Classification of Effective Neutrino Mass Operators

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Abstract

We present a classification of $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariant $\Delta L = 2$ ($L$ being lepton number) effective operators relevant for generating small Majorana neutrino masses. Operators of dimension up to 11 have been included in our analysis. This approach enables us to systematically identify interesting neutrino mass models. It is shown that many of the well-known models fall into this classification. In addition, a number of new models are proposed and their neutrino phenomenology is outlined. Of particular interest is a large class of models in which neutrinoless double beta decays arise at a lower order compared to the neutrino mass, making these decays accessible to the current round of experiments.

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Evidence for small but nonzero neutrino masses has been mounting over the years from experiments on atmospheric [1], solar [2] and accelerator [3] neutrinos. All experiments point toward a spectrum where the neutrino masses are less than or of order 1 eV. This is the first experimental challenge to the Standard Model, for its structure and particle content, taken as a stand–alone renormalizable field theory, do not allow for nonzero neutrino masses. New ingredients must be introduced in order to accommodate massive neutrinos; and the race is on to find the underlying new physics.

While it is possible to accommodate neutrino masses, $m_\nu$, by modifying the low–energy particle content of the Standard Model, the smallness of $m_\nu$ will remain a puzzle in such attempts. Consider, for example, introducing an $SU(2)_L$ triplet scalar field which acquires a vacuum expectation value (VEV) and generates Majorana masses for the left–handed neutrinos [4]. Either the VEV of the triplet or the relevant Yukawa couplings must be exceedingly small in order to generate $m_\nu$ of order 1 eV. Similarly, if right–handed neutrinos are introduced into the low energy spectrum so that Dirac masses can be generated, the smallness of the relevant Yukawa couplings ($\sim 10^{-11}$) would beg for an explanation.

A more natural explanation for the smallness of $m_\nu$ is that they are generated (via some underlying new physics) at a scale $\Lambda$ higher than the electroweak scale\(^1\) and manifest themselves at low energies through effective higher dimensional ($d > 4$) operators which are suppressed by appropriate powers of $\Lambda$. One can then understand on purely dimensional ground why the $m_\nu$ are small, without precise knowledge of the underlying new physics.

As an example of this effective operator description, consider the well–known seesaw mechanism [5]. Here the underlying theory has a modified particle content, viz., the addition of heavy right–handed neutrinos. Upon integrating out these heavy fields, one arrives at an effective theory without the right–handed neutrinos, but with a set of dimension 5 operators.

\(^1\)Typically, $\Lambda$ corresponds to the scale at which lepton number conservation is violated.
[6] which can generate small Majorana masses for the left–handed neutrinos.

While the $d = 5$ seesaw operators are the lowest dimensional effective neutrino mass operators, there may be situations in which they are not the suitable ones. For instance, as exemplified in many of the models discussed in Sec. III, there may be selection rules which forbid their presence. Such selection rules may be necessary for models in which the lepton number (L) breaking scale $\Lambda$ is sufficiently low $^2$ that the $d = 5$ operators would generate too large a neutrino mass (larger than $O(1)$ eV). In such cases, the dominant contributions to the neutrino mass will come from operators with dimension higher than 5, typically through radiative corrections. In view of the current interests in neutrino mass models, it will be useful to identify all potentially relevant neutrino mass operators. We list in this paper all such effective operators and estimate the size of the neutrino mass they generate. In so doing we will reproduce many of the neutrino mass models that already exist in the literature and find a systematic way to arrive at new models. It should be noted that this effective operator approach has been widely used in other studies of possible new physics [7], in particular, baryon number violation and proton decays [6,8].

Since we are interested in operators that can lead to a mass term for the left–handed neutrino fields in the Standard Model, the operators must violate lepton number by two units, i.e., $\Delta L = 2$. They contain only fields which are present in the Standard Model and must be $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant. They are also required to conserve baryon number, otherwise the nonobservation of proton decays will constrain the scale $\Lambda$ to be greater than $10^{14}$ GeV, in which case neutrino masses induced by operators with $d > 5$ will be too small to be of interest. Because limits on lepton flavor violating processes such as $\mu \rightarrow eee$ and $\mu \rightarrow e\gamma$ constrain $\Lambda$ to be larger than a few TeV, we shall focus on effective

$^2$For example, if L is broken due to quantum effects of gravity, $\Lambda$ will be of order the Planck scale which can be as low as a few TeV in some of the scenarios that speculate the existence of large extra dimensions.
operators of dimension 11 and lower: Operators of higher dimension will likely lead to neutrino masses that are too small to satisfy the atmospheric neutrino data which require at least one neutrino to have a mass of about 0.03 eV [1].

The remainder of this paper is organized as follows. Sec. II provides a classification of $\Delta L = 2$ operators with dimension less than 12. In Sec. III, we present various renormalizable models that induce the $\Delta L = 2$ operators of Sec. II. There we also discuss the radiative neutrino mass generation mechanisms in these models and outline their main phenomenological consequences. In Sec. IV, we point out the significance of some of the operators to neutrinoless double beta ($\beta\beta_0\nu$) decays. We identify a number of effective operators which induce $\beta\beta_0\nu$ decays at a lower order compared to the neutrino mass. We offer our conclusions in Sec. V.

II. CLASSIFICATION OF EFFECTIVE $\Delta L = 2$ OPERATORS

We shall use a notation in which all fermion fields are left–handed, denoted by

$$L(1, 2, -\frac{1}{2}), \quad e^c(1, 1, 1), \quad Q(3, 2, \frac{1}{6}), \quad d^c(\bar{3}, 1, \frac{1}{3}), \quad u^c(\bar{3}, 1, -\frac{2}{3}),$$

where the $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers are indicated in parentheses. We shall suppress all generation indices in this section, but will reinstate them in the next section when we discuss renormalizable neutrino mass models. Here $L$ and $Q$ stand for the lepton and quark doublets, respectively; $e^c = C\bar{e}^T_R$ ($C$ being the charge conjugation operator), $u^c$, and $d^c$ stand for the charge conjugates of the right–handed charged leptons, up–type quarks, and down–type quarks, respectively. The Standard Model Higgs doublet is denoted as $H(1, 2, \frac{1}{2})$ and $\bar{H}$ will denote its hermitian conjugate.

The $\Delta L = 2$ operators can be derived systematically using the Standard Model degrees of freedom as follows. There are three basic fermion bilinears that carry two units of lepton number:

$$\{L^i L^j, \ L^i \bar{e}^c, \ e^c \bar{e}^c\}. \quad$$

4
Here $i$ and $j$ are $SU(2)_L$ indices, and $\bar{e}^c$ stands for either the hermitian conjugate or the Dirac adjoint of $e^c$. Any $\Delta L = 2$ effective operator will have one of these basic fermion bilinears accompanied by a product of other fields which is neutral under color and carries a net baryon number of zero. We classify the effective neutrino mass operators according to the number of fermion fields they contain. Three separate groups can be identified: (i) operators containing $L^i L^j$ and no other fermion fields; (ii) operators containing four fermion fields; and (iii) operators containing six fermion fields. Operators containing four or more fermion bilinears have dimension 12 or higher and will not be considered here because, as mentioned above, the neutrino masses generated by such operators will be constrained by limits on lepton flavor violation to be typically much smaller than 0.03 eV and will not be that interesting for the current neutrino oscillation phenomenology. In case (i), neutrino masses will arise at tree level. In case (ii), one pair of fermion fields must be annihilated to generate neutrino masses, which will therefore arise at the one–loop level. And in case (iii), which requires the annihilation of two fermion pairs, neutrino masses will arise as two–loop radiative corrections.

(i) With $L^i L^j$ not accompanied by any more fermion fields, one obtains the well–known dimension five operator for neutrino mass [6]:

$$O_1 = L^i L^j H^k H^l \epsilon_{ik} \epsilon_{jl}$$  \hspace{1cm} (3)

(ii) Operators with four fermion fields are:

$$O_2 = L^i L^j L^k e^c H^l \epsilon_{ij} \epsilon_{kl}$$

$$O_3 = \left\{ L^i L^j Q^k d^c H^l \epsilon_{ij} \epsilon_{kl}, \quad L^i L^j Q^k d^c H^l \epsilon_{ik} \epsilon_{jl} \right\}$$

$$O_4 = \left\{ L^i L^j \bar{Q}_i \bar{u}^c H^k \epsilon_{jk}, \quad L^i L^j \bar{Q}_k \bar{u}^c H^k \epsilon_{ij} \right\}$$

$$O_5 = L^i L^j Q^k d^c H^l H^m \bar{H}_i \epsilon_{jl} \epsilon_{km}$$

$$O_6 = L^i L^j \bar{Q}_i \bar{u}^c H^l H^k \bar{H}_j \epsilon_{ij}$$

$$O_7 = L^j Q^i \bar{e}^c \bar{Q}_k H^k H^l H^m \epsilon_{il} \epsilon_{jm}$$

$$O_8 = L^i \bar{e}^c u^c d^c H^j \epsilon_{ij}$$  \hspace{1cm} (4)
Before we list the operators in group (iii), we wish to make several remarks.

