Preparation of decoherence-free, subradiant states in a cavity

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The cause of decoherence in a quantum system can be traced back to the interaction with the environment. As it has been pointed out first by Dicke, in a system of $N$ two-level atoms where each of the atoms is individually dipole coupled to the environment, there are collective, subradiant states, that have no dipole coupling to photon modes, and therefore they are expected to decay slower. This property also implies that these type of states, which form an $N-1$ dimensional subspace of the atomic subsystem, also decohere slower. We propose a scheme which will create such states. First the two-level atoms are placed in a strongly detuned cavity and one of the atoms, called the control atom is excited. The time evolution of the coupled atom-cavity system leads to an appropriately entangled state of the atoms. By applying subsequent laser pulses at a well defined time instant, it is possible to drive the atomic state into the subradiant, i.e., decoherence free subspace. Up to a certain average number of the photons, the result is independent of the state of the cavity. The analysis of the conditions shows that this scheme is feasible with present day techniques achieved in atom cavity interaction experiments.

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Subradiant states of a system of two-level atoms [1–5] has recently gained wide attention because of their exceptionally slow decoherence [6–8]. This stability of quantum superpositions inside the subradiant subspaces originates from the low probability of photon emission, which means very weak interaction between the atoms and their environment. Hence the subradiant states span a decoherence-free subspace (DFS) [9–11] of the atomic Hilbert-space and consequently can become important from the viewpoint of quantum computation (QC) [10,12]. The scheme we propose can be used to prepare subradiant states in a cavity. Our method is based on second order perturbation theory but the exact results verify the validity of the perturbative approach. We also investigate to what extent our scheme is independent of the state of the cavity field. Finally the requirements needed to prepare subradiant states in the proposed way will be compared with available experimental techniques [13–16].

We investigate a system of $N$ identical two-level atoms in a single mode cavity. Each individual atom is equivalent to a spin-$1/2$ system, and the whole atomic ensemble can be described by the aid of collective atomic operators $J_+$, $J_-$ and $J_z$ obeying the same algebra as the usual angular momentum operators [1]. We consider the following model Hamiltonian:

$$H = H_0 + H_{int} = \hbar \omega_a J_z + \hbar \omega_c a^\dagger a + \hbar g \left( a^\dagger J_- + a J_+ \right),$$

(1)

where $a$ and $a^\dagger$ are the annihilation and creation operators of the cavity mode, $\omega_a$ is the transition frequency between the two atomic energy levels, $\omega_c$ denotes the frequency of the cavity mode, different from $\omega_a$, and $g$ is the coupling constant. We note that the Hamiltonian (1) is written in the framework of Dicke's theory, i.e., with the assumption that all the atoms are subjected to the same field, which is a good approximation when the size of the atomic sample is small compared to the wavelength of the cavity mode. As discussed later in detail, there are experimental situations where this requirement is fulfilled. Our proposed scheme for preparing subradiant states involves a detuned cavity. We shall assume that the detuning is much larger than the resonant Rabi frequency:

$$\omega_c - \omega_a = \Delta \gg g.$$

(2)

Now any state of the atomic system and the cavity field can be expanded as a linear combination of eigenstates of $H_0$. These are tensorial products of collective atomic states and number states of the field: $|j, m, \lambda \rangle \otimes |n \rangle$, where the indices $j, m$ and $\lambda$ label the atomic state (also called Dicke state [18]) while $n$ refers to the $n$th Fock state of the mode. The quantum number $j$ corresponds to the eigenvalues of the operator $J^2 = J^2_+ + (J_+ J_- + J_- J_+)/2$. This index is in one-to-one correspondence with the Young digram [17] that describes the permutation symmetry of the state. The possible values of $j$ is $N/2, N/2 - 1, \ldots$, the smallest value being 0 if $N$ is even and 1/2 if $N$ is odd. The index $m$ of the $|j, m, \lambda \rangle$ Dicke state labels the eigenstates of the collective atomic operator $J_z$, that is essentially proportional to the energy of the atomic subsytem. This is the index that is decreased (increased) by one under the action of the operator $J_-$ ($J_+$);

$$J_- |j, m, \lambda \rangle = \sqrt{j(j+1) - m(m-1)} |j, m-1, \lambda \rangle,$$

(3)
including the case when $m = -j$, when the result is the zero vector. The states with $m = -j$ are the lowest ones of the Dicke ladders [1], they are called subradiant, because they have no dipole coupling to other lower lying states. Finally the index $\lambda$ distinguishes states with the same $j$ and $m$. For more details see Refs. [1,8,17–21].

Besides the collective atomic states $|j, m, \lambda\rangle$, we shall also use the natural basis that assigns a well defined state to each individual atom. These vectors will be labeled by a string of 0-s and 1-s corresponding to the ground and excited states, respectively. E. g., the ground state of the atomic subsystem is written in this basis as $|00\ldots 0\rangle$; this state (as well as the fully excited one) is also an element of the Dicke basis, $|00\ldots 0\rangle = |j = N/2, m = -N/2, \lambda = 1\rangle$.

