POLARIZED STRUCTURE FUNCTIONS IN QCD

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We review the nucleon’s polarized structure functions from the viewpoint of gauge invariant, nonlocal light-cone operators in QCD. We discuss a systematic treatment of the polarized structure functions and the corresponding parton distribution functions. We also address a question of what information on the structure of Nature will be obtained from the future polarized experiments. From this point of view, we will discuss the $W^\pm \gamma$ production at RHIC polarized experiment.

1 Introduction

In the last ten years, great progress has been made both theoretically and experimentally in hadron spin physics. Furthermore, in conjunction with new projects like the “RHIC spin project”, “polarized HERA”, etc., we are now in a position to obtain more information on the spin structure of nucleons. The spin dependent quantity is, in general, very sensitive to the structure of interactions among various particles. Therefore, we will be able to study the detailed structure of hadrons based on QCD. However the purpose of new experiments should not be limited to only the check of QCD. We also hope that we can find some clue to new physics beyond the standard model.

In this talk, we first review a systematic treatment of polarized structure functions and summarize the recent theoretical progress on the QCD evolutions. Secondly, we address a question of what we will be able to learn from the future polarized experiments by considering the phenomena called Radiation Zeros (RAZ) as an example. We reanalyze this phenomena at the realistic RHIC polarized collider. We will point out that the polarization of colliding protons will emphasize the RAZ phenomena in the cross section and a “moderate energy” machine is better than the extremely high energy machines to find this phenomena.
2 Classification of structure functions

To describe a variety of high-energy processes to which the factorization theorem can be applied, it is desirable to have a formulation based on a universal language in QCD. The traditional approach relies on the operator product expansion (OPE), but it can be applied only to a limited class of processes. This calls for an approach based on the factorization as a generalization of the OPE. As a result of factorization, the structure functions (cross section) are given as convolution of the short- and long-distance parts. The former contains all the dependence on the hard scale and the latter is controlled by the nonperturbative dynamics of QCD.

For the long-distance part, the parton distribution functions (PDFs) are introduced and it is now standard to define them as the nucleon's matrix elements of nonlocal light-cone operators in QCD\(^3\). The momentum of nucleon (mass \(M\)) \(P\) will be written in terms of two auxiliary light-like vectors \(p^\mu\) and \(w^\mu\) \((p^2 = w^2 = 0,\ p \cdot w = 0)\), \(P^\mu = p^\mu + \frac{M^2}{2} w^\mu\). The (quark) PDF is defined by the Fourier transform of the nucleon's matrix element of the nonlocal light-cone quark operator as,

\[
\int_{-\infty}^{+\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)[0,\lambda w]\Gamma\psi(\lambda w)|PS\rangle .
\] (1)

Here \(|PS\rangle\) is the nucleon state with spin \(S\) which is decomposed into the longitudinal and transverse parts with respect to \(p^\mu(w^\mu)\) direction, \(S^\mu = S^{\mu \|} + S^{\mu \perp}\). The variable \(x\) can be interpreted as the momentum fraction of parton (Bjorken \(x\)) since \(\lambda x = (xP) \cdot (\lambda w)\). \(\Gamma\) is a generic Dirac matrix and the link operator \([0,\lambda w]\) which makes the operators gauge invariant is given by,

\[
[y, z] = \text{Pexp}
\left(i g \int_0^1 dt (y - z)_\mu A^\mu (ty + (1 - t)z)\right)
\]

Possible choices for \(\Gamma\) and spin states which depend on the processes considered in Eq.(1) lead to various PDFs.

2.1 Quark distributions

Let us first consider the quark distributions. An important observation made in Ref.[4] is that one can generate all quark distribution functions up to twist-4 by substituting all the possible \(\Gamma\). By decomposing Eq.(1) into independent tensor structures, one finds nine independent quark distribution functions associating with each tensor structure\(^1\). Their spin, twist and chiral classifications
are listed in Table 1. The distributions in the first row are spin-independent, while those in the second and third rows correspond to the longitudinally ($S_\parallel$) and transversely ($S_\perp$) polarized nucleons. Each column refers to the twist. The distributions marked with "⋆" are referred to as chiral-odd, because they correspond to chirality-violating Dirac matrix structures $\Gamma = \{\sigma_{\mu\nu}i\gamma_5, 1\}$. The other distributions are chiral-even, because of the chirality-conserving structures $\Gamma = \{\gamma_\mu, \gamma_\mu\gamma_5\}$. In the massless quark limit, chirality is conserved through the propagation of a quark. This means that in the DIS one can measure only the chiral-even distributions up to tiny quark mass corrections because the perturbative quark-gluon and quark-photon couplings conserve the chiralities. On the other hand, in the Drell-Yan and certain other processes, both chiral-odd and chiral-even distribution functions can be measured because the chiralities of the quark lines originating in a single nucleon are uncorrelated.