1. Operators $O_2 - O_6$ are obtained by multiplying $L^i L^j$ with any one of the combinations \{\(Le^c, Q d^c, Qu^c, \bar{L}e^c, \bar{Q}d^c, \bar{Q}u^c\)\} which all conserve lepton as well as baryon number. $O_7$ and $O_8$ are obtained as the product of $L^i \bar{e}^c$ with $\bar{F}F$ where $F = \{L, e^c, Q, u^c, d^c\}$.

For operators without any derivatives or gauge boson fields (see comment 5 below), the number of barred (and unbarred) fields should be even. Note also that there are no group (ii) operators of the type $\bar{e}^c e^c$ that are gauge invariant, owing to $SU(2)_L$ antisymmetry in the Higgs field.

2. We have shown explicitly the $SU(2)_L$ group contractions using the indices \((i,j,k,l,m,n)\). The color indices are, however, suppressed. In operators involving six fermion fields (to follow), when there are two quark and two anti–quark fields, there are two possible color contractions, which will not be shown, but should be assumed.

3. Lorentz indices are not explicitly shown in our operators. All possible Lorentz contractions should be allowed. Operator $O_1$ is a unique Lorentz scalar, viz., $(L^T CL^j) H^k H^l \epsilon_{ijkl}$, where $C$ is the charge conjugation matrix. Operator $O_2$ has the following Lorentz contractions:

\[
(L^T CL^j)(L^k T C e^c) H^l \epsilon_{ijkl}, \quad (L^T CL^k)(L^j T C e^c) H^l \epsilon_{ijkl},
\]

\[
(L^T C e^c)(L^j T C L^k) H^l \epsilon_{ijkl},
\]

along with those obtained from Eq. (5) by replacing $C$ with $\sigma_{\mu \nu}$. In the first entry for the operator $O_4$, the following Lorentz contractions are allowed:

\[
O_4 = (L^T CL^j)(\bar{Q}_i^T C \bar{u}^c) H^k \epsilon_{jk}, \quad (L^T \sigma_{\mu \nu} L^j)(\bar{Q}_i^T \sigma^{\mu \nu} \bar{u}^c) H^k \epsilon_{jk},
\]

\[
(\bar{Q}_i \gamma_\mu L^i)(\bar{u}^c \gamma^\mu L^j) H^k \epsilon_{jk}, \quad (\bar{Q}_i \gamma_\mu L^j)(\bar{u}^c \gamma^\mu L^i) H^k \epsilon_{jk}.
\]

Although Fierz identities exist between several of these operators, they must be counted as independent because the $SU(2)_L$ contractions will not be the same.
4. We have not explicitly written down operators with $\Delta L = 2$ that can be obtained by multiplying the lower dimensional $\Delta L = 2$ operators with one of the gauge invariant, baryon and lepton number conserving operators that appear in the Standard Model Lagrangian. These latter operators include $\bar{H}_i H_i$, $L^i e^c \bar{H}_i$, $Q^i d^c \bar{H}_i$, and $Q^i u^c H^j \epsilon_{ij}$, as well as their hermitian conjugates. Such product operators, while not explicitly displayed, are understood to be present in our list. In the case of product operators containing $\bar{H}_i H_i$, with the Higgs doublets having the trivial $SU(2)_L$ contraction as shown, the presence of such a higher dimensional operator will necessarily imply the existence of the corresponding lower dimensional operator without the $\bar{H}_i H_i$ factor. (This can be seen by closing the $\bar{H}_i$ and $H_i$ lines to form a loop, which will give an infinite contribution to the lower dimensional operator, necessitating its presence.) When the $SU(2)_L$ contraction on the Higgs fields is nontrivial, we do list explicitly such operators (compare, e.g., $O_3$ with $O_5$). Product operators involving one of the Standard Model operators $L^i e^c \bar{H}_i$, $Q^i d^c \bar{H}_i$, or $Q^i u^c H^j \epsilon_{ij}$ (or their hermitian conjugates), and one of the lower dimensional $\Delta L = 2$ operators of Eqs. (3)-(4), while not listed explicitly, can be quite interesting from the point of view of neutrino mass models. Examples of this type of operators are $L^i L^j \bar{L}_k \bar{e}^c H^i H^m H^k \epsilon_{il} \epsilon_{jm}$, $L^i L^j \bar{Q}_k \bar{d}^c H^i H^m H^k \epsilon_{il} \epsilon_{jm}$, and $L^i L^j Q^k u^c H^i H^m H^n \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$. Since all possible Lorentz contractions are to be assumed, these operators can be generated without inducing the lower dimensional operator $O_1$. We shall give an example of this type of neutrino mass models in Sec. III. Note also that the flavor structure of $L^i e^c \bar{H}_i$, etc., in these product operators need not be the same as that of the corresponding Standard Model operators, which implies the possibility of interesting neutrino phenomenology from these operators.

5. We have not included in our list $\Delta L = 2$ operators which involve the Standard Model gauge boson fields. They can arise, for example, through covariant derivatives of the fermion or Higgs fields. It may be more difficult to generate such operators at tree level from an underlying renormalizable theory, making them perhaps less interesting
for the generation of neutrino masses. Nevertheless, we wish to list such $\Delta L = 2$ operators of the lowest dimension, which turns out to be 7. They are $(L^T \sigma_{\mu\nu} L) H H B^{\mu\nu}$, $(L^T \sigma_{\mu\nu} L) H H W^{\mu\nu}$, $(L^T C D_{\mu} D^\mu L) H H$, and $(\bar{e} \gamma^{\mu}_L D^\mu L) H H H$. Here $B^{\mu\nu}$ and $W^{\mu\nu}$ are the $U(1)_Y$ and $SU(2)_L$ field strength tensors and we have suppressed the $SU(2)_L$ indices for simplicity. Although we have only shown operators with the covariant derivative acting on a specific field, it is understood that one should include similar operators with the covariant derivative acting on the other fields. For example, the third operator listed above includes $(L^T C L) (D^\mu H) (D^\mu H)$, and the fourth operator includes $(\bar{e} \gamma^{\mu}_L (D^\mu H) H H$, and so on.

(iii) We now proceed to write down the operators with six fermion fields through dimension 11. The procedure we follow is analogous to the case of the operators containing four fermion fields. There are 12 such operators at the dimension 9 level:

\[
\mathcal{O}_9 = L^i L^j L^k e^c L^l e^c_{ij} \epsilon_{kl} \\
\mathcal{O}_{10} = L^i L^j L^k e^c Q^j d^c e_{ij} \epsilon_{kl} \\
\mathcal{O}_{11} = \{L^i L^j Q^k d^c Q^l d^c e_{ij} \epsilon_{kl}, \ L^i L^j Q^k d^c Q^l d^c e_{ik} \epsilon_{jl}\} \\
\mathcal{O}_{12} = \{L^i L^j \bar{Q}_i \bar{u}^c \bar{Q}_j \bar{u}^c, \ L^i L^j \bar{Q}_k \bar{u}^c \bar{Q}_l \bar{u}^c e_{ij} \epsilon^{kl}\} \\
\mathcal{O}_{13} = L^i L^j \bar{Q}_i \bar{u}^c L^l e^c \epsilon_{jl} \\
\mathcal{O}_{14} = \{L^i L^j \bar{Q}_k \bar{u}^c Q^k d^c e_{ij}, \ L^i L^j \bar{Q}_l \bar{u}^c Q^l d^c e_{jl}\} \\
\mathcal{O}_{15} = L^i L^j L^k d^c \bar{L}_i \bar{u}^c \epsilon_{jk} \\
\mathcal{O}_{16} = L^i L^j e^c d^c \bar{e}^c \bar{u}^c e_{ij} \\
\mathcal{O}_{17} = L^i L^j d^c d^c \bar{e}^c \bar{u}^c e_{ij} \\
\mathcal{O}_{18} = L^i L^j d^c u^c \bar{u}^c e_{ij} \\
\mathcal{O}_{19} = L^i Q^j d^c d^c \bar{e}^c \bar{u}^c e_{ij} \\
\mathcal{O}_{20} = L^i d^c \bar{Q}_i \bar{u}^c \bar{e}^c \bar{u}^c \tag{7}
\]

