ essentially unchanged.

Squark loops cannot induce this kind of shifts in the $\epsilon$ parameters, since their leading effect is a positive contribution to $\epsilon_1$. Thus, we will assume that all squarks are heavy, with masses of the order of one TeV. Since the mass of the lightest Higgs $m_H$ receives a significant contribution from stop loops, we can treat $m_H$ as an independent parameter and, in our analysis, we fix $m_H = 113$ GeV. Varying the pseudoscalar Higgs mass $m_A$ does not modify the results of our fit, and therefore we fix $m_A = 1$ TeV. The choice of the right-handed slepton mass has also an insignificant effect on the fit. Therefore, we are left with four relevant supersymmetric free parameters: the weak gaugino mass $M_2$, the higgsino mass $\mu$, the ratio of the Higgs vacuum expectation values $\tan \beta$ (which are needed to describe the chargino–neutralino sector), and a supersymmetry-breaking mass for the left-handed sleptons, $m_{\tilde{e}_L}$ (lepton flavour universality is assumed). The choice of the $B$-ino mass parameter $M_1$ does not significantly affect our results and, for simplicity, we have assumed the gaugino unification relation $M_1 = 5/3M_2\tan^2 \theta_W$.

We have computed the supersymmetric one-loop contributions to $\epsilon_1$, $\epsilon_2$ and $\epsilon_3$ using the results presented in ref. [4,3], and the package LoopTools [16] for the numerical computation of loop integrals. Figure 2 shows the range of the $\epsilon$ parameters
that can be spanned by varying $M_2$, $\mu$, $\tan \beta$, and $\tilde{m}_{\ell L}$, consistently with the present experimental constraints. We have imposed a limit on charged slepton masses of 96 GeV [18], on chargino masses of 103 GeV [18], and on the cross section for neutralino production $\sigma(e^+e^- \rightarrow \chi^0_1\chi^0_2 \rightarrow \mu^+\mu^-E) < 0.1$ pb. We have also required that the supersymmetric contribution to the muon anomalous magnetic moment, $a_{\mu} = (g-2)/2$, lie within the range $0 < a_{\mu} < 7.5 \times 10^{-9}$. As apparent from figure 2, light particles in the chargino-neutralino sector and light left-handed sleptons shift the values of $\epsilon_i$ in the favoured direction, and by a sufficient amount to obtain a satisfactory fit.

In figure 3 we show an alternative presentation of our results directly in terms of the shifts in the observables $m_W$, $\sin^2 \theta_{\text{eff}}$ and $\Gamma_\ell$ induced by supersymmetry.\footnote{A good approximation of the relations between shifts in the physical observables and in the $\epsilon$ parameters is given by $\delta m_W = (0.53\delta \epsilon_1 - 0.37\delta \epsilon_2 - 0.32\delta \epsilon_3) \times 10^5$ MeV; $\delta \Gamma_\ell = (1.01\delta \epsilon_1 - 0.22\delta \epsilon_3) \times 10^5$ keV; $\delta \sin^2 \theta_{\text{eff}} = -0.33\delta \epsilon_1 + 0.43\delta \epsilon_3$.} For reference, we also display in figure 3 the difference between the measured values of the observables (excluding the hadronic asymmetries) and the corresponding SM predictions for $m_H = 113$ GeV, $m_t = 174.3$ GeV. Supersymmetric contributions can bring the theoretical predictions in perfect agreement with the data. An interesting observation is that sparticle effects can increase $m_W$ by $\delta m_W$ up to $\sim 100$ MeV, which corresponds to approximately three standard deviations, and decrease $\sin^2 \theta_{\text{eff}}$ by $\delta \sin^2 \theta_{\text{eff}}$ up to about $-8 \times 10^{-4}$ ($\sim 4 \sigma$). Note the marked anticorrelation between $\delta m_W$ and $\delta \sin^2 \theta_{\text{eff}}$. $\Gamma_\ell$ is moved upwards, but only by less than 90 keV, or about 1 $\sigma$.