The form of the Hamiltonian (1) implies that the time evolution of the system shall exhibit two time scales: The first characteristic time is due to the self-Hamiltonian $H_0$ and is approximately $2\pi/\omega_c$ (or $2\pi/\omega_k$) and the second is proportional to $2\pi/g$. Generally $g \ll \omega_c \approx \omega_k$ and the faster process induced by $H_0$ can be eliminated by going into an interaction picture. However, if the frequency difference $\Delta$ is large enough, then the energy transfer between two adjacent eigenstates of $H_0$, differing in only one photon number, becomes negligible. This means that the amplitude of the corresponding collective Rabi oscillations will be very small, that is, the process on the second time scale will be unnoticeable and even slower mechanisms will become apparent. The situation is similar to the proposals [22] and [23].

Hereafter we shall focus on the solution of the Schrödinger equation in the case when just a single atom is excited at $t = 0$. This initial state can be prepared by starting from the state $|00\ldots 0\rangle$, and exciting one well defined control atom. This excitation can be achieved via a third much higher lying level, so that the wavelength of the addressing pulse allows to focus it on the desired target atom [24]. For the sake of simplicity we always consider the control atom as being the first, hence the initial state will be written as

$$|\phi(0)\rangle = |100\ldots 0\rangle \otimes |n - 1\rangle. \quad (4)$$

In order to find the complete analytical solution of the Schrödinger equation induced by the Hamiltonian (1), in principle one should calculate all the eigenvalues and the corresponding eigenstates of $H$. Although this problem can be solved analytically [25], more insight is gained by a simple perturbative approach. The exact nonperturbative numerical solution of the Schrödinger equation verifies that results obtained via perturbation theory yield excellent approximations.

The state

$$|1\rangle \equiv \left(\frac{1}{\sqrt{N}} \sum_{k=1}^{N} |00\ldots 010\ldots 0\rangle\right) \otimes |n - 1\rangle =$$

$$= |j = N/2, m = -N/2 + 1, \lambda = 1\rangle \otimes |n - 1\rangle, \quad (5)$$

which is in the completely symmetric subspace, and the subradiant states:

$$|i\rangle \equiv |j = N/2 - 1, m = -N/2 + 1, \lambda = i - 1\rangle \otimes |n - 1\rangle \quad (6)$$

with $i = 2, 3, \ldots N$, have the same unperturbed energy, they span the $N$-fold degenerate eigensubspace of $H_0$ corresponding to the eigenvalue $E^0(n) = \hbar(n\omega_c - N\omega_c/2) - \hbar\Delta$.

It can be seen that first order degenerate perturbation theory is not giving any correction to the energy, because all the matrix elements of $H_{\text{int}}$ between the states above vanish, the action of $H_{\text{int}}$ on vectors $|j, m, \lambda\rangle \otimes |n - 1\rangle$ gives a linear combination of $|j, m - 1, \lambda\rangle \otimes |n\rangle$ and $|j, m + 1, \lambda\rangle \otimes |n - 2\rangle$ that are orthogonal to the states (5) and (6). In order to obtain nonzero energy corrections we have to make a second order degenerate perturbation calculation [26], and find the eigenvalues of the matrix:

$$\sum_{m} \frac{|i|H_{\text{int}}|m\rangle \langle m|H_{\text{int}}|k\rangle}{E^0(n) - E^0_m}, \quad (7)$$

where the sum runs over all eigenstates of $H_0$ with eigenvalue $E^0_m \neq E^0(n)$. The only nonvanishing energy corrections in second order are the following:

$$\delta E_1 = \hbar g^2 \Delta (Nn - 2N - 2n + 2),$$

$$\delta E_i = \delta E_1 + \hbar N\frac{g^2}{\Delta}, \quad i = 2, 3\ldots N. \quad (8)$$

At this point we can formulate the requirements that assure the validity of the perturbation theory: the magnitude of $\delta E_1$ and $\delta E_i$ must be much smaller than $\hbar|\Delta|$, the minimum of the difference between $E^0(n)$ and all other unperturbed energy levels.
The most important consequence of Eqs. (8) is that the Bohr frequencies that determine the time dependences of the subradiant and non-subradiant states are different.

Now we expand the initial state (4) as the linear combination of the fully symmetric (non-subradiant) state \(|1\rangle\), and an appropriate subradiant state:

\[
|2\rangle = \frac{1}{\sqrt{N(N-1)}} \left[ (N-1)|100\ldots0\rangle - \sum_{k=2}^{N} |0\ldots0\ 1\ 0\ldots0\rangle \right] \otimes |n-1\rangle.
\] (9)

By assigning the symbol \(|2\rangle\) to the state in Eq. (9), we have utilized the freedom of choosing a basis in the subradiant subspace. Now the initial state reads

\[
|\phi(0)\rangle = \frac{1}{\sqrt{N}}|1\rangle + \sqrt{\frac{N-1}{N}}|2\rangle.
\] (10)