And there are 40 operators with $d = 11$:
\[ \mathcal{O}_{21} = \{ L^i L^j L^k c Q^l u^c H^m H^n \epsilon_{ij} \epsilon_{km} \epsilon_{ln}, \quad L^i L^j L^k c Q^l u^c H^m H^n \epsilon_{il} \epsilon_{jm} \epsilon_{kn} \} \]

\[ \mathcal{O}_{22} = L^i L^j L^k e^c \bar{Q}_k \bar{e}^c H^i H^m \epsilon_{il} \epsilon_{jm} \]

\[ \mathcal{O}_{23} = L^i L^j L^k e^c \bar{Q}_k \bar{e}^c H^i H^m \epsilon_{il} \epsilon_{jm} \]

\[ \mathcal{O}_{24} = \{ L^i L^j Q^k d^e Q^l d^e H^m \bar{H}_i \epsilon_{jk} \epsilon_{km}, \quad L^i L^j Q^k d^e Q^l d^e H^m \bar{H}_i \epsilon_{jm} \epsilon_{kl} \} \]

\[ \mathcal{O}_{25} = L^i L^j Q^k d^e Q^l d^e H^m H^n \epsilon_{im} \epsilon_{jn} \epsilon_{kl} \]

\[ \mathcal{O}_{26} = \{ L^i L^j Q^k d^e L_i \bar{e}^c H^i H^m \epsilon_{jk} \epsilon_{km}, \quad L^i L^j Q^k d^e \bar{L}_k \bar{e}^c H^i H^m \epsilon_{il} \epsilon_{jm} \} \]

\[ \mathcal{O}_{27} = \{ L^i L^j Q^k d^e Q_i \bar{d}^e H^i H^m \epsilon_{jk} \epsilon_{km}, \quad L^i L^j Q^k d^e Q_k \bar{d}^e H^m H^n \epsilon_{il} \epsilon_{jm} \} \]

\[ \mathcal{O}_{28} = \{ L^i L^j Q^k d^e \bar{Q}_j \bar{u}^e H^i \bar{H}_i \epsilon_{kl}, \quad L^i L^j Q^k d^e \bar{Q}_k \bar{u}^e H^i \bar{H}_i \epsilon_{jl} \}
\]

\[ L^i L^j Q^k d^e \bar{Q}_l \bar{u}^e H^i \bar{H}_i \epsilon_{jk} \]

\[ \mathcal{O}_{29} = \{ L^i L^j Q^k u^c \bar{Q}_k \bar{u}^e H^i H^m \epsilon_{il} \epsilon_{jm}, \quad L^i L^j Q^k u^c \bar{Q}_i \bar{u}^e H^i H^m \epsilon_{ik} \epsilon_{jm} \} \]

\[ \mathcal{O}_{30} = \{ L^i L^j L_i \bar{e}^c Q_k \bar{u}^e H^k H^l \epsilon_{jl}, \quad L^i L^j \bar{L}_m \bar{e}^c \bar{Q}_n \bar{u}^e H^k H^l \epsilon_{ik} \epsilon_{jl} \epsilon_{mn} \} \]

\[ \mathcal{O}_{31} = \{ L^i L^j \bar{Q}_i \bar{d}^e \bar{Q}_k \bar{u}^e H^k H^l \epsilon_{jl}, \quad L^i L^j \bar{Q}_k \bar{d}^e \bar{Q}_n \bar{u}^e H^k H^l \epsilon_{ik} \epsilon_{jl} \epsilon_{mn} \} \]

\[ \mathcal{O}_{32} = \{ L^i L^j \bar{Q}_j \bar{u}^e \bar{Q}_k \bar{u}^e H^k H^l \epsilon_{i,j,k} \epsilon_{mn} \}
\]

\[ \mathcal{O}_{33} = \bar{e}^c \bar{e}^c L^j L^l e^c \bar{e}^c H^k H^l \epsilon_{ik} \epsilon_{jl} \]

\[ \mathcal{O}_{34} = \bar{e}^c \bar{e}^c L^j Q^k e^c \bar{d}^e H^k H^l \epsilon_{ik} \epsilon_{jl} \]

\[ \mathcal{O}_{35} = \bar{e}^c \bar{e}^c L^j e^c \bar{Q}_j \bar{u}^e H^k H^l \epsilon_{ik} \]

\[ \mathcal{O}_{36} = \bar{e}^c \bar{e}^c Q^i d^e Q^j d^e H^k H^l \epsilon_{ik} \epsilon_{jl} \]

\[ \mathcal{O}_{37} = \bar{e}^c \bar{e}^c Q^i d^e \bar{Q}_j \bar{u}^e H^k H^l \epsilon_{ik} \]

\[ \mathcal{O}_{38} = \bar{e}^c \bar{e}^c \bar{Q}_i \bar{u}^e \bar{Q}_j \bar{u}^e H^k H^j \]

\[ \mathcal{O}_{39} = \{ L^i L^j L^k L^l L_i \bar{L}_j H^m H^n \epsilon_{jm} \epsilon_{kl}, \quad L^i L^j L^k L^l L_i \bar{L}_j H^m H^n \epsilon_{ij} \epsilon_{kl} \}
\]

\[ L^i L^j L^k L^l L_i \bar{L}_j H^m H^n \epsilon_{jk} \epsilon_{kl}, \quad L^i L^j L^k \bar{L}_m \bar{L}_q H^m H^n \epsilon_{ij} \epsilon_{km} \epsilon_{ln} \epsilon_{pq} \} \]

\[ \mathcal{O}_{40} = \{ L^i L^j L^k Q^l \bar{L}_i \bar{Q}_j H^m H^n \epsilon_{km} \epsilon_{ln}, \quad L^i L^j L^k Q^l \bar{L}_i \bar{Q}_j H^m H^n \epsilon_{jm} \epsilon_{kn} \}
\]

\[ L^i L^j L^k Q^l \bar{L}_i \bar{Q}_j H^m H^n \epsilon_{jm} \epsilon_{kn}, \quad L^i L^j L^k Q^l \bar{L}_i \bar{Q}_m H^m H^n \epsilon_{jk} \epsilon_{ln}, \quad L^i L^j L^k Q^l \bar{L}_i \bar{Q}_m H^m H^n \epsilon_{jk} \epsilon_{ln}, \quad L^i L^j L^k Q^l \bar{L}_i \bar{Q}_m H^m H^n \epsilon_{jk} \epsilon_{ln} \} \]
\[ L^i L^j L^k Q^l L^m \bar{Q}^n H^m H^n \epsilon_{ijl} \epsilon_{km}, \quad L^i L^j L^k Q^l \bar{L}_m \bar{Q}^n H^m \epsilon_{ijl} \epsilon_{km}, \]

\[ L^i L^j L^k Q^l L^m \bar{Q}^n H^m H^n \epsilon_{ijl} \epsilon_{km}, \quad L^i L^j L^k Q^l \bar{L}_m \bar{Q}^n H^m \epsilon_{ijl} \epsilon_{km} \]

\[ \mathcal{O}_{41} = \{ L^i L^j L^k d^L_i \bar{d}^L \bar{e}^L H^m H^n \epsilon_{ijl} \epsilon_{km}, \quad L^i L^j L^k d^L_i \bar{d}^L \bar{e}^L H^m \epsilon_{ijl} \epsilon_{km} \} \]

\[ \mathcal{O}_{42} = \{ L^i L^j L^k u^L_i \bar{u}^L \bar{e}^L H^m H^n \epsilon_{ijl} \epsilon_{km}, \quad L^i L^j L^k u^L_i \bar{u}^L \bar{e}^L H^m \epsilon_{ijl} \epsilon_{km} \} \]

\[ \mathcal{O}_{43} = \{ L^i L^j L^k d^L_i \bar{d}^L \bar{e}^L H^l \bar{H}_i \epsilon_{jk}, \quad L^i L^j L^k d^L_i \bar{d}^L \bar{e}^L H^l \bar{H}_i \epsilon_{kl} \]

\[ L^i L^j L^k d^L \bar{L}_i \bar{e}^L H^m \bar{H}_n \epsilon_{ijl} \epsilon_{km} \}