Let us now analyse in detail the mass spectrum responsible for this effects on the $\epsilon$ parameters. The most significant contribution is coming from light sneutrinos. The effect is maximal when $\tan \beta$ is large since this allows the smallest possible sneutrino mass compatible with the charged slepton mass bound,

$$m^2_{\tilde{\nu}_L} = m^2_\nu + m^2_W |\cos 2\beta|.$$  (3.1)

Figure 4 shows the supersymmetric contributions to the $\epsilon$ parameters as functions of the charged slepton (or sneutrino) mass, for a (purely gaugino) chargino of 105 GeV and for $\tan \beta = 10$. The steep functional dependence of the $\epsilon$’s on $m_{\tilde{\nu}}$ illustrates why very light sneutrinos are required to improve significantly the electroweak fit. The dependence of the $\epsilon$’s on the lightest chargino mass (again for a purely gaugino state) is shown in figure 5. This dependence is quite milder than in the sneutrino case. Notice from figure 5 that, even in the limit of heavy charginos, in which all the effect is coming from slepton vacuum polarization contributions, we can obtain a significant improvement over the SM fit of electroweak data. Light charginos (mostly because of their contributions to vertex and box diagrams) can improve the situation, especially by making $|\delta \epsilon_3| \lesssim |\delta \epsilon_2|$, as it seems suggested by the data. Next, we show in figure 6 how the supersymmetric contributions to $\epsilon$’s vary with the

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Figure 3: The area inside the dotted curves represents the shifts in the values of $\sin^2 \theta_{\text{eff}}$, $m_W$ and $\Gamma_\ell$ induced by supersymmetric corrections, for the same parameter region as in figure [2]. The shifts necessary to reproduce the central values of the data with $m_t = 174.3$ GeV and $m_H = 113$ GeV are also shown, together with the corresponding experimental errors. The dot-dashed lines are obtained by varying the left slepton masses, with all other supersymmetric particle decoupled. The solid curve is obtained analogously, but also keeping a gaugino-like chargino of 105 GeV. In each curve, the circles correspond to $m_\tilde{\nu} = 60, 70, 80$ GeV from left to right.

The lightest chargino composition (or, in other words, with the parameter $\mu$, for a fixed value of the chargino mass $m_{\chi^+} = 105$ GeV). Part of the region where the lightest chargino is dominantly a gaugino state (i.e. large $\mu$) is preferred by the requirements $\delta \epsilon_1 \lesssim 0$ and $|\delta \epsilon_3| \lesssim |\delta \epsilon_2|$, suggested by the data. For illustration purposes, the bound $0 < \delta a_\mu < 7.5 \times 10^{-9}$ has not been imposed in figure [2]. It would have the effect of excluding the region of negative $\mu$, and the region where the lightest chargino is dominantly a higgsino (small $|\mu|$).

The effect of the light sneutrinos on the electroweak observables is also explicitly shown in figure [3]. The dot-dashed lines show the contribution of light left sleptons, when all other supersymmetric particles are decoupled. It is apparent that left sleptons alone are responsible for the largest part of the effect. When light gauginos are added to the spectrum (see solid lines of figure [3], $\sin^2 \theta_{\text{eff}}$ increases, $\Gamma_\ell$ decreases, and $m_W$ remains constant, bringing the theoretical prediction to an even better agreement with the data. On the other hand, light higgsinos (which appear only in vacuum polarization diagrams) further decrease $\sin^2 \theta_{\text{eff}}$ and increase $m_W$ with respect to the sneutrino contribution.

To summarize, the request of an improved electroweak data fit is making precise demands on the supersymmetric mass spectrum. The left-handed charged sleptons have to be very close to their experimental bounds, the sneutrino mass is selected to be below about 80 GeV, the squarks are in the TeV range, and $\tan \beta \gtrsim 4$, while there
Figure 4: Supersymmetric contributions to the $\epsilon$ parameters as functions of the charged slepton (or sneutrino) mass, for a (purely gaugino) chargino mass of 105 GeV and $\tan \beta = 10$.

Figure 5: Supersymmetric contributions to the $\epsilon$ parameters as functions of the mass of a (purely gaugino) chargino, for a charged slepton mass of 96 GeV and $\tan \beta = 10$.

is no information on right-handed slepton masses. The lightest chargino, preferably a gaugino state with mass below about 150 GeV, further improves the fit. This range of supersymmetric parameters is very adequate in explaining the alleged discrepancy between the experimental and theoretical values of the muon anomalous magnetic moment $\mu$. In practice, requiring the supersymmetric contribution to $g - 2$ to be in the range indicated by the data amounts to selecting a sign (positive in our
Figure 6: Supersymmetric contributions to the $\epsilon$ parameters as functions of the higgsino mass $\mu$, for a charged slepton mass of 96 GeV, a chargino mass of 105 GeV, and $\tan \beta = 10$.

conventions) of the parameter $\mu$. We recall that, for moderately large $\tan \beta$, the negative sign of $\mu$ is disfavoured by the present measurements of the $B \rightarrow X_s \gamma$ branching ratio.