By the aid of this expansion and using the Bohr frequencies resulting from (8), it is easy to calculate the time evolution of the state (10). Discarding an overall phase factor, this time dependent state has the form

\[
|\phi(t)\rangle = \frac{1}{\sqrt{N}} \exp \left(iN\frac{g^2}{\Delta} t\right) |1\rangle + \sqrt{\frac{N-1}{N}}|2\rangle,
\] (11)
or, on using Eqs. (5) and (9):

\[
|\phi(t)\rangle = \left[ (N \cos(\alpha t) - i(N-2) \sin(\alpha t)) |100\ldots0\rangle + 2i \sin(\alpha t) \sum_{k=2}^{N} |0\ldots010\ldots0\rangle \right] \otimes |n-1\rangle/N.
\] (12)

Here we introduced the parameter

\[
\alpha = \frac{N g^2}{2\Delta},
\] (13)
which is independent of \(n\). Because of this latter fact, from now on the state of the cavity field will be omitted in the notation. We also note that the characteristic time of the time evolution, \(2\pi/\alpha\), is much longer than that of the free evolution due to \(H_0\), being the consequence of the fact that the evolution described in Eq. (11) is induced by a weak, nonresonant interaction.

Eq. (12) reveals that in \(|\phi(t)\rangle\) the weight of the state \(|100\ldots0\rangle\) and those of the states with the first atom unexcited changes during the course of time. As we can see, the moduli of the corresponding coefficients in Eq. (12) are

\[
\sqrt{\frac{N^2 \cos^2(\alpha t) + (N-2)^2 \sin^2(\alpha t)}{N}} \quad \text{and} \quad \frac{2|\sin(\alpha t)|}{N},
\]
respectively. Comparing these values to Eq (9), it can be shown that for an arbitrary \(N\) there exists a time instant \(t_m\) when

\[
|\phi(t)\rangle = \frac{1}{\sqrt{N(N-1)}} \left[ (N-1)e^{i\varphi}|100\ldots0\rangle - \sum_{k=2}^{N} |0\ldots0\ 1\ 0\ldots0\rangle \right],
\] (14)

which differs from the subradiant state \(|2\rangle\) only in the phase factor \(e^{i\varphi}\) of the first term. Combination of the previous two equations and Eq. (9) yields the following requirement for \(t_m\):

\[
\sqrt{\frac{N^2 \cos^2(\alpha t_m) + (N-2)^2 \sin^2(\alpha t_m)}{2|\sin(\alpha t_m)|}} = N - 1.
\] (15)
interaction time of the atoms and the cavity must be longer than $t$. We use the fact that the interaction Hamiltonian $H_{int}$ does not mix states with different number of excitation (essentially $n + m$):

$$
\langle j, m, \lambda | \otimes \langle n | H_{int} | n' \rangle \otimes | j, m \pm 1, \lambda \rangle = 0,
$$

unless $n' = n \neq 1$. This implies that the calculations based on second order perturbation theory can be performed for each $N$-fold degenerate energy level of $H_0$ corresponding to different values of $n$. After replacing the state $|n - 1\rangle$ with $|\psi(t)\rangle$ in Eqs. (5) and (6), we obtain the following result:

$$
\delta E_1 - \delta E_0 = \frac{g^2}{\Delta} N \sum_{n} |c_n|^2 = 2\alpha,
$$

which is therefore also valid in this general case. Thus we have proven that our scheme does not require special preparation of the cavity field. However, it should be borne in mind, that the results above are based on perturbation theory. For given $N, g$ and $\Delta$ the validity of the perturbative calculations depends on $\langle n \rangle$, the average number of photons in the cavity field. Hence it is clear that our scheme can not be independent of the average photon number on a very large scale. Nevertheless, until $\frac{g}{\Delta} \sqrt{N(n)} \ll 1$, all the previous statements hold. For the case of 10 atoms, we have performed exact (nonperturbative) numerical calculations for the experimentally realizable ratio $[14]$ of $g/\Delta = 30$, and found that the time evolution follows Eq. (11) within 2% relative error in the coefficients.

Finally we compare the requirements of our scheme with the experimental possibilities. The atom cavity experiments of Haroche and co-workers, as described in the review paper [28], show that the description of the interaction of a number of Rydberg atoms with a single mode cavity is truly described in the framework of the Dicke model. In more recent experiments [14–16] the interaction of a detuned cavity with one and two atoms has been found to be in agreement with theoretical predictions. The parameters realized in these experiments with rubidium Rydberg atoms: $g/2\pi \approx 24kHz$ and detunings as large as $\Delta/2\pi \approx 800kHz$ show that the conditions of the validity of our perturbation approach hold as much as for about hundred atoms, because the average photon number in a cavity can be kept much less than 1. Taking for instance 10 atoms and $g/\Delta = 30$, we have $\alpha = 2.5 \times 10^4/s$ giving for $t_m$ a value of $22\mu s$. The interaction time of the atoms and the cavity must be longer than $t_m$ what can be achieved already with atoms with somewhat less than thermal velocities for centimeter sized cavities. Finally we note that addressing of single atoms is a common problem in almost all of the proposals in QC, but there are promising works indicating future success [24].

In conclusion, we have proposed a method to prepare decoherence-free, subradiant states of a multiatomic system. We have shown that our perturbative approach is compatible with present day techniques in atom cavity experiments.

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