\[ \mathcal{O}_{44} = \{ L^i L^j L^k Q^e e^L i^L \bar{e}^L \bar{H}^l H^m \epsilon_{ijl} \epsilon_{km}, \quad L^i L^j L^k Q^e e^L i^L \bar{e}^L \bar{H}^l H^m \epsilon_{ijl} \epsilon_{km} \}

\[ L^i L^j L^k Q^e \bar{Q}^e \bar{Q}^e H^l H^m \epsilon_{ijl} \epsilon_{km}, \quad L^i L^j L^k Q^e \bar{Q}^e \bar{Q}^e H^l \epsilon_{ik} \epsilon_{jm} \}

\[ \mathcal{O}_{45} = \{ L^i L^j L^k e^L d^L \bar{d}^L \bar{e}^L H^k H^l \epsilon_{ik} \epsilon_{jl} \}

\[ \mathcal{O}_{46} = \{ L^i L^j L^k e^L u^L \bar{e}^L \bar{u}^L \bar{H}^k H^l \epsilon_{ik} \epsilon_{jl} \}

\[ \mathcal{O}_{47} = \{ L^i L^j Q^k Q^l \bar{Q}^j \bar{Q}^j H^m H^n \epsilon_{km} \epsilon_{ln}, \quad L^i L^j Q^k Q^l \bar{Q}^j \bar{Q}^j H^m \epsilon_{jm} \epsilon_{ln} \}

\[ L^i L^j Q^k Q^l \bar{Q}^j \bar{Q}^j H^m H^n \epsilon_{im} \epsilon_{jn}, \quad L^i L^j Q^k Q^l \bar{Q}^j \bar{Q}^j H^m \epsilon_{jk} \epsilon_{ln} \]

\[ L^i L^j Q^k Q^l \bar{Q}^j \bar{Q}^j H^m H^n \epsilon_{jn} \epsilon_{kl}, \quad L^i L^j Q^k Q^l \bar{Q}^j \bar{Q}^j H^m \epsilon_{ij} \epsilon_{ln} \]

\[ L^i L^j Q^k Q^l \bar{Q}^j \bar{Q}^j \bar{H}^m H^n \epsilon_{im} \epsilon_{jn} \epsilon_{kl} \}

\[ L^i L^j Q^k Q^l \bar{Q}^j \bar{Q}^j H^m H^n \epsilon_{ik} \epsilon_{jm} \epsilon_{ln} \}

\[ L^i L^j Q^k Q^l \bar{Q}^j \bar{Q}^j H^m H^n \epsilon_{ik} \epsilon_{jm} \epsilon_{ln} \epsilon_{pq} \]

\[ L^i L^j Q^k Q^l \bar{Q}^j \bar{Q}^j H^m H^n \epsilon_{ik} \epsilon_{jm} \epsilon_{ln} \epsilon_{pq}, \quad L^i L^j Q^k Q^l \bar{Q}^j \bar{Q}^j H^m \epsilon_{im} \epsilon_{jn} \epsilon_{kl} \epsilon_{pq} \}

\[ \mathcal{O}_{48} = \{ L^i L^j d^L \bar{d}^L \bar{d}^L \bar{d}^L H^k H^l \epsilon_{ik} \epsilon_{jl} \}

\[ \mathcal{O}_{49} = \{ L^i L^j d^L \bar{d}^L \bar{d}^L \bar{d}^L H^k H^l \epsilon_{ik} \epsilon_{jl} \}

\[ \mathcal{O}_{50} = \{ L^i L^j d^L \bar{d}^L \bar{d}^L \bar{d}^L \bar{H}_i \epsilon_{jk} \}

\[ \mathcal{O}_{51} = \{ L^i L^j d^L \bar{d}^L \bar{d}^L \bar{d}^L \bar{H}_i \epsilon_{ik} \epsilon_{jl} \}

\[ \mathcal{O}_{52} = \{ L^i L^j d^L \bar{d}^L \bar{d}^L \bar{d}^L \bar{H}_i \epsilon_{jk} \}

\[ \mathcal{O}_{53} = \{ L^i L^j d^L \bar{d}^L \bar{d}^L \bar{d}^L \bar{H}_j \}

\[ \mathcal{O}_{54} = \{ L^i Q^j Q^k d^L \bar{Q}^j \bar{e}^L \bar{H}^l H^m \epsilon_{jk} \epsilon_{km}, \quad L^i Q^j Q^k d^L \bar{Q}^j \bar{e}^L \bar{H}^l H^m \epsilon_{km} \epsilon_{jk} \}

\[ L^i Q^j Q^k d^L \bar{Q}^j \bar{e}^L \bar{H}^l H^m \epsilon_{im} \epsilon_{jk}, \quad L^i Q^j Q^k d^L \bar{Q}^j \bar{e}^L \bar{H}^l H^m \epsilon_{ij} \epsilon_{km} \}

\[ \mathcal{O}_{55} = \{ L^i Q^j Q^k \bar{e}^L \bar{e}^L \bar{H}^k H^l \epsilon_{il}, \quad L^i Q^j Q^k \bar{e}^L \bar{e}^L \bar{H}^k H^l \epsilon_{il} \}

\[ 10 \]
The classification of the effective $\Delta L = 2$ operators given in the previous section can be quite useful in building renormalizable models of neutrino mass. We shall describe in this section how to systematically identify from these operators interesting neutrino mass models. We will see that this method reproduces several well–known models. More interestingly, many new models of neutrino mass will be uncovered. While we will not present an exhaustive discussion of all these new models, we will outline the most interesting features for neutrino mass and phenomenology in several of these models.

A. Tree–level neutrino mass models

The operator $\mathcal{O}_1$ of Eq. (3) can generate small neutrino masses at tree level. The simplest way to induce $\mathcal{O}_1$ is by the seesaw mechanism. As shown in Fig. 1, $\mathcal{O}_1$ will result after the heavy fields $N_{1,3}$ are integrated out. Here $N_i$ denotes the familiar $SU(2)_L$ singlet right–handed neutrinos. It is also possible to induce $\mathcal{O}_1$ using $N_3$, which are $SU(2)_L$ triplets and have zero hypercharge [9].

In Fig. 2, we show an alternate way of inducing $\mathcal{O}_1$ by the exchange of an $SU(2)_L$ triplet scalar $\Phi_3$ which carries $Y = +1$. The neutral component of $\Phi_3$ will receive an induced vacuum expectation value (VEV) through its trilinear coupling with the Standard Model Higgs doublet. This is sometimes referred to as the Type II seesaw mechanism [10], which

$$L^i Q^j \bar{Q}^n \bar{e}^c H^k H^l \epsilon_{ik} \epsilon_{jl} \epsilon^{mn} \{$$

$$\mathcal{O}_{56} = L^i Q^j d^c d^e \bar{e}^c d^e H^k H^l \epsilon_{ik} \epsilon_{jl}$$

$$\mathcal{O}_{57} = L^i d^c \bar{Q}^j \bar{u}^c \bar{e}^c d^e H^j H^k \epsilon_{ik}$$

$$\mathcal{O}_{58} = L^i u^c \bar{Q}^j \bar{u}^c \bar{e}^c \bar{u}^c H^j H^k \epsilon_{ik}$$

$$\mathcal{O}_{59} = L^i Q^j d^c d^e \bar{u}^c H^k H^l \epsilon_{ik}$$

$$\mathcal{O}_{60} = L^i d^c \bar{Q}^j \bar{u}^c \bar{e}^c H^j \bar{H} \epsilon_{ik}$$

(8)

III. RENORMALIZABLE MODELS OF NEUTRINO MASS
FIG. 1. Tree–level neutrino mass generation through $O_1$ via the seesaw mechanism. $N_1$ ($N_3$) is an $SU(2)_L$ singlet (triplet) fermion with zero hypercharge.

FIG. 2. Type II seesaw mechanism where an $SU(2)_L$ triplet scalar acquires an induced VEV. can occur in the absence of right–handed neutrinos. For example, in $SU(5)$ grand unified theories with a 15–plet of scalars which contain the $\Phi_3$ field, the requisite trilinear scalar coupling will arise from the Lagrangian term $15 \bar{5} \bar{5}$, where the 5–plet scalar fields contain the Standard Model Higgs doublet. The phenomenology of $O_1$ as the source of neutrino mass has been studied extensively. Here we simply note that, for $O_1$ to be interesting to the current neutrino oscillation data, the heavy particles $N_{1,3}$ or $\Phi_3$ must have masses of order $10^{12} - 10^{15}$ GeV.
B. One–loop neutrino mass models

The group (ii) operators listed in Eq. (4) which involve two neutrino fields can lead to radiative neutrino masses at the one–loop level. Symmetries or selection rules must forbid the existence of the lower dimensional operator $O_1$ for self–consistency. We shall illustrate five examples of one–loop neutrino mass generation. Two of the five models are well–known and have been thoroughly investigated in the literature, while the other three are apparently new.