4. Phenomenological implications

It is interesting to consider if the requirements obtained in the previous section on the mass spectrum are consistent with predictions from the various theoretical schemes proposed for supersymmetry breaking. Squarks much heavier than sleptons, heavy higgsino states, and large values of $\tan \beta$ are fairly generic consequences of supersymmetric models with heavy gluinos and radiative electroweak symmetry breaking. More unusual is the existence of a sneutrino with mass less than about 80 GeV. For instance, in the supergravity-inspired scheme, in which all sleptons have a common supersymmetry-breaking mass at the GUT scale and gaugino masses are unified, we find

$$m_{\tilde{\nu}}^2 - m_{\tilde{\ell}_R}^2 = 0.56 M_2^2 + \frac{m_Z^2}{2} (1 - 4 \sin^2 \theta_W) |\cos 2\beta| .$$

This relation, together with eq. (3.1), implies that $m_{\tilde{\nu}} \gtrsim m_{\tilde{\ell}_R}$, once we use the chargino mass limit $M_2 \gtrsim 100$ GeV. The experimental limit on $m_{\tilde{\ell}_R}$ rules out the possibility of a very light sneutrino. Therefore, $m_{\tilde{\nu}} < 80$ GeV requires different supersymmetry-breaking masses for left and right sleptons. This could be achieved in supergravity GUT schemes with non-universal soft masses, by giving different scalar mass terms to matter fields in the $\bar{5}$ and in the $10$ representations of $SU(5)$. If we call $m_0$ the left slepton soft mass at the GUT scale, the sneutrino mass is
approximately given by

\[ m_\tilde{\nu}^2 = m_0^2 + 0.78M_2^2 - \frac{m_Z^2}{2} |\cos 2\beta|. \]  

(4.2)

If we impose \( m_0^2 > 0 \), the requirement \( m_\tilde{\nu} < 80 \text{GeV} \) implies \( M_2 < 116 \text{GeV} \), and therefore the chargino should lie just beyond its experimental limit.

Gauge-mediated supersymmetry breaking models \[19\] always predict \( m_{\tilde{\ell}_R} < m_{\tilde{\ell}_L} \), and exclude the existence of a very light sneutrino. On the other hand, this is a possibility in anomaly-mediated models \[20\] with an additional universal supersymmetry-breaking scalar mass, since the right and left charged sleptons turn out to be nearly degenerate in mass \[21\]:

\[ m_{\tilde{\ell}_L}^2 - m_{\tilde{\ell}_R}^2 \simeq 0.04 \left( m_Z^2 |\cos 2\beta| + M_2^2 \ln \frac{m_{\tilde{\ell}_R}}{m_Z} \right). \]  

(4.3)

In the case of anomaly mediation, the relation between gaugino masses is \( M_1 = 11M_2 \tan^2 \theta_W \), but this does not give any sizeable modification of the results shown in figures 2, 6. Therefore, both GUT supergravity schemes with non-universal mass terms and anomaly mediation can give mass spectra compatible with the requirements discussed in the previous section.

The selected supersymmetric mass spectrum, with sleptons and possibly charginos just beyond the present experimental bounds, is certainly very encouraging for the next generation of experiments. Future hadron and linear colliders can fully probe this parameter region. However, the phenomenology may be slightly unconventional. Indeed, the lightest supersymmetric particle (LSP) is either a neutralino (most probably a B-inos state) or the sneutrino. In gravity mediation with gaugino unification, the neutralino can be the LSP only if \( m_{\tilde{\chi}_\pm} < 110–120 \text{GeV} \). In anomaly mediation, there is the possibility that an almost mass-degenerate SU(2) triplet of gaugino states is the LSP, and the LEP bound on chargino masses is evaded. Otherwise, the sneutrino is the LSP.