### 2.2 Gluon distributions

The gluon distribution functions can be defined in the similar way as for the quark distributions. The gauge-invariant definition of the gluon distribution functions is provided by

$$\frac{2}{x} \int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle PS|w_\alpha G^\alpha\mu(0)[0, \lambda w]w_\beta G^\beta\nu(\lambda w)|PS\rangle .$$

Corresponding to independent tensor structures, we have four gluon distribution functions summarized in Table 2. The gluon distributions mix through renormalization with the flavor singlet chiral-even quark distributions. On the other hand, there exists no gluon distributions that mix with the chiral-odd quark distributions.
2.3 Twist-3 three particle distributions

Coherent many-particle contents of the nucleon are described by multi PDFs. In this talk, we mention only the twist-3 quark-gluon correlation functions,

\[ \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x + i\zeta(x' - x)} \langle PS| \bar{\psi}(0) \Gamma[0, \zeta w] g G^{\mu\nu}(\zeta w)[\zeta w, \lambda w] \psi(\lambda w)|PS \rangle. \]

Similarly to the quark distributions, one can define the multiparton distributions by considering possible Dirac matrices for \( \Gamma \) which are listed in Table 3.

<table>
<thead>
<tr>
<th>( S ) ave.</th>
<th>( S_\parallel )</th>
<th>( S_\perp )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi(x, x')^* )</td>
<td>( \Psi(x, x')^* )</td>
<td>( \Psi(x, x'), \bar{\Psi}(x, x') )</td>
</tr>
</tbody>
</table>

Table 3: Spin and chiral classification of quark-gluon correlations at twist-3.

The treatment here can be extended to the case of three-gluon correlation functions, which are relevant to the (singlet) quark distribution \( g_T(x) \) and the gluon distribution \( \zeta_{3T}(x) \). The details have been discussed in Refs. [6,7].

3 QCD evolution

In the case of the lowest twist, their QCD evolution can be easily estimated using the standard techniques. The calculation for the higher twist terms is, however, generally very complicated due to the presence of multiparton distributions. Although it is possible to generalize the DGLAP approach to the three-body case, one is forced to use some particular techniques, e.g. the use of the light-like axial gauge etc. [7,9]. On the other hand, it has been known that one can construct a set of “local composite operators” from the nonlocal one by taking the moment of PDFs. Therefore we can take an approach based on these local composite operators [10]. The advantage of this approach is that all the relevant steps can be worked out based on the standard and familiar field theory techniques in any gauge.

The common feature for the higher twist operators is that the number of a set of local operators with the same quantum numbers increases with spin (moment) and these operators are related through the QCD equation of motion [11,12]. However, once a complete set of operators is identified, the anomalous dimensions which derive QCD evolution is easily calculated following the theorem [13] explained now. This theorem tells us that in non-abelian gauge theory, three kinds of operators will mix under renormalization of gauge invariant operators. (I) the gauge invariant operator itself. (II) the BRST invariant operators. (III) the operators which are proportional to the equation of motion (EOM). The EOM operators involve both BRST “invariant” and
“variant” ones $^{10,14}$. Although the physical matrix elements of the BRST invariant and EOM operators vanish $^1$, it is necessary to consider these operators to complete the renormalization. At the lowest twist level, these complexities do not come into play because there exists neither EOM nor BRST invariant operator of twist-2. The EOM as well as the BRST invariant operators always have smaller spin by at least one unit than the possible highest spin operators of the same dimension.

The explicit calculations at the one-loop level appear in Refs.$^{[10,8,15]}$ for $g_T$ and in Ref.$^{[16]}$ for $h_L$. Much more recent progress on the twist-3 parton densities can be found in Ref.$^{[17]}$.

4 $W^\pm\gamma$ production at RHIC

The polarized PDFs will be measured precisely by the future polarized experiments and we will be able to check if the QCD predictions are consistent or not. However, we believe that the polarized experiments will provide us with a great store of knowledge on the structure of all interactions. From this point of view, the final part of this talk is devoted to a discussion on the radiative weak boson production $pp \to W^\pm\gamma$ at RHIC polarized experiment. We point out two reasons why we reanalyze this process. (1) Due to the $V-A$ structure of the $W$ boson interaction, only the initial quarks which have definite helicities can participate in the process. Therefore, an experiment with the polarized beams will be more efficient to study this process $^{18}$ and an information on $\Delta q(x)$ is relevant. (2) Many works so far assumed rather high energy collisions $^{19}$. However, a realistic experiment with the polarized beams becomes available at RHIC whose center of mass energy is around $\sqrt{s} \sim 500$ GeV.

4.1 RAZ at the partonic level

Radiative weak boson ($W\gamma$) production in hadronic collisions has been the subject of much theoretical interests since this process contains the gauge boson trilinear coupling and develops the so-called radiation zero (RAZ) $^2$. The RAZ is a typical example which is sensitive to the structure of electroweak interaction.