In this class of models, the induced neutrino masses are suppressed by a loop factor, and usually also by ratios of light fermion masses to the scale of new physics $\Lambda$. Because of the additional suppression factors, the scale $\Lambda$ in this class of models may be much smaller than the corresponding scale in the seesaw models, and may even be close to the electroweak scale.

Notation:

In the models we shall present in the remainder of this section, heavy scalar bosons will be integrated out to generate the corresponding effective $\Delta L = 2$ operators. We shall use the following notation to denote these scalars. $\Phi$ will generically denote color singlet scalars, $\Omega$ will denote color triplet scalars, and $\bar{\Omega}$ color anti–triplet scalars. Note that $\bar{\Omega}$ is not the hermitian conjugate of $\Omega$. The $SU(2)_L$ quantum numbers of these scalar fields will be indicated as a subscript. Their hypercharge quantum numbers can be inferred from the interaction Lagrangians, and will not be displayed explicitly. For example, $\Phi_1$ will transform as $(1, 1)$ under $SU(3)_C \times SU(2)_L$, while $\bar{\Omega}_3$ transforms as $(3, 3)$. $SU(2)_L$ indices will be denoted by $(i, j, ...)$, while the subscripts $(a, b, ...)$ will denote generation indices.

1. Operator $O_2$:

Consider the following renormalizable Lagrangian:

$$\mathcal{L}^{O_2} = f_{ab} L_i^a L_j^b \epsilon_{ij} \Phi_1 + \mu \bar{H}_i \Phi_2 e^{ij} + y_a L_i^a e^c \Phi_2 c + h.c. \tag{9}$$
where the Yukawa couplings $f_{ab} = -f_{ba}$ due to $SU(2)_L$ symmetry, $y_a$ are the (diagonal) Yukawa couplings of the charged leptons, and $\Phi_2$ is a second scalar doublet. The hypercharges of $\Phi_1$ and $\Phi_2$ are $+1$ and $-\frac{1}{2}$, respectively. The simultaneous presence of the three terms in Eq. (9) will result in lepton number violation, leading to the operator $O_2$, as depicted in Fig. 3. Upon closing the $Le^c$ line (indicated by the thin solid line in Fig. 3) with the insertion of a Higgs field, finite neutrino masses will result.

![Diagram](image_url)

**FIG. 3.** Diagram that induces the operator $O_2$ as in the Zee model.

The model just described is the Zee model for neutrino mass [11]. The phenomenology of this model has been extensively studied, and we have nothing more to add here.

2. **Operator $O_3$:**

The operator $O_3$ can be induced from the following renormalizable Lagrangian:

$$
\mathcal{L}^{O_3} = f_{ab}L^i_aQ^j_b\epsilon_{ij}\bar{\Omega}_1 + g_{ab}L^i_a\bar{d}^c_b\Omega_{2i} + \mu\bar{\Omega}_1\Omega^i_2\bar{H}_i + h.c.
$$

(10)

where $\bar{\Omega}_1$ has hypercharge $\frac{1}{3}$, and $\Omega_2$ has $Y = \frac{1}{6}$. It is also possible to write an analogous Lagrangian with $\bar{\Omega}_1$ replaced by $\bar{\Omega}_3$, which is a triplet of $SU(2)_L$, rather than a singlet. Either of these Lagrangians will lead to $O_3$ via the diagram in Fig. 4. Upon closing the $Q$ line with the $d^c$ line (indicated by the thin solid line in Fig. 4) and inserting a Higgs field, this diagram will induce a neutrino mass at the one-loop level.
FIG. 4. Diagram that induces the operator $O_3$ through the Lagrangian in Eq. (10). This model has a realization in supersymmetric models with $R$–parity violation.

The induced neutrino masses can be estimated from Fig. 4 to be

$$m_{\nu_a} \sim \frac{fg}{16\pi^2} (m_d)_a \left(\frac{\mu v}{M^2}\right),$$

where $(m_d)_a$ is the mass of the down–type quark with flavor index $a$, $v$ is the VEV of $H$, and $M$ is an average mass of the $\Omega$ fields. Using $f = g = 10^{-3}$, $v = \mu = 174$ GeV and $M = 3$ TeV, we get $m_{\nu_3} \sim 0.06$ eV, which is in the interesting range for atmospheric neutrino oscillations. Assuming minimal flavor structure in the matrices $f$ and $g$, we also see that $m_{\nu_2} \sim (m_s/m_b)m_{\nu_3}$, which is in the interesting range for solar neutrino oscillations.

A specific realization of this model (with $\bar{\Omega}_1$ in Fig. 4) is the supersymmetric Standard Model with $R$–parity violation [12]. $\bar{\Omega}_1$ is identified with $\bar{d}c$ and $\Omega_2$ with $\tilde{Q}$ in this case. The phenomenology of this specific realization has been well studied. We simply note that $O_3$ has other realizations as well, for example, via $\Omega_3$ or by using leptoquarks without supersymmetry.

3. Operator $O_4$:

While it is possible to induce $O_4$ by integrating out scalar fields, it turns out neutrino masses will arise in that case either at tree level as in Fig. 2 or at one–loop level as in the Zee
model depicted in Fig. 3. Consider $O_4$ arising through the couplings $LL\Phi_3 + Qu^c H + HH\Phi_3^\dagger$, where $\Phi_3$ transforms as $(1, 3, +1)$ under $SU(3)_C \times SU(2)_L \times U(1)_Y$. In this case, the neutral component of $\Phi_3$ will acquire a tree–level VEV, and therefore a dimension 5 Majorana mass term for the neutrinos will be allowed at tree level, as in Fig. 2. If $\Phi_3$ is replaced by an $SU(2)_L$ singlet $\Phi_1$ and a second scalar doublet $\Phi_2$ is introduced so that lepton number is broken, then the model becomes identical to the Zee model.

A more interesting way of generating $O_4$ is to integrate out leptoquark gauge bosons, as shown in Fig. 5. The Lagrangian of the model has the form

$$L^{O_4} = \bar{Q}_i \gamma^\mu L^i G_1^{\mu} + \bar{u}^c \gamma^\mu L^i G_2^{\mu} + G_1^{\mu} G_2^{\mu} \epsilon_{ij} + h.c.$$  \hspace{1cm} (12)

Here $G_1^{\mu}$ is a leptoquark gauge boson which is a singlet of $SU(2)_L$ and has electric charge $+\frac{2}{3}$. $G_2^{\mu}$ is a leptoquark gauge boson that is an $SU(2)_L$ doublet with its $I_3 = +\frac{1}{2}$ member carrying an electric charge of $+\frac{2}{3}$. Upon spontaneous symmetry breaking, $G_1^{\mu}$ and $G_2^{\mu}$ will mix, leading to the generation of neutrino mass via the loop shown in Fig. 5. We have also allowed in Fig. 5 for the possibility that $G_1^{\mu}$ may be replaced by an $SU(2)_L$ triplet gauge boson $G_3^{\mu}$.

![FIG. 5. Leptoquark gauge bosons in SO(10) GUT inducing operator $O_4$.](image)

In Grand Unified Theories (GUT) which unify $(Q, L, \nu^c)$ into a common multiplet, the diagram of Fig. 5 will arise naturally. Such is the case in $SO(10)$ GUT (but not in $SU(5)$
GUT). Consider the $SU(2)_L \times SU(2)_R \times SU(4)_C \equiv G_{224}$ subgroup of $SO(10)$. The 45 gauge bosons of $SO(10)$ contain a $(1, 1, 15)$ and a $(2, 2, 6)$ under $G_{224}$. $G_1$ of Fig. 5 is contained in $(1, 1, 15)$, while $G_2$ is in $(2, 2, 6)$. The covariant derivative term for the $16$–dimensional Higgs fields will contain a piece $45^\dagger 45 16^\dagger 16$, where $45$ denotes the gauge fields. The Standard Model Higgs doublet $H$ may be contained partly in $16$, as in currently popular $SO(10)$ models [13], $16$ also contains a Standard Model singlet component which acquires a large GUT scale VEV. As a consequence, $G_1^\mu$ and $G_2^\mu$ will mix through this covariant derivative term.