A light spectrum of electroweak interacting sparticles is promising for early discovery. Hard leptons generated from the decay chains of supersymmetric particles are the generic signature of our scenario with light sleptons. This is particularly promising for searches at the Tevatron that rely on trilepton events. The trilepton topology is generated by production of a \( \chi_1^+ \chi_2^0 \) pair with a subsequent fully leptonic decay. In our case, we expect that the dominant decay modes of the (W-inolike) next-to-lightest neutralino are \( \chi_2^0 \to \tilde{\ell}_L^\pm \ell^\mp \) and \( \chi_2^0 \to \tilde{\nu} \nu \) and, for the chargino, \( \chi_1^\pm \to \tilde{\nu}_L^\pm \ell^\mp \) and \( \tilde{\chi}_1^\pm \to \tilde{\ell}_L^\pm \nu \). The decay modes into \( \tilde{\ell}_R \) are strongly suppressed in the pure W-inolimit, while an excess of \( \tau \) in the final state is present for a significant gaugino–higgsino mixing.

The slepton decay modes depend on the nature of the LSP. However in either case \( \tilde{\ell}_L^\pm \to \ell^\pm \chi_1^0 \) and \( \tilde{\nu} \to \nu \chi_1^0 \) for \( \chi_1^0 \) LSP or \( \tilde{\ell}_L^\pm \to \nu^\prime \ell^\pm \nu, \tilde{\ell}_L^\pm \to \tilde{\nu}_L \bar{f} f' \) for \( \tilde{\nu} \) LSP, the
final states are rather similar. Notice however that, for a sneutrino LSP, the charged slepton can decay also into a charged lepton of a different flavour, \( \tilde{\ell}_L^\pm \to \ell^\pm E \), or into quarks. The branching ratio into a single trilepton channel is approximately

\[
BR(\chi_2^0 \to \mu^+ \mu^- E) \times BR(\chi_1^\pm \to \mu^\pm E) = \frac{1}{9} \left[ 1 + \left( \frac{m_{\chi_2^0}^2 - m_{\tilde{\nu}}^2}{m_{\chi_1^0}^2 - m_{\tilde{\nu}}^2} \right)^2 \right]^{-1}.
\]

At present the experimental limit on the cross section of a single trilepton channel is \( \sigma(3\mu) < 0.05\,\text{pb} \) for \( m_{\chi_1^\pm} = 100–120\,\text{GeV} \) [22]. Since the cross section for production of gaugino-like \( \chi_1^\pm \chi_2^0 \) at \( \sqrt{s} = 2\,\text{TeV} \) is 0.3 pb (0.2 pb) for \( m_{\chi_1^\pm} = 100\,\text{GeV} \) (120 GeV), the signal rate (which is obtained by multiplying eq. (4.3) by the cross section) is not far beyond the present limit, and within reach of the Tevatron upgrading.\(^2\)

Let us now make some remarks on the relic abundance of the LSP in the scenario discussed here. Sneutrinos rapidly annihilate with antin neutrinos in the early universe through \( Z^0 \) exchange in the \( s \)-channel. Even in case of a cosmic lepton asymmetry, their number density would still be depleted by the process \( \tilde{\nu} \tilde{\nu} \to \nu \nu \) via neutralino \( t \)-channel exchange. This annihilation process is efficient, having an \( s \)-wave contribution, and it leads to a present sneutrino relic density

\[
\Omega_{\tilde{\nu}} h^2 \simeq 10^{-3} \left( \frac{M_2}{100\,\text{GeV}} \right)^2 \left( 1 + \frac{m_{\tilde{\nu}}^2}{M_2^2} \right).
\]

Values of \( \Omega_{\tilde{\nu}} \) interesting for the dark matter problem would require \( M_2 \gtrsim 1\,\text{TeV} \). At any rate, since the sneutrino-nucleon scattering cross section, in the non-relativistic regime, is 4 times larger than the cross section for a Dirac neutrino of the same mass, the case of a sneutrino with halo density in our galaxy has been ruled out by nuclear recoil detection searches. Nevertheless, it has been suggested [23] that a cold dark matter sneutrino could be resurrected in presence of a lepton-number violating interaction that splits the real and imaginary parts of the sneutrino field, since this would lead to a vanishing coupling of the LSP to the \( Z^0 \) boson.