The RAZ is a phenomena that the cross section (amplitude) for some process develops zero in the some point of the phase space. To understand this phenomena in the simplest way, although not rigorous, let us use the soft photon approximation. Since the soft photon factorizes as the eikonal factor, the amplitude which contains one photon can be written as, $M_\gamma \simeq e J \cdot \epsilon(k) M$ where $J^\mu = \sum_i Q_i \eta_i \frac{e^\mu}{p_i^\mu}$ and $\epsilon^\mu(k)$ is the polarization vector of photon. $Q_i$
is the charge of \( i \)-the particle and \( \eta_i = (+) \) for the incoming (outgoing) particle. The sum is taken over all external particles. If \( \frac{Q_i}{p_i} = \text{const.} \) for all \( i \), the energy-momentum conservation \( \sum_i \eta_i p_i^\mu = 0 \) implies \( J^\mu = M_\gamma = 0 \). The well-known example is the process, \( u(p_1) + d(p_2) \rightarrow W^+(p_3) + \gamma(k) \). In this case, the identity \( \frac{4}{3} p_1 \cdot k = \frac{1}{3} p_2 \cdot k = \frac{1}{3} p_3 \cdot k \) is satisfied at \( \cos \theta_\gamma = -\frac{1}{3} \).

### 4.2 Hadronic cross section

To obtain a realistic (hadronic) cross section, we must convolute the partonic cross section with the PDFs \(^{20}\). Fig.1 shows the cross section in \( Pb \) for the various proton’s spin configurations at \( \sqrt{s} = 500 \text{Gev} \). Denoting the helicity of initial protons by \( p(\pm) \), Figs.(1.a1),(1.a2),(1.b1) and (1.b2) correspond to \( p(-)p(+) \), \( p(+)p(-) \), \( p(+)p(+) \) and \( p(-)p(-) \) respectively. We have chosen the minimum cut-off energy for the photon to be 5 Gev.

![Figure 1: Distribution of \( \cos \theta_\gamma \) in the laboratory frame for the reaction \( pp \rightarrow W^+\gamma + X \).](image)

One can see a rather clear dip in the cross section when the initial proton’s helicities are parallel each other. This is because the helicity distributions of quarks depend on the spin of the parent protons. Note that if the proton contains the equal parton densities of both helicity states as for the unpolarized case, the convolution completely smears out the RAZ. How about is the energy dependence of this smearing effect coming from the convolution? It is easily supposed that the dip will be more smeared as the energy becomes higher. It is because the small \( x \) partons start to participate in the process at higher energy and those sea partons carry less information of the parent proton’s spin. Namely the contribution from the small \( x \) partons is almost the same for the polarized and unpolarized protons.

We also define an asymmetry by,

\[
A = \frac{d\sigma (p(-)p(+) \rightarrow W^+\gamma) - d\sigma (p(+)p(-) \rightarrow W^+\gamma)}{d\sigma (p(-)p(+) \rightarrow W^+\gamma) + d\sigma (p(+)p(-) \rightarrow W^+\gamma)},
\]
and plot it in Fig. 2. This asymmetry amounts to 40%.

A comment is in order concerning the higher order corrections. Since the RAZ occurs only at the partonic tree level, the higher order (QCD) corrections might give an important effect and it is possible that RAZ is completely smeared out in the physical hadronic cross sections. There has been much effort to estimate higher order corrections in the standard model and phenomenological analyses. The QCD radiative corrections can be classified into three effects: (1) virtual corrections and soft gluon emissions (2) hard gluon emissions (3) gluon initiated processes $q_1 g \rightarrow V \gamma q_2$ and $g q_2 \rightarrow V \gamma q_1$. Among these effects, the first one leaves the RAZ intact. Its main effect is an almost constant $K$ factor. The second and third effects, on the other hand, will completely wash out the RAZ phenomena. These effects, however, have been known to be important at high energies. It is, therefore, expected that in the RHIC energy region, it is sufficient to include only the first effect which will not change the shape of the cross section from the tree level one except for some multiplicative enhancement. In particular, the asymmetry, Fig. 2, remains the same.

5 Conclusions

We have surveyed the polarized structure functions from a viewpoint of gauge invariant nonlinear operators. We explained an approach to obtain the QCD evolution which preserves maximal (BRST and Lorentz) symmetries of the theory at every step of investigation. We also presented the radiative weak boson production at RHIC which strongly depends on the helicity distribution of quarks inside the proton. We have pointed out that the experiments in the RHIC energy region will be very efficient to study this process.

We hope that various kinds of new experiments and theoretical investigations will be able to clarify not only perturbative and nonperturbative aspects of QCD but also the full structure of all interactions in Nature.

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