An interesting consequence of Fig. 5 is the modification of how the light neutrino masses scale with the quark masses in $SO(10)$ models. While the conventional seesaw contributions to neutrino masses scale quadratically with the up–type quark masses, the new contributions arising from Fig. 5, given by

$$m_{\nu_a} \sim \frac{g^2}{16\pi^2}(m_u)_a \left( \frac{v}{M_G} \right),$$

where $g$ is the $SO(10)$ coupling constant and $M_G$ is the common mass of the leptoquark gauge bosons, scale linearly with the up–type quark masses. Although $m_{\nu_a}$ here is suppressed by a loop factor, it may be the more dominant contribution for the lighter generations of neutrinos because of this linear scaling.

4. Operator $O_5$:

Operator $O_5$ is different from $O_3$ in that an additional $H \bar{H}$ has been added. Notice that the $SU(2)_L$ contraction acting on the $H \bar{H}$ is non–trivial. Here we present a renormalizable model which generates $O_5$ without inducing $O_3$. The Lagrangian of the model is

$$L^{O_5} = f_{ab} L_a Q_b \bar{\Omega}_1 + g_{ab} L_a d_b \bar{\Omega}_2 + \lambda \bar{\Omega}_2 \bar{H} H + \mu \bar{\Omega}_1 \bar{\Omega}_2 \bar{H} + h.c.$$  

(14)

We have not explicitly shown the $SU(2)_L$ contractions, nor the hypercharges of the colored scalar fields, both of which should be obvious from Eq. (14). This Lagrangian leads to operator $O_5$ through the diagram of Fig. 6. It is also possible to replace the $\bar{\Omega}_1$ field by an $SU(2)_L$ triplet field $\bar{\Omega}_3$, which is indicated in Fig. 6.
Upon closing the $Qd^c$ loop, we obtain a finite neutrino mass, which can be estimated to be

$$m_{\nu_a} \sim \frac{fg}{32\pi^2} (m_d)_a \left( \frac{\lambda\mu v^2}{M^4} \right),$$

where $M$ is the assumed common mass of the colored scalars and $(m_d)_a$ stands for the mass of the down-type quark with flavor index $a$. We can obtain interesting neutrino masses from Eq. (15). For example, choose $f = g = 10^{-2}$, $\lambda = 1$, and $\mu = M = 5$ TeV to get $m_{\nu_3} \sim 0.04 \text{ eV}$.

5. Neutrino mass model with a product operator:

As noted in Sec. II, operators such as $\tilde{O}_1 = L_i^a L_j^b Q^k u^c H^l H^m H^n \epsilon_{lid} \epsilon_{jme_{kn}}$, which is a product of the Standard Model $d = 4$ operator $Q^k u^c H^m H^n \epsilon_{kn}$ and $O_1$, can lead to interesting neutrino mass models. Here we illustrate this possibility with $\tilde{O}_1$. The dimension five $\Delta L = 2$ operator will not be induced in this model, since we make use of a nontrivial Lorentz contraction.

Consider the following Lagrangian which breaks lepton number by two units:

$$\mathcal{L}^{\tilde{O}_1} = f_{ab} L_a Q_b \tilde{\Omega}_3 + g_{ab} L_a u_b \tilde{\Omega}_2 + \mu_1 \Omega_2' \tilde{\Omega}_3 H$$
$$+ \mu_2 \Omega_2' \tilde{\Omega}_3' H + \mu_3 \Omega_3' \tilde{\Omega}_3 H + h.c.$$ (16)
Here $(\Omega_2, \Omega'_2)$ are $SU(2)_L$ doublet leptoquark scalars with $Y = (7/6, 1/6)$ respectively, while $(\Omega_3, \Omega'_3)$ are $SU(2)_L$ triplet leptoquarks with $Y = (1/3, -2/3)$ respectively. We have suppressed $SU(2)_L$ indices for simplicity.

The diagram shown in Fig. 7 will generate $\tilde{O}_1$ with this Lagrangian. Upon closing the $Q$ and $u^c$ lines, as shown by the light solid line in Fig. 7, neutrino masses will be induced as one-loop radiative corrections. We can estimate the induced neutrino masses to be

$$m_{\nu_a} \sim \frac{fg}{48\pi^2} (m_u)_a \left( \frac{\mu_1 \mu_2 \mu_3 v^2}{M^6} \right),$$

where we have ignored any generation dependence in the coupling matrices $f$ and $g$, and assumed a common mass $M$ for the leptoquark scalars, taken to be much heavier than the weak scale. In Eq. (17), $(m_u)_a$ denotes the mass of the up-type quark with flavor index $a$. Take, as an example, $f = g = 10^{-2}$, $\mu_1 = \mu_2 = \mu_3 = M = 10$ TeV. In this case, $m_{\nu_3} \sim 0.2$ eV, which is in the interesting range for atmospheric neutrino oscillations. Clearly, other choices of parameters are possible with interesting neutrino phenomenology even with the scale of new physics being relatively low.

Another feature worth mentioning is that the lowest dimensional $\Delta L = 2$ operator that arises in this model is $\tilde{O}_1$ as shown in Fig. 7. We can assign lepton number of $-1$ to both $\Omega_2$ and $\Omega_3$ that have Yukawa couplings to leptons. It is the mixing term $\Omega_2 \Omega'_3$ that violates
L by two units. However, this mixing can occur in the model only after inserting three $H$
fields, as in the figure. As a result, no lower dimensional $\Delta L = 2$ operators are induced.

If we replace the color anti–triplet scalars in Fig. 7, which are triplets of $SU(2)_L$, by
color anti–triplets that are $SU(2)_L$ singlets, the contribution to $\tilde{O}_1$ will vanish, since the
effective scalar mixing term $\Omega_2^+\Omega_1^+HHH$ is identically zero due to $SU(2)_L$ symmetry.

C. Two–loop neutrino mass models

Let us now turn to renormalizable models wherein neutrino masses are induced as two–
loop radiative corrections. A generic feature of this class of models is that, for interesting
neutrino mass phenomenology, the scale of new physics tends to be much lower than the
Corresponding scale in one–loop mass generation models. In addition to an extra loop
suppression factor, we will see that the induced neutrino masses in this class of models scale
quadratically with charged fermion masses, as opposed to the typical linear scaling in the
one–loop models.

1. Operator $O_9$:

Neutrino mass models induced via $O_9$ have been well studied [14,15]. The relevant
Lagrangian is

$$\mathcal{L}^{O_9} = f_{ab}L_a L_b \Phi_1 + g_{ab}e_a \epsilon_b^c e_c \Phi_1 F^- + \mu \Phi_1 \Phi_1 F^- + h.c.$$  

where $\Phi_1$ is a singly charged $SU(2)_L$ singlet scalar, while $\Phi_1^F$ is a doubly charged singlet.
Operator $O_9$ is induced through the diagram of Fig. 8. (Such a two–loop diagram for
neutrino mass generation was first discussed in Ref. [4].)

Upon closing the $Le^c$ loops on both sides, indicated by the light solid lines in Fig. 8, we
see that the induced neutrino mass matrix has a structure given by

$$m_\nu = \frac{1}{(16\pi^2)^2} \left( f M_\ell g M_\ell f^T \right) \left( \frac{\mu}{M^2} \right).$$  

Here $M_\ell$ stands for the diagonal charged lepton mass matrix. An interesting feature of this
matrix structure is that, owing to $SU(2)_L$ symmetry, $f_{ab} = -f_{ba}$, which means that, for
three generations of neutrinos, \( \text{Det}(m_\nu) = 0 \). This indicates that one of the neutrinos will remain massless at the two–loop level [15], but it will acquire a mass at the three–loop level.

To see the numerical magnitude of the induced neutrino masses, let us set \( f = g = 0.09, \mu = M = 3 \text{ TeV} \). The heaviest neutrino will then have a mass \( m_{\nu_3} \sim 0.03 \text{ eV} \).

It should be noted that \( \mathcal{O}_9 \) can also arise if \( \Phi_1 \) in Fig. 8 is replaced by \( \Phi_3 \), an \( SU(2)_L \) triplet scalar.