Cosmologically more interesting is the case of a neutralino LSP. For a \( B \)-ino LSP and for \( m_{\tilde{\ell}_R} \lesssim 2m_{\tilde{\nu}} \), the neutralino annihilation rate in the early universe is dominated by \( \tilde{\ell}_R \) exchange, and its relic abundance is approximately given by

\[
\Omega_{\chi_1} h^2 \simeq \frac{m_{\tilde{\ell}_R}^4}{\text{TeV}^2 m_{\chi_1^0}^2} f \left( \frac{m_{\chi_1^0}^2}{m_{\tilde{\ell}_R}^2} \right),
\]

where \( f(x) = (1+x)^4/(1+x^2) \). For instance, for \( m_{\chi_1^0} = 60\,\text{GeV} \) and \( m_{\tilde{\ell}_R} = 130\,\text{GeV} \), we obtain \( \Omega_{\chi_1} = 0.3 \) (for a Hubble constant \( h = 0.7 \)). If \( m_{\tilde{\ell}_R} \gtrsim 2m_{\tilde{\nu}} \), then \( t \)-channel sneutrino and left charged slepton exchange dominate the annihilation cross section.

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\(^{2}\)We thank G. Polesello for help in the numerical calculation.
Since the hypercharge of left sleptons is half the hypercharge of right sleptons, even in this case we obtain an appropriate value of the neutralino relic abundance. For instance, for $m_{\tilde{\chi}_1^0} = 60\,\text{GeV}$ and $m_{\tilde{\nu}} = 70\,\text{GeV}$, we find $\Omega_{\chi} = 0.5$. However, we recall that coannihilation effects between $\tilde{\nu}$ and $\chi_1^0$ could significantly reduce the estimate of the relic abundance given here. Nevertheless, we can conclude that the supersymmetric mass spectrum selected by our analysis of electroweak data can predict the correct $\chi_1^0$ relic density to explain dark matter.

5. Conclusions

The long era of precision tests of the SM is now essentially completed. The result has been a confirmation of the SM to a level that was hardly believable apriori. In fact, on conceptual grounds, we expect new physics near the electroweak scale. The fitted Higgs mass from the radiative corrections is remarkably light. This fact is in favour of a picture of electroweak symmetry breaking in terms of fundamental Higgs fields like in supersymmetric extensions of the SM. A light Higgs in the MSSM should be accompanied by a relatively light spectrum of sparticles so that it would be natural to expect some of the lightest supersymmetric particles to be close to their present experimental limits. Although it is well known that the supersymmetric corrections to the relevant electroweak observables are rather small for sparticles that obey present experimental limits, still it is possible that some of these effects distort the SM quantitative description with shifts of a magnitude of the order of the present experimental errors. So it is interesting to look at small discrepancies in the data that could be attributed to supersymmetric effects. One such effect is the small excess of the measured value of $m_W$ with respect to the SM prediction for the observed value of $m_t$ and $m_H$ in agreement with the present direct lower bound. Alternatively, the same effect is manifested by a corresponding deficit of the $\epsilon_2$ parameter. Another effect could be hidden by the fact that unfortunately there is an experimental discrepancy between the values of $\sin^2 \theta_{\text{eff}}$ measured from leptonic and hadronic asymmetries. If eventually the true value will be established to be more on the side of the leptonic asymmetries, then an effect of the same order of that present in $\epsilon_2$ will also be needed in $\epsilon_3$ to better reconcile the fitted value of $m_H$ with the direct limits on the Higgs mass.

We have shown in this note that negative shifts in $\epsilon_2$ and $\epsilon_3$ of a comparable size would indeed be induced by light sleptons and moderately large $\tan \beta$. Charged sleptons near their experimental limit of about 100 GeV could well be compatible at large $\tan \beta$ with sneutrinos of masses as low as $55 - 80\,\text{GeV}$. If accompanied by light charginos and neutralinos one can obtain shifts in the radiative corrections of precisely the right pattern and magnitude to reproduce the description of the data that we discussed. Light sleptons and large $\tan \beta$ are also compatible with the recent indication of a deviation in the muon $g - 2$. We have discussed the phenomenological
implications of this situation. Interestingly, the discovery of supersymmetric particles at the Tevatron in the next few years could be possible in channels with three hard isolated leptons.

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