2. Operator \( \mathcal{O}_{10} \):

We shall present two renormalizable models that induce \( \mathcal{O}_{10} \). The effective operators are depicted for the two models in Figs. 9 and 10. The renormalizable Lagrangians for the two models can be readily written down from these figures. They are

\[
\mathcal{L}_{1}^{\mathcal{O}_{10}} = f_{ab} L_a L_b \Phi_1 + g_{ab} L_a Q_b \bar{\Omega}_1 + h_{ab} e_c d^c_a d^c_b \Omega_1 + \mu \bar{\Omega}_1 \bar{\Omega}_1 \Phi_1 + h.c. \\
\mathcal{L}_{2}^{\mathcal{O}_{10}} = f_{ab} L_a L_b \Phi_1 + g_{ab} L_a d^c_b \Omega_2 + h_{ab} e_c Q_b \bar{\Omega}_2 + \mu \bar{\Omega}_2 \bar{\Omega}_2 \Phi_1 + h.c. 
\]

(20)

Here the two Lagrangians are meant to be taken separately, and not simultaneously. As shown in Figs. 9 and 10 in light solid lines, upon annihilating the \( L \) and \( e^c \) fields as well as the \( Q \) and \( d^c \) fields, two–loop neutrino masses will result. The estimate of the masses is analogous to that in Eq. (19), with one difference, namely, one of the charged lepton mass matrices must be replaced by the down–type quark mass matrix.
It is possible to substitute an $SU(2)_L$ triplet scalar $\Phi_3$ for the $SU(2)_L$ singlet $\Phi_1$ in these models. This is also indicated in Figs. 9 and 10. In this case, a symmetry must be used to prevent the coupling $\Phi_3 \bar{H} H$ so that the neutral component of $\Phi_3$ does not acquire an induced VEV at tree level, or else the dimension 5 term of Fig. 2 will contribute more dominantly to neutrino masses. Clearly, both models will lead to interesting neutrino masses, e.g., with $m_{\nu_3} \sim 0.03$ eV, if the scale of new physics is around a few TeV.

3. Operator $O_{11}$:

Operator $O_{11}$ contains the fields $(LLQQd^c\bar{d}^c)$. We shall present three models for inducing this effective operator. The three models correspond to the following different contractions: (1) $(LL)(QQ)(d^c\bar{d}^c)$, (2) $(LQ)(LQ)(d^c\bar{d}^c)$, and (3) $(Ld^c)(Ld^c)(QQ)$. Consider the
(LL)(QQ)(d^c d^c) contraction first. The required couplings in the renormalizable model are shown in Fig. 11. The relevant Lagrangian for this model is

$$\mathcal{L}_{11}^{O_{11}} = f_{ab} L_a L_b \Phi_1 + g_{ab} Q_a Q_b \Omega_1 + h_{ab} d^c_a d^c_b \bar{\Omega}_1 + \mu \Phi_1 \bar{\Omega}_1 \Omega_1 + h.c. \quad (21)$$

As shown in Fig. 11, the scalar \( \Phi_1 \) may be replaced by an \( SU(2)_L \) triplet scalar \( \Phi_3 \). In this case, the coupling \( \Phi_3 \bar{H} \bar{H} \) must be prevented by a symmetry so that the neutral component of \( \Phi_3 \) does not acquire an induced VEV at tree level, or else the dimension 5 operator of Fig. 2 will be the more dominant effective neutrino mass operator. An example of such a symmetry is a \( Z_4 \), under which \( \Phi_3 \rightarrow -\Phi_3 \), \( L \rightarrow iL \), \( e^c \rightarrow ie^c \), with other fields being neutral. The cubic scalar coupling term in Eq. (21) breaks this \( Z_4 \) symmetry, either softly or spontaneously.

![Diagram of neutrino mass generation](image_url)

FIG. 11. An example of two–loop radiative generation of neutrino masses from the operator \( O_{11} \).

Upon closing the quark lines in Fig. 11, the effective scalar coupling \( \Phi_3 \bar{H} \bar{H} \) will be generated so that the neutral member of \( \Phi_3 \) will get a VEV. This VEV will be suppressed by two–loop factors and two powers of down–type quark masses, and will scale as \( v^2/M \), where \( M \) is the assumed common mass of the heavy scalars. The neutrino masses that are induced will be approximately
\[ m_{\nu_a} \sim \frac{fgh}{(16\pi^2)^2} \left( \frac{\mu(m_d)_a^2}{M^2} \right). \quad (22) \]

The numerical estimate of the masses parallels that of Eq. (19).

The case of using an SU(2)_L singlet scalar \( \Phi_1 \), as in Eq. (21), has an interesting consequence. The effective \( d = 9 \) operator will be of the form \((\nu e)(ud)(d^c d^c)\). In order to convert it into a neutrino mass term, we can contract one of the \( d^c \) fields with the \( d \) field, but we will be left with \((\nu e)(ud^c)\). Further exchange of a W boson can convert the \( e \) into a \( \nu \) and the \( u \) into a \( d \). This is shown in Fig. 12. Note that only the longitudinal component of \( W \) contributes to this diagram, and we have indicated it as \( H^- \) in Fig. 12. To see this, it is convenient to work in the Landau gauge, where the \( W \) boson propagator is purely transversal. In this gauge, it is clear that the amplitude for the transition of an off–shell vector boson into a scalar boson must vanish. It can, however, arise through the longitudinal component of the vector boson. (Such higher loop diagrams for neutrino masses have been considered in Ref. [16].)

\[ \text{FIG. 12. A three–loop diagram that induces neutrino masses from the operator } \mathcal{O}_{11} \text{ through the Lagrangian of Eq. (21).} \]

In addition to an extra loop suppression factor, the three–loop diagram of Fig. 12 is suppressed by couplings of the longitudinal \( W \) to charged fermions. We estimate the
induced neutrino masses to be

\[ m_\nu \sim \frac{fgh}{(16\pi^2)^3} \left( \frac{\mu m_\ell^2}{M_W^2} \right)^{3/2} \mu \bar{M}^2 \]  

(23)

where \( m_\ell \) and \( m_d \) denote the respective charged lepton and down–type quark masses, and \( M_W \) is the mass of the \( W \) boson. With this estimate, we see that, for neutrino masses to be in the interesting range, the masses of the scalars should be of the same order as the weak scale.

In Fig. 13, we display the renormalizable model where the fermion fields in \( \mathcal{O}_{11} \) are contracted as \((LQ)(LQ)(d^c d^c)\). The Lagrangian terms of this model are

\[ \mathcal{L}^{\mathcal{O}_{11}}_2 = f_{ab} L_a Q_b \bar{\Omega}_1 + g_{ab} d^c_a d^c_b \bar{\Omega}'_1 + \mu \bar{\Omega}_1 \bar{\Omega}_1 \bar{\Omega}'_1 + \text{h.c.} \]  

(24)

We can also replace the \( \bar{\Omega}_1 \) field by \( \bar{\Omega}_3 \), an \( SU(2)_L \) triplet.

![Diagram](image)

**FIG. 13.** Diagram that induces the operator \( \mathcal{O}_{11} \) through Eq. (24).

Fig. 14 shows the fermion field contraction \((Ld^c)(Ld^c)(QQ)\) that induces \( \mathcal{O}_{11} \). The Lagrangian for this case is

\[ \mathcal{L}^{\mathcal{O}_{11}}_3 = f_{ab} L_a d^c_b \bar{\Omega}_2 + g_{ab} Q_a Q_b \bar{\Omega}_1 + \mu \bar{\Omega}_2 \bar{\Omega}_2 \bar{\Omega}_1 + \text{h.c.} \]  

(25)

In both Figs. 13 and 14, the induced neutrino masses are of the same order, given approximately by

\[ m_{\nu_\alpha} \sim \frac{f^2 g}{(16\pi^2)^2} (m_d)_\alpha^2 \left( \frac{\mu}{M^2} \right). \]  

(26)
FIG. 14. Two–loop neutrino mass generation from $O_{11}$ through the renormalizable Lagrangian in Eq. (25).

4. Operator $O_{12}$:

$O_{12}$ can be obtained in a way very similar to $O_{11}$. The neutrino mass generation mechanism is shown in Fig. 15. Note that this diagram is very similar to Fig. 11, with the $d^c$ fields in Fig. 11 replaced by the $u^c$ fields here. We simply write down the Lagrangian for this model:

$$\mathcal{L}^{O_{12}} = f_{ab} L_a L_b \Phi_1 + g_{ab} Q_a Q_b \Omega_1 + h_{ab} u^c_a u^c_b \Omega_1 + \mu \Omega_1 \Omega_1 \Phi_1^\dagger + h.c.$$  \hspace{1cm} (27)

Compared to Fig. 11, the only difference in the neutrino mass estimate is that here it will be proportional to $m_u^2$, rather than $m_d^2$.

5. Operator $O_{21}$:

$O_{21}$ is a dimension 11 operator. We shall illustrate various ways of building renormalizable models of neutrino mass with this operator. The field content of $O_{21}$ is $(L L L e^c Q u^c H H)$. As in the previous examples, we can contract the fermion fields in different ways. Three specific models are obtained by the contractions (1) $(L L) (e^c u^c) (Q L)$, (2) $(L Q) (L e^c) (L u^c)$, and (3) $(L L) (L u^c) (Q e^c)$. The Lagrangians for these three choices are as follows:

$$\mathcal{L}_1^{O_{21}} = f_{ab} L_a L_b \Phi_1 + g_{ab} e^c_a u_b^c \Omega_1 + h_{ab} L_a Q_b \Omega_1 + \mu \Omega_1 \Omega_1 \Phi_1^\dagger + h.c.$$ 

$$\mathcal{L}_2^{O_{21}} = f_{ab} L_a Q_b \Omega_1 + g_{ab} L_a e^c_b \Phi_2 + h_{ab} L_a u_b^c \Omega_2 + \mu \Omega_1 \Omega_1 \Phi_1^\dagger + h.c.$$
Here we have not explicitly written down the renormalizable terms that result in the scalar mixings. These scalar mixing terms can be read off from Figs. 16, 17 and 18.

Neutrino masses arise in all three models upon closing the $Le$ lines and the $Qu$ lines. The order-of-magnitude estimates for the masses in the case of Figs. 17 and 18 are

$$m_{\nu_a} \sim \left( \frac{fgh}{(16\pi^2)^2} \right) \left( \frac{(m_\ell)_a(m_u)_a\mu^3v^2}{M^6} \right).$$

Here we have denoted by $\mu$ all cubic scalar couplings entering the diagrams. $(m_\ell)_a$ and $(m_u)_a$ denote, respectively, the masses of the charged leptons and the up-type quarks with flavor index $a$. If we choose $f = g = h = 0.15$ and $\mu = M = 3$ TeV, we obtain $m_{\nu_3} \sim 0.05$ eV, in the interesting range for atmospheric neutrino oscillations. The same estimate will apply to Fig. 16 if the $\Phi_3$ scalar is used. On the other hand, if $\Phi_1$ is used, there is an additional loop suppression, along with a suppression factor of $m_\ell^2/M_W^2$ as in Fig. 12.
IV. NEUTRINOLESS DOUBLE BETA DECAYS AND NEUTRINO MASS

In many of the $\Delta L = 2$ effective operators, there is a tree-level contribution to neutrinoless double beta ($\beta\beta_0\nu$) decay amplitudes, while neutrino masses arise only as two-loop radiative corrections.\(^3\) Such models will have an exciting phenomenological consequence, namely, that $\beta\beta_0\nu$ decays might be observable in the current round of experiments, while the induced neutrino masses are quite consistent with the atmospheric and solar neutrino data.

The effective operators that mediate $\beta\beta_0\nu$ processes will have the form $[uu\bar{d}\bar{d}ee]$. Operators of this form can be easily identified from our list. The $d = 9$ operators from Eq. (7) that have this form are:

$$\mathcal{O}_{d=9}^{\beta\beta_0\nu} = \{ \mathcal{O}_{11}^{(ii)}, \mathcal{O}_{12}^{(i)}, \mathcal{O}_{14}^{(ii)}, \mathcal{O}_{19}, \mathcal{O}_{20} \}. \quad (30)$$

\(^3\)Gauge models where this phenomenon occurs have been noted in specific contexts [17].
FIG. 17. Generation of the $d = 11$ operator $\mathcal{O}_{21}$ through the Lagrangian $\mathcal{L}_{2}^{O_{21}}$ of Eq. (28).

FIG. 18. Generation of the $d = 11$ operator $\mathcal{O}_{21}$ through the Lagrangian $\mathcal{L}_{3}^{O_{21}}$ of Eq. (28).

Here the superscript $(ii)$ stands for the second entry in the respective operator, and so on.

From the set of $d = 11$ operators in Eq. (8), the following operators will induce $\beta\beta 0\nu$ decays at tree level:

$$
\mathcal{O}_{\beta\beta 0\nu}^{d=11} = \{ \mathcal{O}_{24}^{(i)}, \mathcal{O}_{28}^{(i),(iii)}, \mathcal{O}_{32}^{(i)}, \mathcal{O}_{36}, \mathcal{O}_{37}, \mathcal{O}_{38}, \mathcal{O}_{47}^{(i),(iv)}, \mathcal{O}_{53}, \mathcal{O}_{54}^{(i),(iv)}, \mathcal{O}_{55}, \mathcal{O}_{59}, \mathcal{O}_{60} \}. 
$$

(31)

In order to estimate the strength of these $\beta\beta 0\nu$ operators, we compare their amplitudes to that arising from light Majorana neutrino exchanges. This latter amplitude is given by
A_{\beta\beta 0} \sim G_F^2 m_\nu \langle \frac{1}{q^2} \rangle$, where $\langle \frac{1}{q^2} \rangle$ is the average of the inverse squared Fermi momentum and is approximately equal to $1/(100 \text{ MeV})^2$. The current experimental constraint on light Majorana neutrino masses is $m_\nu < 0.3 \text{ eV}$, derived from the nonobservation of $\beta\beta 0$ decays [18]. Hence, $A_{\beta\beta 0} < 10^{-18} \text{ GeV}^{-5}$. Consider now the amplitude arising from $O_{11}^{(ii)}$. Since this operator has dimension 9, it carries a suppression factor of $\Lambda^{-5}$, where $\Lambda$ denotes the scale of new physics above which the effective description becomes invalid. Comparing the two amplitudes, we see that, if $\Lambda \sim 10^{3.6} \text{ GeV} \sim 4 \text{ TeV}$, the induced $\beta\beta 0$ decay amplitude will be near the current experimental limit. From the diagram of Fig. 11 (with $\Phi_3$ field rather than $\Phi_1$ field), we also see that $m_\nu \sim (1/16\pi^2)^2 m_d^2/\Lambda \sim 3 \times 10^{-4} \text{ eV}$, which is extremely tiny and well consistent with solar and atmospheric neutrino data. (Here we considered only the first generation couplings and assumed the relevant $f \sim g \sim h \sim 1$.) Similar estimates apply to the other $d = 9$ operators as well. For the $d = 11$ operators, there is an additional suppression factor of $(v/\Lambda)^2$ due to the presence of two additional Higgs fields. The scale $\Lambda$ will have to be somewhat smaller than 4 TeV in these cases for $\beta\beta 0$ decays to be observable in the near future. The induced neutrino masses will also have this same additional suppression factor. Thus, it is possible to have negligible neutrino masses while having sizable $\beta\beta 0$ processes.

Of course, our estimates here have been rather crude. We have ignored important differences in nuclear matrix elements. However, we think that the above order-of-magnitude estimates suffice to demonstrate the potential importance of these operators. It will be interesting to investigate in more detail the rates for $\beta\beta 0$ processes in this class of models.

V. CONCLUSIONS

We have presented a classification of effective $\Delta L = 2$ operators for the Standard Model that may be relevant for generating small neutrino Majorana masses. The lowest dimensional ($d = 5$) such operators are provided by the seesaw mechanism. If these $d = 5$ operators are absent from the underlying renormalizable gauge theory due to selection rules, higher
dimensional operators will become relevant. These higher dimensional $\Delta L = 2$ operators will induce neutrino masses through radiative corrections. The scale of new physics, $\Lambda$, can be as low as a few TeV in this class of models.

We have presented a list of all $\Delta L = 2$ operators through $d = 11$ which contain no derivatives or gauge boson fields. We can readily construct from this list various renormalizable models for neutrino masses. We are able to identify several of the well-known radiative neutrino mass models as specific realizations of some of these effective operators. Furthermore, we can identify a large class of new models where neutrino masses arise as one-loop, two-loop or even three-loop radiative corrections. We have given several examples of these new models, and have outlined their main features for neutrino mass phenomenology. Of special interest is a class of operators of dimension 9 and 11 which contribute directly to neutrinoless double beta decays, while generating only very tiny neutrino masses at the two-loop level. Models of this class have the exciting prospect that neutrinoless double beta decays will be observable in the ongoing round of experiments, while being fully consistent with solar and atmospheric neutrino oscillations. The operators presented here will have interesting consequences for cosmology as well. For example, the observed baryon asymmetry in the universe may have been generated by electroweak sphaleron processes which converted a primordial lepton asymmetry into baryon excess. The $\Delta L = 2$ operators listed here will be constrained if we require that the observed baryon asymmetry is produced correctly by this mechanism [19]. We plan to revisit our models in a future publication and analyze this and other issues such as the expectation for flavor changing neutral current processes in the leptonic as well as in the hadronic sectors.
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